

## Heat Exchange between Film Condensation and Porous Natural Convection across a Vertical Wall

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**Abstract:** Conjugate heat transfer across a vertical solid wall separating natural convection in a cold fluid-saturated porous medium and film condensation in a saturated-vapour medium is analyzed. The analysis reveals that this thermal interaction process is mainly controlled by the thermal resistance ratio of wall to porous-side natural convection and that of condensate film to natural convection. Asymptotic and numerical results of interest are obtained for the local and mean overall Nusselt number as functions of these two thermal resistance ratios.

### Nomenclature

$g$	gravitational acceleration
$l_c$	scale of convection layer thickness
$l_f$	scale of condensate film thickness
$H$	wall height
$Ja$	Jacob number, cf., Eq. (7)
$k$	thermal conductivity
$K$	permeability of porous medium
$Nu$	mean overall Nusselt number
$Nu_x$	local overall Nusselt number
$Pr$	Prandtl number, $= \nu / \alpha$
$Ra_c$	Rayleigh number of porous-side natural convection, cf., Eq. (22)
$Ra_f$	Rayleigh number of condensate film, cf., Eq... (6)
$T$	temperature
$T_{c\infty}$	free temperature of cold porous medium
$T_s$	saturation temperature
$u, v$	dimensional velocity components
$U, V$	dimensionless velocity components

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$x, y$	dimensional vertical and horizontal coordinates
$X, Y$	dimensionless vertical and horizontal coordinates
$w$	wall thickness

### Greek letters

$\Delta_f$	dimensionless thickness of condensate film
$\alpha$	thermal diffusivity
$\beta$	thermal expansion coefficient
$\Gamma$	thermal resistance ratio of condensate film to natural convection layer
$\eta$	inverse Oseen function
$\nu$	kinematic viscosity
$\theta$	dimensionless temperature
$\delta_f$	dimensional thickness of condensate film
$\varepsilon_w$	thermal resistance ratio of wall to natural convection

### Subscripts

$c$	cold fluid/convection
$f$	condensate film
$w$	wall
$wc$	wall side facing natural convection layer
$wf$	wall side facing condensate film

## 1 Introduction

Conjugate heat transfer between two fluid media separated by a solid wall is a phenomenon of common occurrence in many engineering applications, such as heat exchangers, thermal isolations, nuclear reactors, *etc.* Therefore, this phenomenon has received a considerable interest in the recent heat transfer research at the fundamental and applied level. However, the analysis of such a two-fluid conjugate heat transfer problem is considered more complicated than that of a one-fluid problem. The assumption of constant temperature or heat flux frequently applied at the solid boundary to solve the classical one-fluid problem is not appropriate for this type of two-fluid problems.

Many studies were reported on thermal interaction between two natural convection systems separated by a vertical wall (e.g., [Kimura (2003); Anderson and Bejan

(1980); Cheng and Minkowycz (1977); Bejan and Anderson (1983)]. Other studies were published on thermal conjugation between forced and natural convection separated by a vertical wall (e.g., [Shu and Pop (1999); Mosaad and Al-Hajeri (2006)]).

Regarding thermal coupling between forced convection and film condensation, only a few studies were published in the literature [Faghri and Sparrow (1985); Chen and Chang (1996); Méndez and Treviño (1996); Bautista et al. (2000); Luna and Méndez (2004)]. Faghri and Sparrow [Faghri and Sparrow (1985)] treated numerically the thermal coupling between forced convection inside a vertical tube and film condensation on the external surface. Later, Chen and Chang [Chen and Chang (1996)] used the local non-similarity method to solve the same problem for a vertical plane wall. Few other studies were reported on the thermal interaction between film condensation and natural convection separated by a vertical wall. Poulikakos [Poulikakos (1986)] treated analytically this problem applying the Nusselt assumptions for the condensate film. Char and Lin [Char and Lin (2001)] used the cubic Spline method to study the same problem however for two porous media.

