 FDMP, vol.6, no.4, pp.385-398, 2010

Inclination Impact on the Mass Transfer Process Resulting from the Interaction of Twin Tandem Jets with a Crossflow

A. Radhouane[1], N. Mahjoub Said[1], H. Mhiri[1], G. Le Palec[2] and P. Bournot[2]

Abstract: "Twin jets in crossflow" is a complex configuration that raises an increasing interest due to its presence in various common applications such as chimney stacks, film cooling, VSTOL aircrafts, etc. . . In the present paper, the twin jets were arranged inline with an oncoming crossflow; they were also inclined which resulted in similar elliptic cross sections of the nozzles' exits. The exploration of the flows in interaction was carried out numerically by means of the finite volume method together with the second order turbulent closure model, namely the Reynolds stress Model (RSM), and a non uniform grid system particularly refined near the injection nozzles. Once validated, the model was upgraded by introducing a temperature gradient and a non reacting fume within the emitted jets in order to follow its dispersion stages. Such a procedure is likely to give an extensive understanding of the resulting flowfield's mixing, expanding and dispersion. In a primer step, we showed the qualitative similarity of the different species' behaviors due to the adopted non reactive property of the handled fume. Then, we evaluated the impact of the jets' initial inclination angle on the emitted fume as well as on the environing air dispersion over the domain. A close dependence of both progressions is observed. Straightening the jets proved to enhance significantly the vertical ascension of the fume, postponing the homogenization of the resulting flowfield composition.

Keywords: Twin jets, crossflow, inclination, mass transfer, pollutants' dispersion.

1 Introduction

"Jets in crossflow" is a common configuration that finds application in more than a field (academic, industrial, etc. . .). The earliest studies focused rather on single

[1] ENIM, Route de Ouardanine, Monastir, Tunisie
[2] IUSTI, UMR CNRS 6595, Marseille, France.

jet configurations and on the impact of several parameters on the mixing process. The variation of the number of the handled jets, namely its augmentation, proved to be a viable solution for the enhancement of the applications' efficiencies. As a consequence, an increasing interest is dedicated to the multiple jets' configurations. The "twin jets in crossflow" is an intermediate configuration. Its exploration is determinant in the way it highlights the contribution of duplicating the handled jets and predicts the behavior of the multiple jets' configuration that is simply an extension of this intermediate one.

When we dispose of twin jets, we can arrange them differently relatively to the oncoming crossflow: they can be inline with the mainstream, side by side or opposed. In applications like film cooling of turbine blades, Vertical and short Take Off and Landing (VSTOL) aircraft engines, injection within combustion chambers of chimney stacks, the first arrangement, namely the tandem (inline) arrangement, is widely adopted. This is actually the one we propose to consider in the present paper.

Too little work was exclusively devoted to the inline jets in crossflow. Ohanian and Rahai (2000) are pioneers in the domain as they first considered this arrangement in 2000. Their configuration consisted in twin planar, inline and turbulent jets under different jet velocity ratios (0.5, 1, and 2) and jets' spacing (d, 2d, and 3d), where d is the jet diameter. The jets' coupling disappeared under the highest jets' spacing. When the jets are brought closer, increasing the exhaust momentum of the downstream jet above the free stream and the upstream jet velocities enhances the throw distance in the cross flow beyond the wall boundary layer thickness. It's only later that the second jet tilts in the direction of the freestream. Bringing the jets closer also enhances the turbulent kinetic energy which in turn promotes the mixing and diffusion processes.

Further consideration was recently dedicated to "double jets in crossflow" by Radhouane et al. (2009-a,b). In the first work, the authors [Radhouane et al. (2009-a)] focused on the dynamics of the resulting flowfield by tracking the evolution of the different velocity components as well as the fluctuating velocities. In the second paper [Radhouane et al. (2009-b)], interest was rather oriented toward heat and mass transfer between the different interacting flows. The temperature distribution was evaluated along the different orientations and over different levels in the domain. A quite similar interest was devoted to the generated mass transfer by tracking the evolution of one of the pollutants' mass fraction. Both works were conducted numerically by means of the finite volume method together with the Reynolds Stress Model (RSM) closure model.

Further authors dealt with the tandem twin jets in crossflow in the context of comparative studies with further arrangements of the same number of jets, namely side by side and opposite [Kolar et al. (2006), Kolar and Savory (2007)].

