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Natural Convection in an Inclined T-Shaped Cavity

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Abstract: This article presents a numerical study on natural convection in a bidimensional inclined "T"-shaped cavity. The governing equations are solved in the framework of a control-volume method resorting to the SIMPLEC algorithm (for the treatment of pressure-velocity coupling). Special emphasis is given to the investigation of the effect of inclination on the heat transfer and mass flow rate. Results are discussed for Prandtl number Pr=0.72, geometry with: opening width C=0.15, blocks gap D=0.5, blocks height, B=0.5 and different values of the Rayleigh number ($10^4 \leq Ra \leq 10^6$).

Keywords: Numerical study/ inclination / "T" form cavity/ isothermal blocks/ Natural convection

Nomenclature

B	dimensionless block height (h'/H')
C	dimensionless opening width (l'/H')
d'	space between adjacent blocks
D	dimensionless space between adjacent blocks (d'/H')
g	gravitational acceleration
h'	block height
H	dimensionless channel height
l'	opening diameter
L'	length of the calculation domain (cavity)
M	mass flow rate (eq. 5)
n	normal coordinate

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Nu	mean Nusselt number (eq.6)
Nu_h	mean Nusselt number along the horizontal planes of the blocks
Nu_v	mean Nusselt number along the vertical planes of the blocks
P'	pressure of fluid
P	dimensionless pressure
Pr	Prandtl number ($Pr = \nu/\alpha$)
Ra	Rayleigh number ($Ra = g\beta\Delta T H'^3/(\alpha\nu)$)
T'	temperature of fluid
T'_H	imposed temperature on the blocks
T'_C	temperature of the cold surface
T	dimensionless temperature of fluid $[= ((T' - T'_C)/(T'_H - T'_C)]$
U', V'	velocities in x' and y' directions
U, V	dimensionless velocities in x and y directions $[= (U', V') * H'/\alpha]$
x', y'	Cartesian coordinates
x, y	dimensionless Cartesian coordinates [=(x', y')/H']
α	thermal diffusivity volumetric coefficient of thermal expansion
λ	thermal conductivity of fluid
ν	cinematic viscosity of fluid
ρ	fluid density
Ψ	dimensionless stream function

Subscripts

C	cold
H	hot
max	maximum

1 Introduction

In general, the study of natural convection in cavities has a variety of possible engineering applications (Achoubir et al., 2008; Mechighel et al., 2009; Semma et al., 2010; Bouabdallah and Bessaih, 2010; Islam et al., 2008; Aouachria, 2009; Bennamoun and Belhamri, 2008; Ben-Arous and Busedra, 2008; Accary et al., 2008; El Alami et al., 2009; Djebali et al., 2009; Mezrhaband and Naji, 2009; Bucchignani, 2009; Meskini et al., to appear, etc.). In particular, the passive character of cooling by natural convection makes it very attractive for application in electronic devices. Along these lines, the search for methods of intensification of natural convection, especially for geometries typically used for electronic devices, has become over the years a topic of great interest.

Cooling of computer boards can be studied by idealizing them as forming horizontal or vertical channels. In practice, packaging constrains and electronic considerations, as well as specific device or system operating modes, are known to lead to a variety of heat dissipation profiles along the channel walls. Accordingly, many types of thermal wall conditions have been proposed in the literature to yield approximate conditions for predicting the thermal performance of such configurations [Bar-Cohen and Rohsenow,1984].

Generally, components are arranged on vertical [Kwak and Song,1998], tilted [Kholai *et al.* 2007] or horizontal channels [Najam *et al.* 2002] and submitted to natural ventilation (thermosiphon effect) to evacuate generated heat.

The case for which the electronic cards are horizontal, has been rarely studied with natural convection [El Alami at al. 2005]. The literature shows that in this case, the space between the components (often modeled by rectangular blocks) remains not well ventilated, even when cooling is provided by a forced flow. Indeed, the latter, being parallel to the planes containing components, often leads to a recirculating flow between them. The small space prevents forced flow from entering these zones. The result is an accumulation of heat between the components which may cause an important increase in surface temperature.