Recently, the problem of thermal interaction between film condensation in a non porous medium and natural convection in a porous medium separated by a vertical wall was treated by Mosaad [Mosaad (1999)], who assumed the separating wall as an interface of zero thermal resistance. However, to achieve better modeling for the physical reality of this thermal conjugation phenomenon, the effect of wall conduction should to be considered in the modeling. Therefore, this problem is retreated here in another way which allows the modeling of the wall conduction effect in the conjugate solution. The analytic Oseen technique is employed for the natural convection layer and the Nusselt-Rohsenow model for the condensate film. The wall conduction is considered only significant in the crosswise direction. The main advantage of such an analytical approach is that the parametric dependence of interactive heat transfer mechanisms are more visible than in a numerical model. Results of interest are presented for the local and mean Nusselt numbers to highlight the wall conduction effects.

## 2 Analysis

The physical model under consideration is illustrated schematically in Fig. 1. An impermeable vertical wall of height  $H$ , thickness  $w$  and constant thermal conductivity  $k$  separates two fluid media at different temperatures. The hot fluid is a dry vapour at saturation temperature  $T_s$ , while the cold medium on the opposite wall side is a fluid-saturated porous medium at bulk temperature  $T_{\infty} < T_s$ . As a result of heat transfer from the hot to cold medium, a thin condensate film flowing down-

wards forms on the hot wall side and a natural convection layer flowing upwards is created on the opposite cold wall side. For clarity in presentation, the subscripts “c”, “f” and “w” are used to recognize variables and parameters belong to the convection, condensate film and wall, respectively.

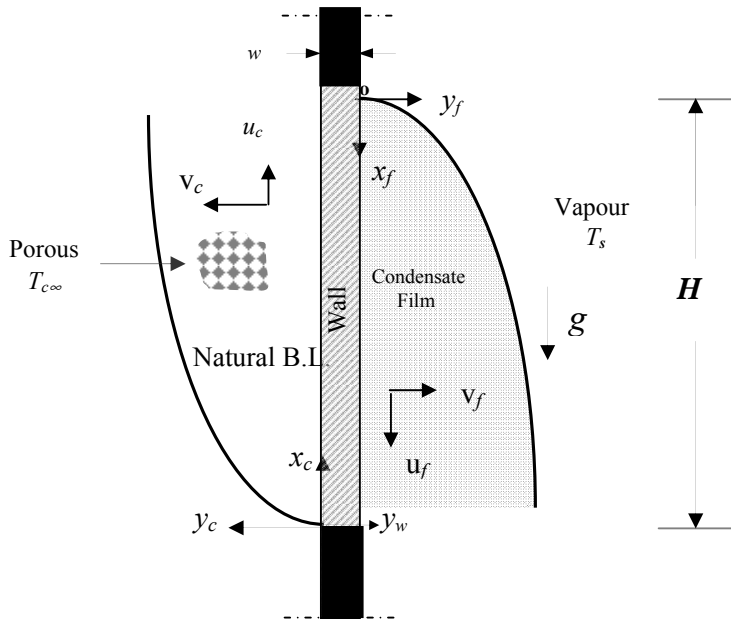


Figure 1: Model Illustration

### 2.1 Film condensation side

For steady, laminar condensate film of constant physical properties, the governing equations of mass, momentum and energy can be expressed, respectively, in the dimensionless forms:

$$\frac{\partial U_f}{\partial X_f} + \frac{\partial V_f}{\partial Y_f} = 0 \quad (1)$$

$$\frac{Ja}{Pr_f} \left( U_f \frac{\partial U_f}{\partial X_f} + V_f \frac{\partial U_f}{\partial Y_f} \right) = -1 + \frac{\partial^2 U_f}{\partial Y_f^2} \quad (2)$$

$$Ja \left( U_f \frac{\partial \theta_f}{\partial X_f} + V_f \frac{\partial \theta_f}{\partial Y_f} \right) = \frac{\partial^2 \theta_f}{\partial Y_f^2} \quad (3)$$