Some further authors chose to compare the twin jets' configuration to the single and/or the multiple one whether in tandem or different arrangements. The most recent work in this context was carried out by Ibrahim and Gutmark (2006). It consists in PIV experiments aiming at investigating the blowing ratio's effect on both arrangements' behavior. The blowing ratios used for the single jet tests were 3.2, 4.8 and 8; and that of the twin jets was 3. The study concerned the jet trajectories and penetration, the jet trajectory deflection, the mass entrainment approximation based on the jet trajectory, the windward and leeward jet spread, the size, location and magnitude of the reverse flow region and the turbulent kinetic energy (TKE). This examination showed mainly the resemblance of the trend of a double tandem jet to that of a single jet of a higher blowing ratio.

Thus, we clearly see that exclusive studies on inline double jets are scarce. The most examined parameters in this configuration are the injection ratio and the jets' spacing while further ones may be explored like the jet nozzles' height, the geometry of the jets' cross section, the initial jets' streamwise inclination, etc... In the present paper, we propose to consider the injection inclination in order to extend the work of Radhouane et al. [Radhouane et al. (2009-a,b)]. Actually, we intend to give a deeper evaluation of this parameter on the dispersion of the different species contained within the handled fume and compare it to the behavior of the environing air. Detailing this aspect of the problem is likely to lighten several joint issues. We can mention as an example the incessantly threatening atmospheric pollution problem or the enhancement and control of the mixing processes met in the combustion and further chemical chambers.

In order to find out plausible explanations and /or optimizing solutions to these issues, numerical simulations of double inline jets in crossflow were conducted under a variable initial inclination angle (30, 45, 60 and 90°). The different geometric assumptions and boundary conditions were deduced from an experimental replica that was extensively described in [Radhouane et al. (2009-a,b)]. Once validated, the model was upgraded by introducing a temperature gradient between the interacting flows. Finally, a non-reactive fume was introduced within the jet nozzles. Each of its components were tracked under the different tested inclinations and compared to the environing flow dispersion.

2 Computational set up

Fig. 1 represents the experimentally handled configuration that was reproduced numerically. The jets are placed three diameters apart from one another according to the oncoming crossflow direction. The center of the upstream jet coincides with the origin of the Cartesian coordinate system that was adopted in order to scale the interacting flows. The choice of the Cartesian coordinate system is motivated by the asymmetry of the jets' distribution within the domain in spite of the symmetry of the configuration (Smith (1998)). Finally, the resulting flow is supposed to be steady, three-dimensional, incompressible and turbulent.

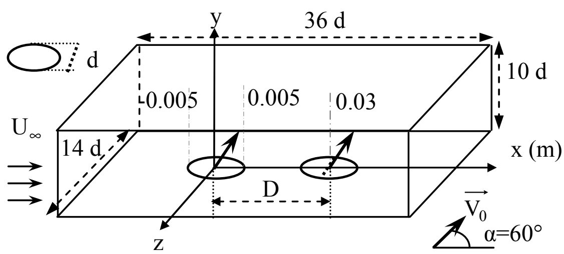

Figure 1: Scheme of the numerical configuration [Radhouane et al. (2009-a,b)]

The handled equations are written in the present case as follows:

$$\frac{\partial\left(\overline{\rho}\tilde{u}_i\right)}{\partial x_i}=0 \tag{1}$$

$$\frac{\partial\left(\overline{\rho}\tilde{u}_i\tilde{u}_j\right)}{\partial x_j}=-\frac{\partial\overline{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\mu\frac{\partial\tilde{u}_i}{\partial x_j}-\overline{\rho u_i''u_j''}\right)+\left(\overline{\rho}_\infty-\overline{\rho}\right)g\delta_{ij} \tag{2}$$

$$\frac{\partial\left(\overline{\rho}\tilde{u}_j\tilde{T}\right)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{Pr}+\frac{\mu_t}{\sigma_t}\right)\frac{\partial\tilde{T}}{\partial x_j}\right] \tag{3}$$

$$\frac{\partial\left(\overline{\rho}\tilde{u}_j\tilde{f}\right)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{Sc}+\frac{\mu_t}{\sigma_f}\right)\frac{\partial\tilde{f}}{\partial x_j}\right] \tag{4}$$