The evaluation of heat transfer increase induced by natural convection in an inclined channel is the objective of the present study.

To counteract the undesired effect discussed above, we have arranged coaxial and identical openings at the upper and lower walls of the cavity (to enhance heat exchange between the vertical sides of the blocks and fresh air and ensure a continued flow of air ambient in the system). The calculation domain is reduced here to a “T” form cavity because of the periodic geometry of the channel.

We investigate the effect of the cavity inclination on the flow structure and the heat transfer characteristics. The aim of this work is to examine the natural convection contribution to heat transfer and mass flow rate. The mass flow rate is considered as a fundamental unknown of the problem and not imposed at the inlet opening.

2 Physical problem and governing equations

The geometry of the problem herein investigated is depicted in fig. 1 (a). The system is made of an inclined parallel-plate channel with rectangular heated blocks placed on the lower plate. Openings are adjusted on the plates as shown in this figure. The studied domain is a "T" form cavity, fig. 1 (b). The blocks are heated with a constant temperature T'_H. The upper plate is cold at a temperature T'_C, while the other sides of the cavity are insulated.

The flow is considered steady, laminar, and incompressible and the Boussinesq

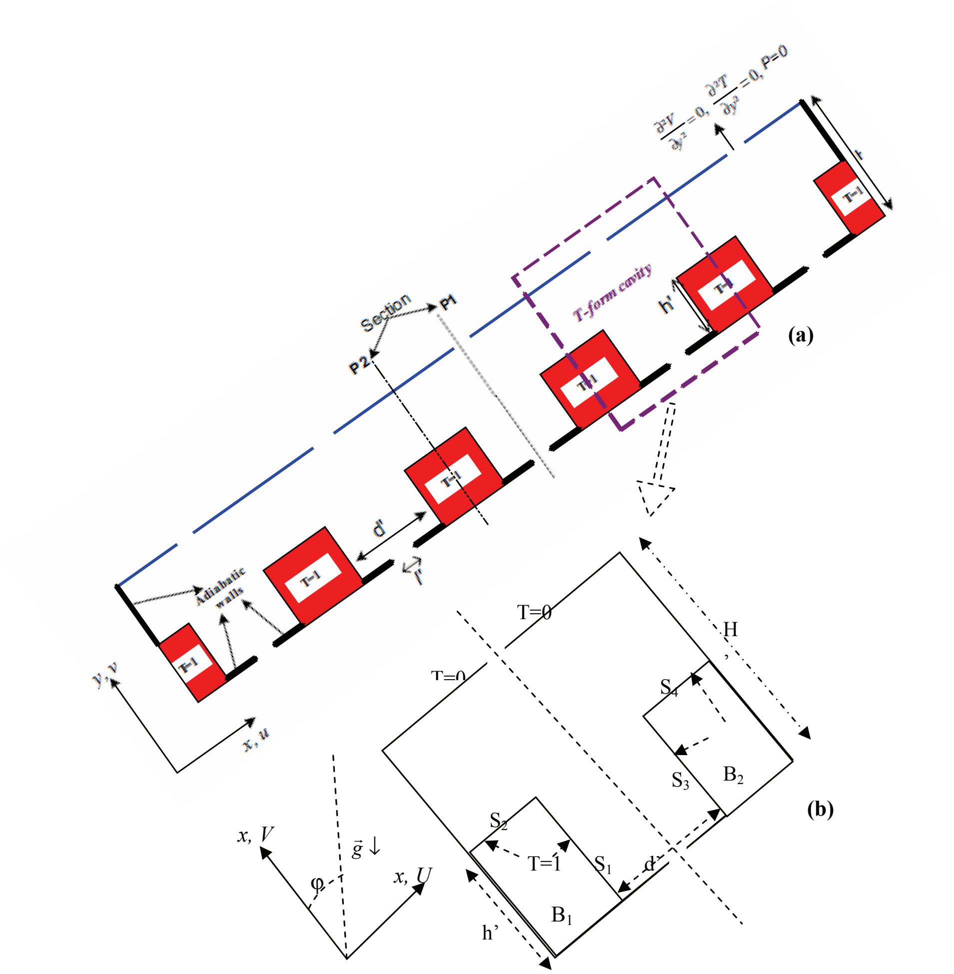


Figure 1: Studied configuration, (a) Inclined channel, (b) Calculation domain

approximation is applied. The dimensionless governing equations can be written as:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \tag{1}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+V\frac{\partial U}{\partial y}=-\frac{\partial P}{\partial x}+\Pr(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}) \tag{2}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial x}+V\frac{\partial V}{\partial y}=-\frac{\partial P}{\partial y}+\Pr(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2})+\Pr RaT \tag{3}$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial x}+V\frac{\partial T}{\partial y}=\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2} \tag{4}$$