The dimensionless parameters and variables introduced above are defined,

$$Y_f = y_f/l_f, \quad X_f = x_f/H, \quad U_f = u_f/(Ja \alpha_f H/l_f^2), \quad V_f = v_f/(Ja \alpha_f/l_f), \quad (4)$$

$$\theta_f = (T_f - T_{c\infty})/(T_s - T_{c\infty}), \quad \Delta_f = \delta_f/l_f, \quad \theta_{wf} = (T_{wf} - T_{c\infty})/(T_s - T_{c\infty}).$$

where  $\theta_f$  is the dimensionless condensate film temperature,  $\theta_{wf}$  is the dimensionless local temperature of the hot wall side facing the condensate film, and  $\Delta_f$  is the dimensionless film thickness. The symbol  $l_f$  denotes to the film thickness scale defined by

$$l_f = HRa_f^{-1/4}. \quad (5)$$

$Ra_f$  is the film Rayleigh number defined as a function of wall height and total temperature drop as

$$Ra_f = gH^3(\rho_f - \rho_v)h_{fg}/(k_f\nu(T_s - T_{c\infty})) \quad (6)$$

The dimensional parameter  $Ja$  is Jacob number defined by

$$Ja = Cp_f(T_s - T_{c\infty})/h_{fg} \quad (7)$$

The similarity solution [Nield and Bejan (2006)] indicated that for the ordinary fluids of  $Pr \geq 1$ , encountered in most practical applications, neglecting the inertia term in the moment equation of condensate film has insignificant effect on the solution. In addition, for most water applications,  $Ja$  is in the range:  $10^{-4} < Ja < 10^{-2}$  [Méndez and Treviño (1996)]. For these ranges of  $Pr$  and  $Ja$ , the terms multiplied by  $Ja$  in Eqs. (2) & (3) can be neglected. Hence, these two equations reduce, respectively, to:

$$\frac{\partial^2 U_f}{\partial Y_f^2} - 1 = 0, \quad (8)$$

$$\frac{\partial^2 \theta_f}{\partial Y_f^2} = 0. \quad (9)$$

The appropriate boundary conditions are:

$$Y_f = 0; \quad U_f = V_f = 0 \quad \text{and} \quad \theta_f = \theta_{wf},$$

$$Y_f = \Delta_f; \quad \partial U_f / \partial Y_f = 0, \quad \text{and} \quad \theta_f = 1, \quad (10)$$

$$x_f = 0 \quad \Delta_f = 0 \quad \text{and} \quad \theta_f = 1.$$

Solving Eqs. (8)& (9) subject to boundary conditions (10) gives, respectively,

$$U_f = 0.5Y_f(2\Delta_f - Y_f) \quad (11)$$

$$\theta_f = \theta_{wf} + (1 - \theta_{wf}) Y_f / \Delta_f \quad (12)$$

Integrating continuity Eq. (1) across the condensate film yields for  $Y_f$  from 0 to  $\Delta_f$ ,

$$V_f|_{Y_f=\Delta_f} = \left( U_f \frac{d\Delta_f}{dx_f} \right)_{Y_f=\Delta_f} - \frac{d}{dx_f} \int_0^{\Delta_f} U_f dY_f \quad (13)$$

The continuity of heat flux at the liquid-vapour interface gives

$$\frac{\partial \theta_f}{\partial Y_f} \Big|_{Y_f = \Delta_f} = \left( U_f \frac{d\Delta_f}{dx_f} - V_f \right)_{Y_f = \Delta_f} \quad (14)$$

Inserting Eqs. (11)-(13) into Eq. (14) yields,

$$\frac{\partial \Delta_f^4}{\partial X_f} = 4(1 - \theta_{wf}) \quad (15)$$

## 2.2 Porous-side natural convection

The following assumptions are made for simplifying the analysis of this part: The flow is steady, laminar, incompressible and two-dimensional; the Boussinesq approximation and Darcy's law are applicable; the fluid-saturated porous medium is isotropic and homogeneous; and the fluid is in local thermal equilibrium with the porous matrix. Under these simplifications, the two-dimensional equations governing the conservation of mass, momentum and energy in the porous-side natural convection layer can be written in the dimensionless forms [Bejan and Anderson (1983)]:

$$\frac{\partial U_c}{\partial X_c} + \frac{\partial V_c}{\partial Y_c} = 0 \quad (16)$$

$$\frac{1}{Ra_c} \frac{\partial V_c}{\partial X_c} - \frac{\partial U_c}{\partial Y_c} = - \frac{\partial \theta_c}{\partial Y_c} \quad (17)$$

$$U_c \frac{\partial \theta_c}{\partial X_c} + V_c \frac{\partial \theta_c}{\partial Y_c} = \frac{1}{Ra_c} \frac{\partial^2 \theta_c}{\partial X_c^2} + \frac{\partial^2 \theta_c}{\partial Y_c^2} \quad (18)$$

Equations (16)-(18) are subject to the boundary conditions:

$$\begin{aligned} Y_c = 0; & \quad V_c = 0 \quad \text{and} \quad \theta_c = \theta_{wc}, \\ Y_c \rightarrow \infty; & \quad U_c = 0, \quad \text{and} \quad \theta_c = 0. \\ x_c = 0; & \quad \theta_c = 0. \end{aligned} \quad (19)$$

The dimensionless parameters introduced above are defined as:

$$\begin{aligned} Y_c &= y_c/l, X_c = x_c/H \quad U_c = u_c/(\alpha_c H/l^2), V_c = v_c/(\alpha_c/l), \\ \theta_c &= (T_c - T_{c\infty})/(T_s - T_{c\infty}), \theta_{wc} = (T_{wc} - T_{c\infty})/(T_s - T_{c\infty}). \end{aligned} \quad (20)$$

where  $\theta_c$  is the temperature of cold porous medium, and  $\theta_{wc}$  is the dimensionless local temperature of the wall side facing the porous medium. The symbol  $l_c$  is the thickness scale of convection layer, defined as

$$l = H/Ra_c^{1/2}. \quad (21)$$

wherein  $Ra_c$  is a modified Rayleigh number defined by

$$Ra_c = g\beta_c KL(T_s - T_{c\infty})/(v_c \alpha_c). \quad (22)$$

The requirement of boundary layer theory that the ratio  $l/H$  should be much less than unity, is according to relation (21) satisfied only for moderate and high  $Ra_c$ . For this  $Ra_c$  limit, the terms divided by  $Ra_c$  in Eqs. (17) & (18) can be neglected. Hence, combining the resultant reduced energy and momentum equations gives

$$\frac{\partial^2 U_c}{\partial Y_c^2} + [V_c] \frac{\partial U_c}{\partial Y_c} + \left[ \frac{\partial \theta_c}{\partial X_c} \right] U_c = 0 \quad (23)$$

The Oseen technique used in previous relevant studies (e.g., [Bejan and Anderson (1983); Mosaad and Al-Hajeri (2006)]) can be applied to solve analytically the above nonlinear differential equation. In this technique, the horizontal velocity component  $V_c$  and temperature gradient ( $\partial \theta_c / \partial X_c$ ) in Eq. (23) are assumed functions of  $Y_c$ -coordinate only. This assumption converts Eq. (23) to an ordinary differential equation, which can be solved analytically subject to boundary conditions (19) to yield:

$$U_c = \theta_{wc} e^{-Y_c/\eta} \quad (24)$$

$$\theta_c = \theta_{wc}(x_c) e^{-Y_c/\eta} \quad (25)$$

The above velocity and temperature solutions involve two unknown parameters, which are inverse Oseen function  $\eta$  and wall surface temperature  $\theta_{wc}$ . Investigating the two above profiles indicates that  $\eta$ -parameter plays the role of convection layer thickness. From the condition that the above velocity and temperature solutions should satisfy the integration of energy equation across the boundary layer, one gets the relation:

$$\frac{d}{dX_c} (\eta \theta_{wc}^2) = \frac{2\theta_{wc}}{\eta}. \quad (26)$$

### 2.3 Solid-fluid interface conditions

The heat conduction in the separating vertical wall of  $H \gg w$  is assumed significant only in the transverse direction. Thus, the continuity of heat flux and temperature at the solid-fluid interface on both sides can be expressed in the dimensionless equality:

$$\frac{\partial \theta_f}{\partial Y_f} \Big|_{Y_f=0} = -\Gamma \frac{\partial \theta_c}{\partial Y_c} \Big|_{Y_c=0} = \Gamma \frac{(\theta_{wf} - \theta_{wc})}{\varepsilon_w} \quad (27)$$