The introduction of the fluctuating variables requires the use of a turbulence closure model. Herein, we adopted the Reynolds Stress Model (RSM), a second order turbulent model, for its ability to model three dimensional complex flows. The introduction of this turbulence model leads to the resolution of the following equation:

$$\underbrace{\frac{\partial}{\partial x_k}\left(\overline{\rho}\tilde{u}_k\overline{u_i''u_j''}\right)}_{C_{ij}}=\underbrace{\frac{\partial}{\partial x_k}\mu\frac{\partial}{\partial x_k}\left(\overline{u_i''u_j''}\right)}_{D_{ij}^L}-\underbrace{\overline{\rho}\left[\overline{u_i''u_k''}\frac{\partial\tilde{u}_j}{\partial x_k}+\overline{u_j''u_k''}\frac{\partial\tilde{u}_i}{\partial x_k}\right]}_{P_{ij}}+D_{ij}^T+G_{ij}+\phi_{ij}+\varepsilon_{ij} \tag{5}$$

C_{ij} being the convective term, and D^L_{ij}, P_{ij} , D^T_{ij} , G_{ij} , ϕ_{ij}, ε_{ij}, respectively, the molecular diffusion, the stress production, the turbulent diffusion, the buoyancy production, the pressure strain and the dissipation rate (Schieste (1993)).

The equations of the turbulent kinetic energy (k) and of the dissipation rate of the kinetic energy (ε) associated with the second-order model are defined as follows:

$$\frac{\partial\left(\overline{\rho}\tilde{u}_j k\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial \mathrm{k}}{\partial \mathrm{x}_j}\right] + \frac{1}{2}\left(P_{ii}+G_{ii}\right) - \overline{\rho}\varepsilon \tag{6}$$

$$\frac{\partial\left(\overline{\rho}\tilde{u}_j \varepsilon\right)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial \mathrm{x}_j}\right] + C_{\varepsilon 1}\frac{1}{2}\left(P_{ii}+C_{\varepsilon 3}G_{ii}\right)\frac{\varepsilon}{k} - C_{\varepsilon 2}\overline{\rho}\frac{\varepsilon^2}{k} \tag{7}$$

For more information concerning the constants introduced in the different equations see Mahjoub et al. (2003) as jets undergo the same conditions in both configurations. The comparison of the experimental and computational results has already been conducted in the work of Radhouane et al. (2009-b). It assumed an injection ratio equivalent to R = 1.29, a jet nozzles' spacing of D=3d and an inclination angle of α=60°: these conditions correspond to the experimental ones. A good level of agreement was obtained.

To fit better the reality, two further conditions were adopted. First, we introduced a temperature gradient of 100° between the interacting flows. Then, we injected a non-reacting fume within the jet nozzles. The handled fume composition and the different adopted boundary conditions are summarized in Tab. 1.

Table 1: Boundary conditions

Injection Nozzles	$u=V_0\cos\alpha$, $v=V_0\sin\alpha$ T_0=403.15K	$k=10^{-3}V_0^2$ $\varepsilon = k^{3/2}/0.5d$
Crossflow	$u=V_0$, v=0 T_∞=303.15K	k= ε=0
Fume Composition	N_2:76.9%, CO_2:20.9%, O_2:1.8%, SO_2:0.4%	

3 Results and discussion

To give consistence to our work, we first need to evaluate the efficiency of the crossflow modeling. For that, we propose to track the evolution of the mainstream velocity in the upstream zone located between the domain entrance and the rear

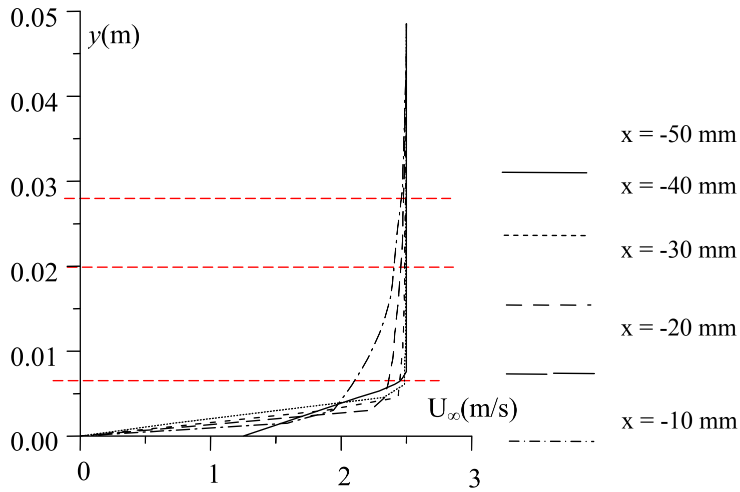

Figure 2: Vertical distribution of the mainstream velocity upstream of the rear jet nozzle under a given inclination (α=60°) on the symmetry plane (z=0)

edge of the first jet. Fig. 2 represents this evolution under an initial inclination angle of 60°, at different longitudinal positions on the symmetry plane (z = 0).