From fig. 1, the dimensionless variables are:

$$x=\frac{x'}{H'},\quad y=\frac{y'}{H'},\quad U=\frac{U'H'}{\alpha},\quad V=\frac{V'H'}{\alpha},\quad T=\frac{T'-T'_C}{T'_H-T'_C},\quad P=\frac{(P'+\rho g y')H'^2}{\rho\alpha^2}$$

$Ra=\frac{g\beta\Delta T H'^3}{\alpha\nu}, Pr=\frac{\nu}{\alpha}$ with $(\Delta T=T'_H-T'_C)$

The imposed boundary conditions, in terms of temperature and velocity, are similar to those of the natural convection flow in a vertical channel [El Alami *et al.* , 2005; Yücel *et al.* , 1980].

T =1 on the block surfaces and T=0 on the upper wall of the cavity

$U=V$=0 on all the rigid walls

$T=U=\frac{\partial V}{\partial y}=0$, $P=-\frac{M^2}{2}$ at the inlet opening

$\frac{\partial T}{\partial x}=0$ for $0.5\leq y\leq 1$; x=0 and x=1

$\frac{\partial T}{\partial y}=0$ for y =0 ; $0.25\leq x\leq x_0$ and $x_1\leq x\leq x_2$

$P=0$ for y=1; $x_0\leq x\leq x_1$ (outlet opening)

Where $x_0=\frac{D+0.35}{2}$; $x_1=\frac{D+0.65}{2}$ and $x_2=D+0.25$;

At the outlet opening, $\frac{\partial^2 T}{\partial x^2}=0$ and $\frac{\partial^2 V}{\partial x^2}=0$.

M is the induced mass flow rate. It is calculated as:

$$M=\int_{x_0}^{x_1} V\Big|_{y=1}^{d} x \tag{5}$$

The mean Nusselt number over the active walls of the blocks is:

$$Nu=\underbrace{\int_0^{0.25}\frac{\partial T}{\partial y}\Big|_{y=0.5}^{d} x}_{(i)}+\underbrace{\int_0^{0.5}\frac{\partial T}{\partial x}\Big|_{x=0.25}^{d} y}_{(ii)}+\underbrace{\int_0^{0.5}\frac{\partial T}{\partial x}\Big|_{x=x_2}^{d} y}_{(j)}+\underbrace{\int_{x_2}^{1}\frac{\partial T}{\partial y}\Big|_{y=0.5}^{d} x}_{(jj)} \tag{6}$$

(i) and (jj) represent the mean Nusselt number along the active planes S_2 and S_4 of the blocks. The terms (ii) and (j) represent that along the surfaces S_1 and S_2, respectively.