The dimensionless parameters appeared above are defined,

$$Y_w = y_w/w, \quad \varepsilon_w = \frac{w k_c}{H k_w} Ra_c^{1/2}, \quad \Gamma = \frac{k_c Ra_c^{1/2}}{k_f Ra_f^{1/4}}. \quad (28)$$

The wall parameter  $\varepsilon_w$  relates the thermal resistance of wall to porous-side convection layer, and conjugation parameter  $\Gamma$  represents the thermal resistance ratio of condensate film to natural convection layer. Substituting the temperature derivatives in Eq. (27) by using Eqs. (12)& (25), this gives after variables separation the two relations:

$$\theta_{wf} = \frac{\eta + \varepsilon_w}{\eta + \varepsilon_w + \Gamma \Delta_f}. \quad (29)$$

$$\theta_{wc} = \frac{\eta}{(\eta + \varepsilon_w + \Gamma \Delta_f)} \quad (30)$$

It is noted that relation (29) for  $\Delta_f=0$  gives  $\theta_{wf}=1$ , while relation (30) for  $\eta=0$  gives  $\theta_{wc}=0$ . The two above relations verify the initial conditions that  $\theta_{wf}=1$  as  $X_f=0$  and  $\theta_{wc}=0$  at  $X_c=0$ .

Now, inserting  $\theta_{cw}$  &  $\theta_{wf}$  from Eqs. (29) & (30) into Eqs. (15) & (26) with substituting  $X_f = 1 - X_c$  gives

$$\frac{d\Delta_f^3}{dX_c} = \frac{-3\Gamma}{(\eta + \varepsilon_w + \Gamma \Delta_f)} \quad (31)$$

$$\frac{d\eta^3}{dX_c} = \frac{6(\eta + \varepsilon_w + \Gamma \Delta_f)^3 \Delta_f^2 - 6\eta^3 \Gamma^2}{\Delta_f^2 (\eta + 3\varepsilon_w + 3\Gamma \Delta_f) (\eta + \varepsilon_w + \Gamma \Delta_f)}. \quad (32)$$

Relations (29)-(32) are considered the more important analysis results, which involve four unknown parameters:  $\Delta_f$ ,  $\eta$ ,  $\theta_{wc}$  and  $\theta_{wf}$ , all are functions of  $x$ -coordinate only.



However, it is also considered of practical and theoretical importance to calculate local and mean Nusselt numbers. The local Nusselt number of porous convection side  $Nu_x(h_x x/k_c)$ , defined based on the local heat flux at the convection wall side, is determined by

$$\frac{Nu_x}{Ra_{cx}^{1/2}} = \frac{\sqrt{X_c}}{\eta} \theta_{wc} \quad (33)$$

Similarly, the local Nusselt number of film condensation side is found by

$$\frac{Nu_x}{Ra_{fx}^{0.25}} = \frac{\sqrt[4]{X_f}}{\Delta_f} (1 - \theta_{wf}) \quad (34)$$

The overall Nusselt number averaged over the entire wall height and based on the average heat flux across the wall and total temperature difference  $(T_s - T_{c\infty})$ , is calculated by

$$\frac{Nu}{Ra_{fx}^{0.25}} = \int_0^1 \frac{\partial \theta_f}{\partial Y_f} \Big|_{Y_f=0} dX_f \quad (35)$$