Within the three first longitudinal locations (x = -50, -40 and -10 mm), the boundary layer of the oncoming crossflow adopts a quasi-constant thickness as soon as we are placed 7 mm far from the injection plane. As we get closer to the upstream jet, the boundary layer thickness is sensibly raised and covers the first stages of the jets' dispersion within the domain. In such a way, the jets are "shielded" from the entire "strength" of the oncoming crossflow and undergo the unique influence of their proper initial inclination enabling them to evolve freely, at least during the primary instants. Once they leave the boundary layer, the jets accuse the full tilting and flattening of the uniform oncoming crossflow.

The velocity profiles at the jet nozzles' exit cross-section are also uniform and this uniformity is justified. In fact, in the reality the jets are fed from similar cylindrical pipes along which the velocity adopts a quasi-plane trend giving consistence to our uniform assumption.

Finally, we need to evaluate the impact of the initially imposed turbulent condition on the development of both jets as the dispersion of the jets is tightly related to their trajectories' progression. Fig. 3 illustrates the evolution of the potential cores (representation of the velocity magnitude) of both jets on the symmetry plane under different initial turbulence intensities. Increasing the initial turbulence of the oncoming crossflow from 0% to 5% does not bring any significant variation of both jet trajectories as the corresponding profiles match totally. Doubling the initial turbulence percentage (10%) brings a slight variation on the both jet trajectories; this variation is however weaker on the development of the downstream jet. In fact, the second jet is further shielded by the presence of the rear one. The mainstream turbulence is then weakened before reaching it. Under the highest initial turbulence condition, we observe the most significant deviation of the jets and the deviation is still more pronounced on the rear jet for the same reason.

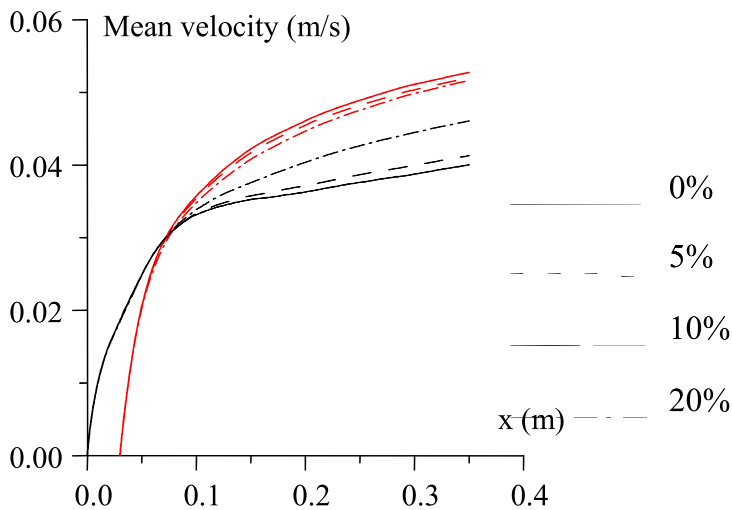

Figure 3: Impact of the initial Freestream turbulence percentage on the upstream (black line) and downstream (red line) jet trajectories on the symmetry plane (z=0)

Now that all the conditions (geometry, dynamic conditions and mass composition) are specified, we can move to the discussion of the obtained results, namely the resulting dispersion of the fume jets within the environing air mainstream. To get a complete idea about this mechanism, we need to track the progression of each of the pollutants contained within the handled fume. Due to the non-reactive character of the fume, we may suppose that all pollutants would behave similarly. Fig. 4 compares precisely the vertical distribution of the different pollutants' mass fraction on the symmetry plane (z = 0) and within two random longitudinal locations. Let's consider the rear nozzle center that will reflect the behavior of the corresponding jet. The second evaluation position was placed at mid distance between the twin jet columns due to the interesting phenomena it shields: the extension of the rear plume, the ascension of the second one and the creation of a trapped flow between the evolving fluid columns. We effectively note a similar behavior of the different considered species at both longitudinal locations. In fact, within the rear jet nozzle (fig. 4-a) for example, we assist to a decreasing profile starting from an initial value. This initial value is reasonably different as the pollutants are injected at different rates within the fume as shown in tab. 1.