3 Numerical method

The governing equations of the problem were solved, numerically, using a control volume method [Patankar, 1980]. The QUICK scheme [Leonard, 1979] was adopted for the discretization of all convective terms of the advective transport equations (eqs. (2-4)). The final discretized equations (1-4) were solved by using the SIMPLEC (SIMPLE consistent) algorithm [Van Doormaal and Raithby, 1984]. As a result of a grid independence study, a grid size of 81x81 was found to model accurately the flow fields described in the corresponding results. Time steps considered are ranging between 10^{-5} and 10^{-4}. The accuracy of the numerical model was verified by comparing results from the present study with those obtained by De Vahl Davis (1983) for natural convection in differential heated cavity, table 1, and then with the results obtained by Desrayaud and Fichera (2002) in a vertical channel with two ribs, symmetrically, placed on the channel walls, table 2. We note that good agreement was obtained in Ψ_{max} and Mterms. When a steady state is reached, all the energy furnished by the hot walls to the fluid must leave the cavity through the cold surface (with the opening). This energy balance was verified by less than 3% in all cases considered here.

Table 1: Comparison of our results and those of De Val Davis (1983)

	De Val Davis 1983	Present study	Maximum deviation
Ra=10^4	Ψ_{max}=5.098	Ψ_{max}=5.035	1.2%
Ra=10^6	Ψ_{max}=17.113	Ψ_{max}=17.152	0.2%

Table 2: Comparison of our results and those of Desrayaud and Fichera (2002)

Ra = 10^5 (A=5)	Desrayaud and Fichera, 2002	Present study	Maximum deviation
Ψ_{max}	151.51	152.85	0.9%
M	148.27	151.72	2.2%

4 Results and discussion

In this work, the heat transfer rate across the hot walls and the flow and temperature fields are examined for a cavity inclination range($0 \le \varphi \le \frac{\pi}{2}$), a Rayleigh number range $10^4 \le$Ra$\le 10^6$ and the following other parameters (B=0.5; C=0.15, Pr=0.72).

The peculiarity of this problem is the emergence of different solutions when varying these parameters.The flow structure is, essentially, composed by open lines, which represent the aspired air by vertical natural convection (thermal drawing), and closed cells which are due to the recirculating flow or to Rayleigh-Bénard convection.

4.1 Flow structure

Numerical results have been obtained for several values of the Rayleigh number and inclination angle. The fields are presented by the streamlines and isotherms in figs. 2, 3 and 4.

We can notice that convective cells appear just above the blocks or inside the jet of aspired air. In the first case, the cells existence is due to the inclined thermal gradient developed between the planes S_2 and S_4 of the blocks (called: *upper active walls UAW*) and the cold wall of the cavity. In the second case the cells appear inside the jet because of the recirculating flow. Hence, three kinds of the problem solution are obtained in the range of inclination angle $0 \leq \varphi \leq \frac{\pi}{2}$.

For $Ra = 5 \times 10^4$, a symmetric solution with cells inside the principal convective flow appears, fig 2.a, for φ=0. This solution called *Intra Cellular flow* (ICF) was found by El Alami *et al.* (2009). The corresponding isotherms show that the *UAW* of the blocks are not well ventilated and the major part of heat exchange exits through the cold wall of the cavity. When varying the inclination angle, the symmetry of the problem solution is destroyed. Hence, for φ =18°, an important convective cell appears out of the fresh air jet and up of the block in the left zone (B_1). Streamlines fill the space between blocks.

The related isotherms show that the walls S_1, S_2 and S_3 are well ventilated and we have good heat exchange through these planes. Contrarily, isotherms are widely spaced near S_4. This latter is, practically, not ventilated. When increasing φ, the convective cell above B_1 becomes stronger with an important size, fig.2c, for φ = 72°. A recirculating cell appears near the block B_2. It constrains fresh air to move away from the wall plane S_3. It is not in favor of B_2 planes ventilation. So, heat exchange is poor along this latter, as shown by the corresponding isotherms. Number of open streamlines is reduced because of the *thermal drawing* as well as the gravity effect are decreased when increasing the cavity inclination.

For extremely inclinating the cavity, $\varphi = \pi$vy, fig.2d, there is no “thermal drawing” and the cavity is filled with two recirculating cells. The fresh air jet disappears because the gravity effect is eliminated: in this situation, g is perpendicular to the axis passing in the middle of the openings. This case is analogue to that of closed cavity. Isotherms show that only S_1 is well ventilated.