### 3 Solution

#### 3.1 Asymptotic results

In this context, asymptotic results are deduced for the simple problem case of  $\varepsilon_w \rightarrow 0$ , wherein the separating wall is considered very thin with negligible thermal resistance. In this thin-wall case, the wall acts as a partition of zero resistance between the two media. Hence, the wall assumes a uniform temperature  $\theta_w$  function of  $x$ -coordinate only. For this thin-wall limit  $\varepsilon_w \rightarrow 0$ , Eqs. (27), (29) & (30) show that for  $\Gamma \rightarrow \infty$ ,  $\theta_{wh} = \theta_{wc} \rightarrow 0$  and the fluid temperature gradient at the wall side facing the porous medium goes to zero. This means that the convection layer will disappear, and the conjugate problem will reduce to the classical one of film condensation on an isothermal vertical surface. Solving the result (15) of film condensation analysis for  $\theta_{wh} = 0$  yields  $\Delta_f = \sqrt[4]{4X_f}$ . Substituting this outcome in Eq. (34) gives the local Nusselt number by

$$Nu_x/Ra_{fx}^{0.25} = 0.707 \quad (36)$$

and, consequently, the mean Nusselt number by

$$Nu/Ra_f^{0.25} = 0.943 \quad (37)$$

The above results are the same known Nusselt solution of film condensation on an isothermal vertical surface.

On the opposite limit  $\Gamma \rightarrow 0$ , the same Eqs. (27), (29) & (30) indicate also that fluid temperature gradient at the wall side facing the condensate film goes to zero, and  $\theta_{wh} = \theta_{wc} \rightarrow 1$ . This means that on this  $\Gamma \rightarrow 0$  limit, the thin wall will assume the high extreme value 1 of condensation side temperature, and the condensate film will disappear. Hence, the two-fluid problem collapses to the classical one-fluid problem of natural convection on an isothermal vertical surface embedded in a porous-fluid medium. Solving the result (26) of natural convection analysis for  $\theta_{wc} = 1$  yields  $\eta = 2\sqrt{X_c}$ . Substituting this outcome in Eq. (33) gives the local Nusselt number by

$$Nu_x/Ra_x^{1/2} = 0.5 \quad (38)$$

and, consequently, the mean Nusselt number by

$$Nu/Ra^{1/2} = 1 \quad (39)$$

The similarity solution [Nield and Bejan (2006)] of natural convection on an isothermal vertical surface embedded in a fluid-saturated porous medium yields the same result (39), however with a constant coefficient of 0.888 instead of 1, i.e., the error is less than 12%. This error may be attributed to the approximations of the Oseen technique applied to model this free convection part of the treated conjugate problem. The asymptotic results (36)-(39) confirm validity of the present model.

### 3.2 Numerical results

The two main differential equations (30-31) should be solved numerically to determine the distributions of  $\Delta_f$  and  $\eta$  along the wall as functions of  $\Gamma$  and  $\varepsilon_w$  parameters. The symbol  $\Delta_f$  is the dimensionless thickness of condensate film, and  $\eta$  represents the dimensionless thickness of natural convection layer. The fourth-order Runge-Kutta numerical technique was employed to integrate simultaneously these two equations. The integration starts at  $X_f = 0$ , where  $\Delta_f = 0$  while  $\eta$  is unknown maximum value (cf., Fig. 1). Therefore, a guess is made on this unknown  $\eta_{max}$ -value at the start point of solution. Then, the integration advances in small steps  $\Delta X_f$  until  $X_f = 1$ . Once the predicted  $\eta$ -value at  $X_f = 1$  is found different from zero, the procedure is repeated using a new adjusted value for  $\eta_{max}$  at  $X_f = 0$ , until, eventually, the predicted  $\eta$ -value at  $X_f = 1$  becomes very close to zero. Solution trials are stopped when predicted  $\eta$ -value at  $X_f = 1$  is found less than  $10^{-6}$ . In the preliminary tests of employed numerical technique, the asymptotic results (36)-(39) were used as a reference to adjust the accuracy of the numerical solution as well

as to insure its reliability. It was found that the solution for  $\Delta X_f = 0.001$  produces stable and accurate results.