Once emitted all pollutants' distributions record a similar stage expressing the passage through the just emitted rear jet. The variation of the initial inclination angle affects here similarly the pollutants' dispersion. In fact, it similarly enlarges the attained stage and postpones the diffusion and vanishing phase within the surrounding air. A similar decreasing slope is also adopted later during the vanishing phase.

The final vanishing begins indeed similarly at the vicinity of y = 8 mm under the weakest inclination angle. Generally speaking, the homogenization step takes place at the same location under the different inclination angles and for the different injected pollutants.

Between the jet nozzles (fig. 4-b), the same phenomenon is observed as the plotted distributions relative to the different species discharged are similar under a given inclination angle. The jets' inclination affects then similarly the progression of the different components of the non-reactive fume: a similar number of peaks and stages is recorded at the same locations and over the same extent.

The non-reactivity of the handled fume allows us then considering one single pollutant progression over the domain; the observations detected on the behavior of one single species will remain true for the others.

Thus, we propose in the following to consider one single pollutant as a tracer of the evolving fume in order to track its progression over the domain μ. Let us consider the carbon dioxide (CO_2) for example. As a first step, we propose to track the impact of the jets' initial inclination on the iso-surfaces of the CO_2 mass fraction contained within the fume. These iso-surfaces were represented in fig. 5 together with five different slices of the same feature (CO_2). The slices were equally spaced and placed at the same locations (between 0 and 0.15 m) under the different inclination cases in order to detect the plume extent discrepancies. A primary and global observation of the resulting flowfield reveals the high impact of the inclination factor on two major aspects: the separate ascension of each of the jets and then the progression of their combined plume.

The proper development of the jets is first affected by the jet nozzles' spacing. It is then either comforted or weakened by the initial emission inclination. In fact, a weaker inclination angle together with the direct flattening of the oncoming crossflow keeps the jet plumes close to the injection plate. The proper evolution of each of the jets (encircled in fig. 5) is then hardy discernible. Under the weakest incli-