Figures 3.a and 3.b present the flow structure and the isothermal field for $Ra = 10^5$. We note that the particularity, for this case, is the appearance of the so-called *chimney effect* for lower values of the cavity inclination, fig.3a, (φ =18°). A multi-cellular flow structure emerges for this value of φ. We note the existence of convective cell above B_1 and two recirculating cells inside the fresh air jet. All the block walls are well ventilated except S_4.

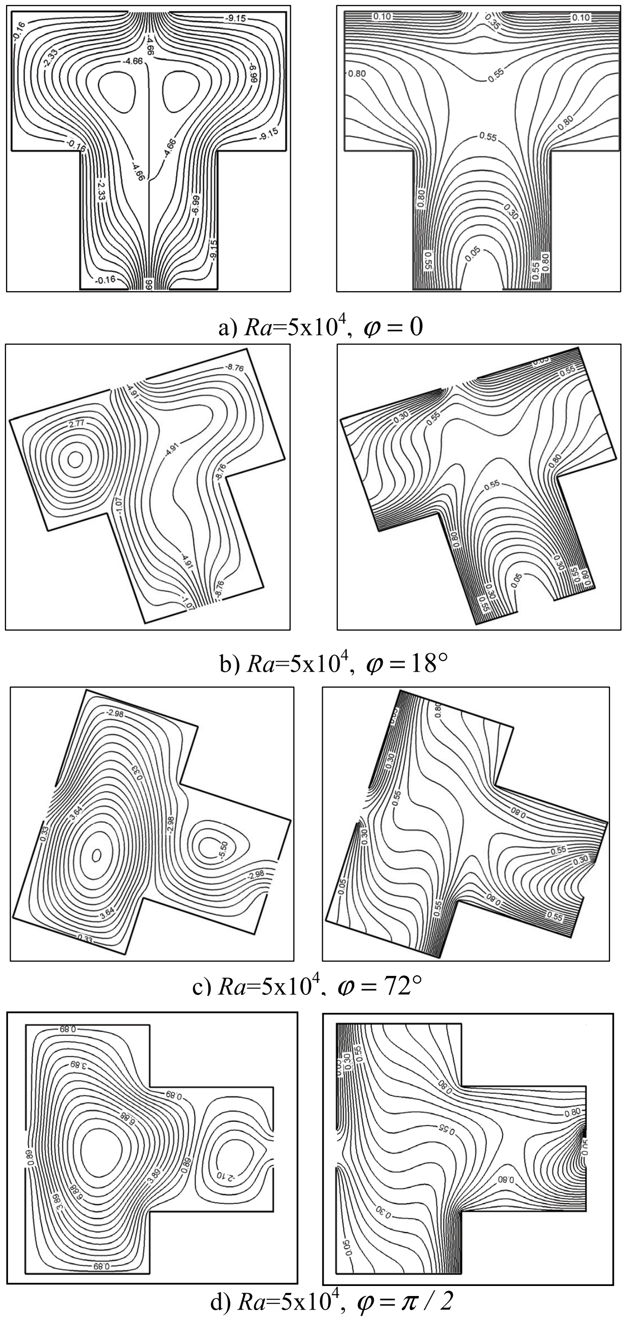
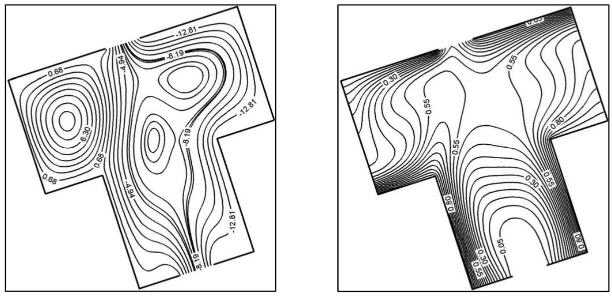
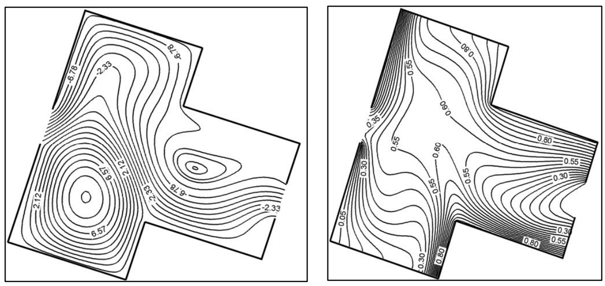