At first, the numerical results of the thin-wall case of  $\varepsilon_w \rightarrow 0$  are discussed. In this case, the wall temperature  $\theta_w(x)$  varies only in the longitudinal direction. Figure 2 shows the distribution of  $\theta_w(x)$  as a function of  $\Gamma$ -parameter. The plotted results indicate that for a fixed  $\Gamma$ -value,  $\theta_w(x)$  varies almost linearly along the wall except at the two ends. It is also noted that  $\theta_w(x)$  approaches the high extreme temperature of the hot vapour side as  $\Gamma \rightarrow \infty$ , while approaches the low extreme temperature of the cold porous side as  $\Gamma \rightarrow 0$ . This behavior can be more clearly seen from the results displayed in Fig. 3, where  $\theta_w(x)$  at the wall mid-height is plotted versus  $\Gamma$ -parameter.

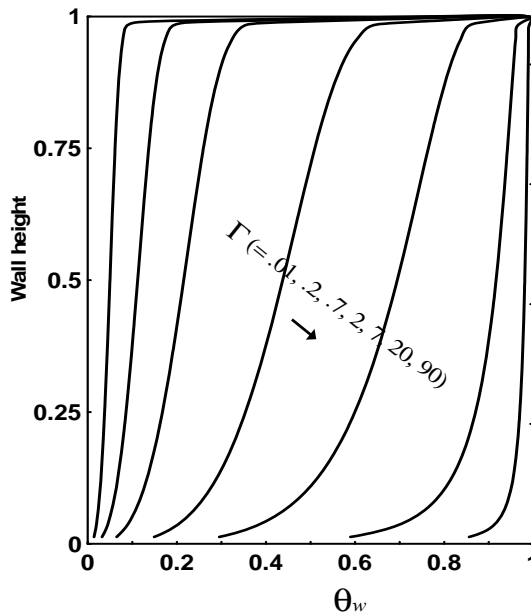


Figure 2: Wall temperature profile as a function of  $\Gamma$  - parameter; for  $\varepsilon_w \rightarrow 0$ .

The dependence of the local Nusselt number on  $\Gamma$ -parameter is shown in Fig. 4. It is clear that  $Nu_x$  approaches the asymptote of film condensation solution as  $\Gamma \rightarrow \infty$ , while approaches the asymptote of porous-side convection as  $\Gamma \rightarrow 0$ . Similar conclusion on the effect of  $\Gamma$ -parameter on the mean Nusselt number can be concluded from the results plotted in Fig. 5. For the purpose of a comparison with other study, data from ref. [Mosaad (1999)] are plotted in the same graph. The compared results indicate that the present model verifies the previous simplified model [Mosaad

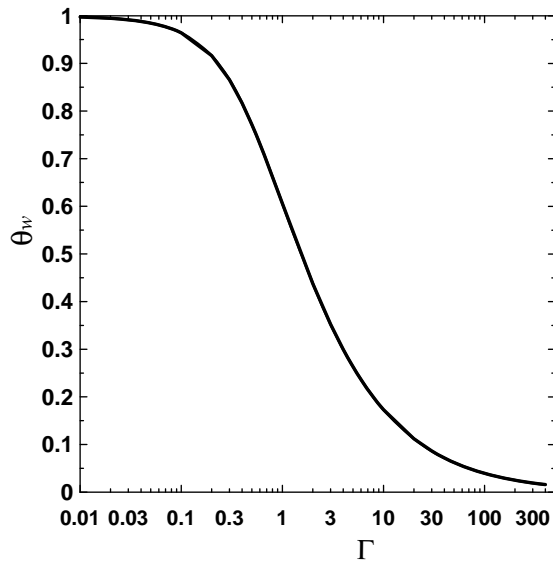


Figure 3: Wall temperature at mid height as a function of  $\Gamma$  - parameter; for  $\varepsilon_w \rightarrow 0$ .

(1999)] of negligible wall resistance case.