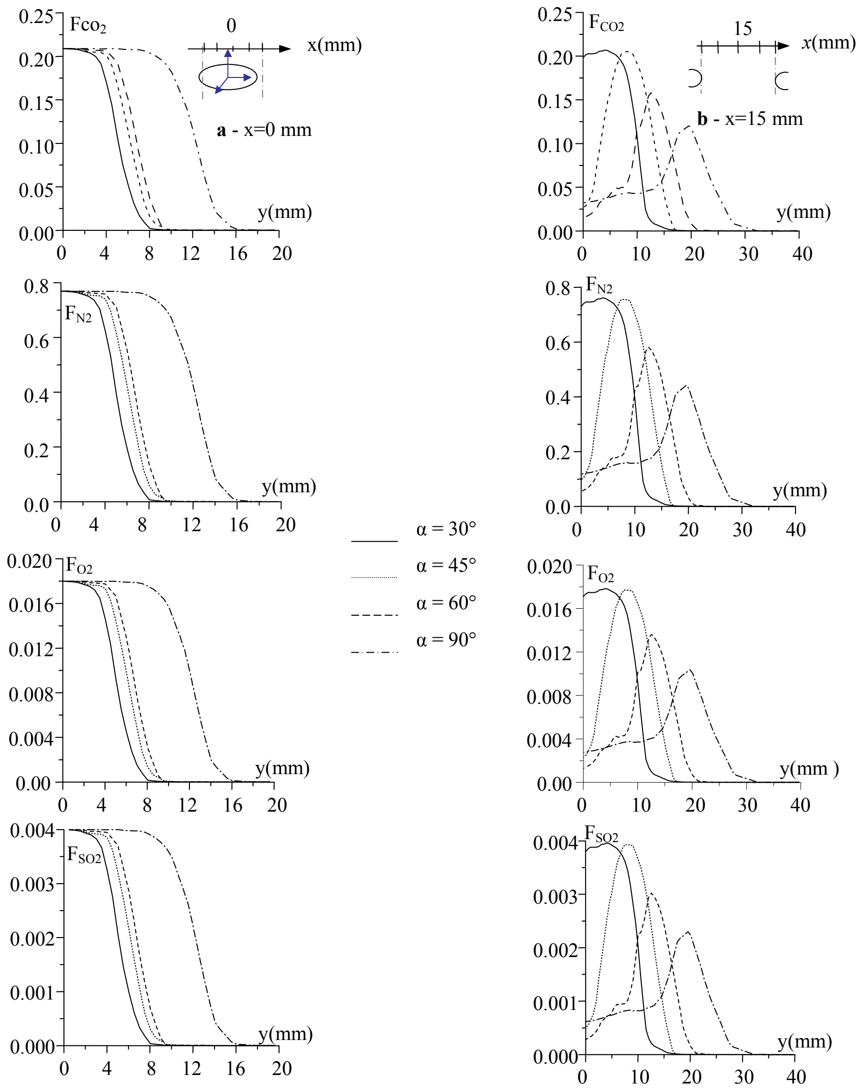

Figure 4: Similarity of the inclination impact on the different species distributions over the domain: a) within the 1^{st} jet, b) between both jet nozzles

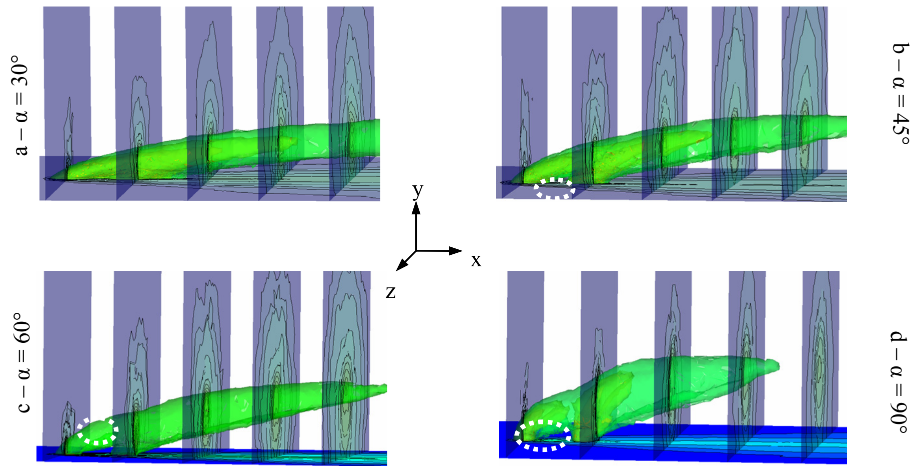

Figure 5: Impact of the initial jet inclination on the dispersion of the CO_2 mass fraction is-surfaces: a- $\alpha = 30^{\circ}$, b- $\alpha = 45^{\circ}$, c- $\alpha = 60^{\circ}$, d- $\alpha = 90^{\circ}$

nation case, for example, it seems as if we have one single jet (fig. 5-a). When the jets are straightened, on the contrary, the jets are more eager to move away from the injection wall before bending, even when directly confronted with the mainstream. The jet columns are consequently more clearly observed and join farther high under the highest inclination case (fig. 5-d).

The more pronounced progression of the most straightened jets originates actually from the more significant impulse brought to them while discharging. This more significant initial impulse gives consistence to the evolving jets which maintains longer their initial column width before being flatted and reduces their elongation. The vertical ascension is consequently largely enhanced with regards to the longitudinal one. This mechanism affects also the moment at which the separate jet columns combine to form one single jet plume with more significant dimensions. In fact, when the jets are maintained close to the injection plane, they almost immediately join and keep on progressing close to the wall. When they are straightened on the contrary, they join farther high and downstream.

The combined plume evolves however higher even if not too far longitudinally. This leads to suppose that the lateral expansion of the jet plumes is further enhanced too when the initial inclination increases. Increasing the inclination factor allows then a deeper vertical dispersion of the emitted fume and reduces its longitudinal extent.