Figure 2: Streamlines and isotherms for $Ra = 5 \times 10^4$ and different values of φ



a) Ra =10^5, $\varphi = 18°$

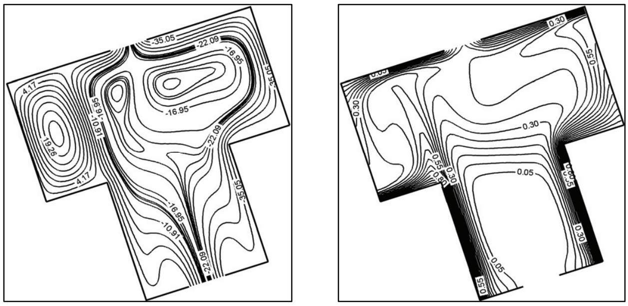


b) Ra =10^5, $\varphi = 72°$

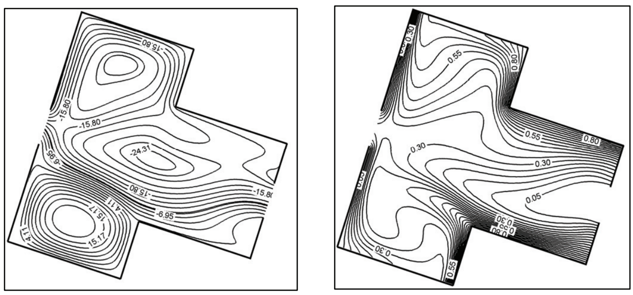
Figure 3: Streamlines and isotherms for $Ra = 10^5$ and different values of φ

Further increase of φ, leads to reduce of the open streamlines number which represent the aspired fresh air, fig.3b, for φ =72°. Corresponding isotherms are too tight near B_1 and they indicate that heat exchange is more developed along the B_1 surfaces than B_2.

Further increase of the Rayleigh number leads to a different flow structure. For $Ra = 10^6$, the aspired air jet is very strong. The *chimney effect* is more developed than in the last case ($Ra = 10^5$), fig.4a, φ =18°. The recirculating cells are very strong and constrain fresh air to go along all active walls. Isotherms in this figure show a good heat exchange through each B_1 and B_2 planes. Even for high values of φ, we note that the jet of aspired air is intense and all the active planes of the blocks are well ventilated, fig. 4b. We note that up of $Ra = 10^6$, the problem solution is not stable and oscillations of numerical values appear.



a) Ra=10^6, $\varphi = 18°$



b) Ra=10^6, $\varphi = 72°$

Figure 4: Streamlines and isotherms for $Ra = 10^6$ and different values of φ

4.2 Heat transfer and mass flow rate

The mean Nusselt number variation with Ra is presented in figure 5, for different values of the cavity inclination. Generally, Nu increases as a power law according to Ra for all selected values of φ. Nu increases according to the following formula:

$$Nu_{\varphi} = 0.54\sin(0.48\varphi + 1.78)Ra^{0.27}$$

The maximum deviation is less than 4%.

We note that heat exchange along the blocks surfaces decreases with the cavity inclination and reaches its minimal value for $\varphi = \pi$vy when the gravity is perpendicular to the opening axis. The Ra increase leads to the appearance of a boundary layer flow along S_1 and S_3 for low values of φ. So, there is an important temperature gradient near all these faces. Another outcome of the problem is the rate of induced mass flow, M. Its value is an important parameter in design and control of electronic equipments. In figure 6, we present the Mvariation with Ra for different values of φ.