We propose to superpose now the vertical progression of the carbon dioxide (CO_2) mass fraction versus that of the air mass fraction in a try to compare and maybe even correlate the behavior of both features (Fig. 6). The already chosen locations in addition to two further ones will be considered for this comparison. One will be placed at the downstream jet nozzle center to compare the twin jet behaviors and the second will be placed far downstream of both jet nozzles to track the progression of the combined plume. The same inclination angles will be considered in order to highlight the impact of this parameter on both mass fraction distributions. A primer global observation of the different plotted profiles reveals a complementary influence of the inclination impact on CO_2 and air mass fractions. Within the first zone that corresponds to the rear jet location (fig. 6-a), the different CO_2 profiles initiate with a common value equivalent to 21% which coincides with the quantity of CO_2 initially introduced within the jet nozzles. Since the latter contain initially only the fume, it is reasonable to find no pure air at the vicinity of the injection plane. It is only when the fume begins to disperse within the domain that the pollutant mass fraction profiles decline, and that the air mass fraction profiles increase regardless of the inclination conditions. The inclination factor affects then only the extent over which the profiles vary and not the way they do it. In fact, the vanishing of the CO_2 mass fraction actually yields the way to the air to settle down and that from y = 8 mm under the weakest inclination angle. This mechanism takes place at approximately y = 16 mm under the highest inclination angle.

This is actually the explanation for the similarity of the impact of the inclination factor on both distributions. In fact, the later the pollutant disperses, the later the pure air settles down. Within the downstream jet location (fig. 6-c), the pollutant's mass fraction behaves the same: a progressive dispersion is observed and expressed by a decreasing profile.

The decreasing portion of the profiles contains however a fluctuation as we are no longer in presence of a single plume but two: the just emitted second jet and slightly farther the extent of the already discharged rear one. This fluctuation on the pollutant distributions is simultaneously and reasonably reflected on the air profiles since the dispersion of the pollutant leaves space to be occupied by the environing air. The more the jets are straightened, the quicker the twin plums join; and they join so rapidly under the highest inclination angle that we practically no longer distinguish any fluctuation along the distribution of the pollutant's mass fraction. This phenomenon is naturally reflected on the air profile as we cross the combined jet plumes to directly discharge into the surrounding main air flow. Between the twin jet nozzles (fig. 6-b), the flow progression is quite more delicate to analyze due to the different initial values of the different profiles. This variation in the initial values is actually generated by the inclination factor. In fact and as already said, the more the jets are straightened, the farther they discharge from the injection plate and the less they let the pollutant deposit against the ground. Under the weakest

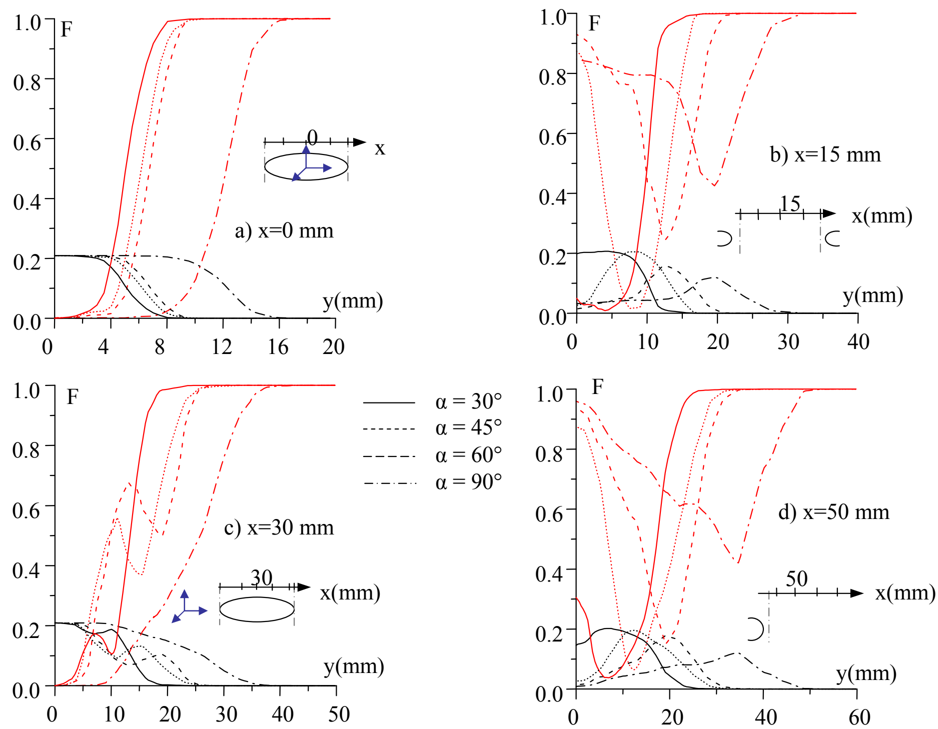

Figure 6: Vertical distributions of the CO_2 (black line) versus air (red line) mass fractions within the different zones of the domain (a- x = 0 mm, b-x = 15 mm, c-x = 30 mm, d-x = 50 mm) in the symmetry plane (z=0) and under a variable initial inclination angle