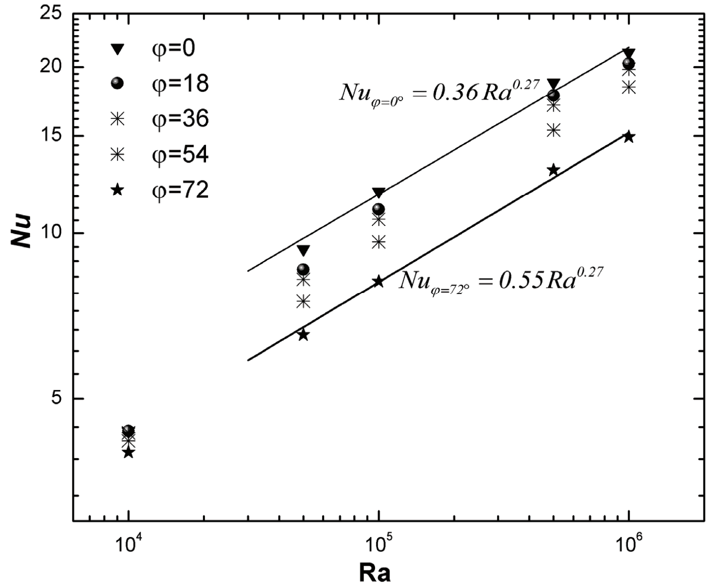


Figure 5: Variation of Nusselt according to the Rayleigh number for various values of φ

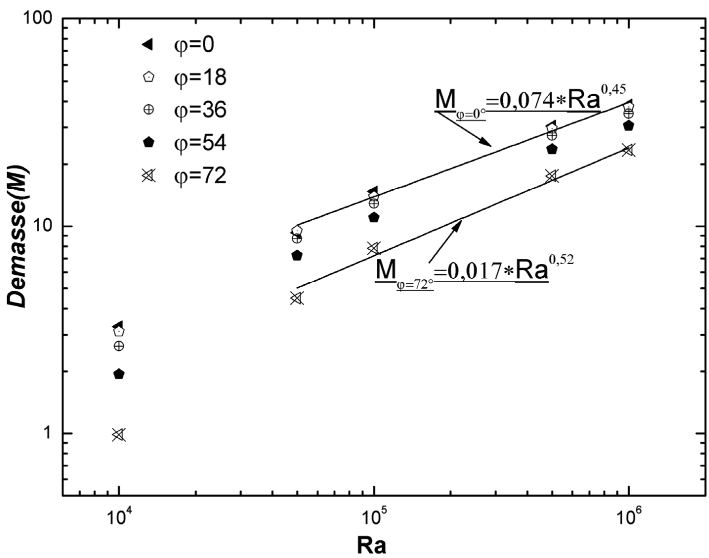


Figure 6: Variation of the mass flow rate according to the Rayleigh number for various values of φ

Contrarily to the Nu evolution, the mass flow rate is absent for maximum inclination of the cavity: M=0 for $\varphi = \pi$vy. Except for this value of φ, Mvariation with Ra is similar to that of Nu.

The extreme correlations of M with Ra are:

$M_{\varphi=72^\circ} = 0.017Ra^{0.52}$ for $\varphi = 72^\circ$ And $M_{\varphi=0} = 0.074Ra^{0.45}$ for $\varphi = 0$

5 Conclusion

A numerical investigation has been conducted to study the enhancement of heat transfer in a cavity with heated rectangular blocks. The results show the existence of different solutions, in the range of $0 \leq \varphi \leq \frac{\pi}{2}$, on which the resulting heat transfer and mass flow rate depend significantly. The mean Nusselt number variation with Ra has been correlated with a power law. Nu decreases when increasing φ. The mass flow rate variation with φ, is similar to that found for Nu, except for $\varphi = \pi/2$, M=0: for this value of φ, the cavity openings are perpendicular to gravity and so there is no induced mass flow rate. M Correlations show that it increases with Ra and inversely, decreases with φ.

In this paper we conclude that inclination of the cavity is not in favor of enhancing heat transfer. Natural convection aspiration of fresh air can be reduced by increasing φ.

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