angle on the contrary the jets almost directly bend against the injection plate which justifies the highest corresponding initial pollutant quantity. This engenders automatically the least quality of air present in this zone: which is comforted by the corresponding air distribution (almost 2%).

The same phenomenon is observed downstream of the twin jet nozzles (fig. 6-d) as different initial values are adopted and the highest one is relative to the immediately bent jets (weakest inclination angle). After that, only one single peak is recorded. It is a maximum in the case of the pollutant profiles and a minimum in that of the air distributions. This peak is attained farther as the jet inclination increases due to their further vertical ascension which allows them to progress further longitudinally.

We can then confirm that the mass fraction distributions of both the pollutants and the surrounding air are complementary as stated previously. This complementarity is only qualitative for the moment since we considered only one single species; the CO_2. Including the others and summing their mass fractions would complete the quantitative corresponding since as soon as the pollutants disperse, the air settles down and replaces them all over the domain. The initial inclination of the jets affects all components' dispersion as the jets are sent further vertically. This is likely to maintain the initial quantity of pollutant longer when moving away from the injection plane and to postpone the final homogenization of the resulting flowfield composition in that direction. In terms of air mass fraction, this is expressed through a later installation of pure air along the vertical direction as well as along the other directions.

4 Conclusion

This work aimed at the examination of two elliptic tandem differently inclined jets within an oncoming cooler crossflow. This examination was conducted numerically by means of the finite volume method together with the RSM (Reynolds stress model) second order turbulent model and a non uniform mesh system. The numerical model was first validated by experimental data and then upgraded by introducing a non-reactive fume and a temperature gradient between the interacting flows. The non-reactivity of the emitted fume resulted in a similar dispersion of the different components of the handled fume regardless of the imposed inclination condition.

Increasing the initial inclination angle of the emitted jets proved to straighten the jet plumes allowing the pollutants to cross farther vertically the domain. However it limited their longitudinal extent. This is to consider according to the handled application and its principal aims. In terms of air mass fraction, this means a later settlement of pure air as the decline of pollutants concentration yields the way to the air to settle down.

References

Ibrahim, I.M., Gutmark., E.J. (2006): Dynamics of single and twin circular jets in crossflow. 44^{th} *AIAA Aerospace Sciences Meeting and Exhibit*, Reno, Nevada.

Kolar, V. et al. (2006): Vorticity and Circulation Aspects of Twin Jets in Cross-Flow for an Oblique Nozzle Arrangement, *Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering*, Vol. 220, No. 4, pp. 247-252.

Kolar, V., Savory E. (2007): Dominant flow features of twin jets and plumes in crossflow, *Journal of Wind Engineering and Industrial Aerodynamics*, Vol. 95, pp.1199–1215.

Mahjoub Said, N. et al. (2003): Three dimensional numerical calculations of a jet in external crossflow: application to pollutant dispersion, *J. of heat transfer*, Vol. 125, ASME.

Ohanian, T., Rahai, H.R. (2001): Numerical investigations of multi turbulent jets in a crossflow, *AIAA 2001-1049*.

Radhouane, A., et al. (2009): Numerical and experimental study of a double jet inclination variation on its dynamic evolution within a crossflow, *Heat and Mass Transfer Journal*, Vol. 45, No. 12, pp. 1597-1616.

Radhouane, A., et al. (2009): Impact of the initial streamwise inclination of a double jet emitted within a cool crossflow on its temperature field and pollutants dispersion, *Heat and Mass Transfer Journal*, Vol. 45, No. 6, pp. 805-823.

Schieste, R. (1993): Modélisation et simulation des écoulements turbulents, Herme‘s, Paris, 1993.

Smith S.H., Mungal M.G. (1998): Mixing, structure and scaling of the jet in crossflow. *J Fluid Mech*, Vol. 357, pp. 83–122.