Results obtained for the more real and practical case of  $\varepsilon_w > 0$  are demonstrated in Figs. 6-7. The fluid temperature gradient at wall sides is plotted as a function of  $\varepsilon_w$ -parameter in Fig. 6. The plotted results indicate also that the fluid temperature gradient assumes a lower value on both sides for a higher value of  $\varepsilon_w$ . This means that the wall acts as a thermal insulator between the two thermal media and relaxes their interaction. In Fig. 7, the mean overall Nusselt number is plotted as a function of  $\varepsilon_w$  and  $\Gamma$  parameters. The displayed results indicate that mean Nusselt number increases with  $\Gamma$  and decreases with  $\varepsilon_w$ .

Here, it is importance to state that the present results are generally in agreement with those reported in the open literature; however, the accuracy of these results can be further evidenced by means of experiments.

#### 4 Conclusions

A theoretical model has been developed to predict the wall temperature profiles and Nusselt number for the conjugate heat transfer process between film condensation and natural convection in a porous medium, separated by a vertical conducting wall with finite thermal conductivity. The main points deduced from this study are:

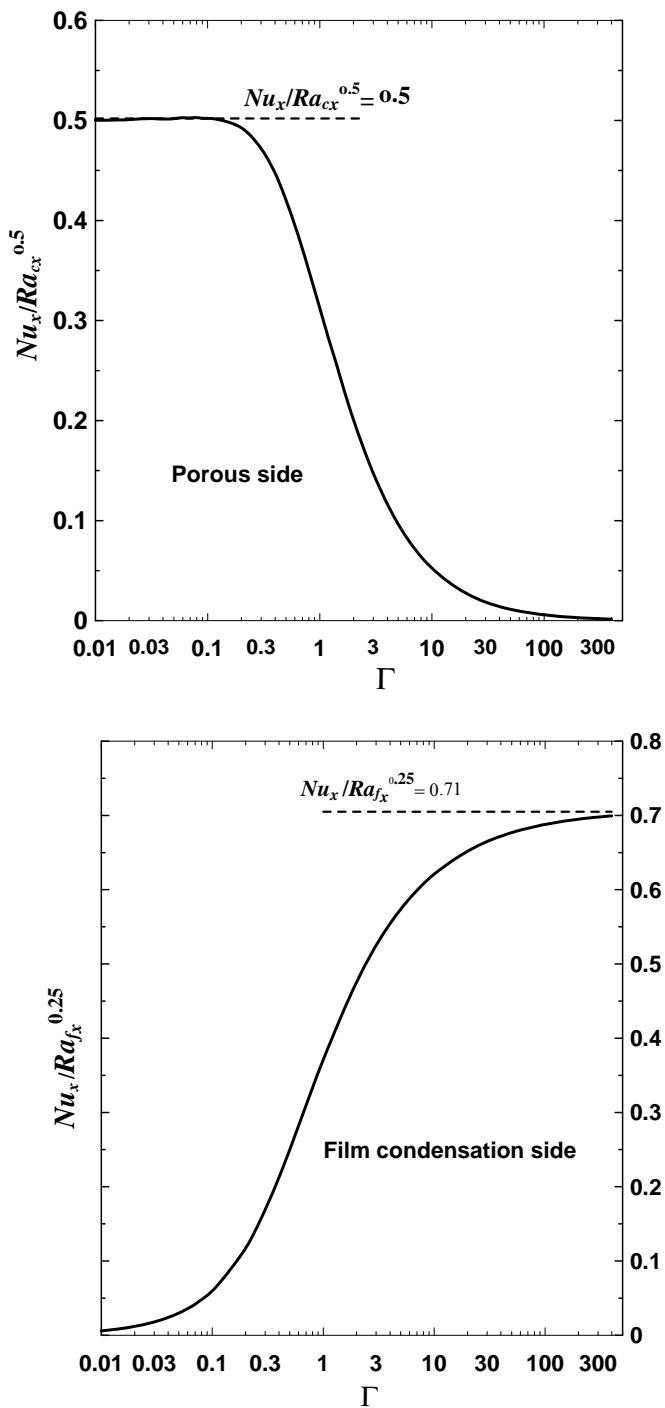


Figure 4: Local Nusselt number on wall sides as function of  $\Gamma$  - parameter; for  $\varepsilon_w \rightarrow 0$ .

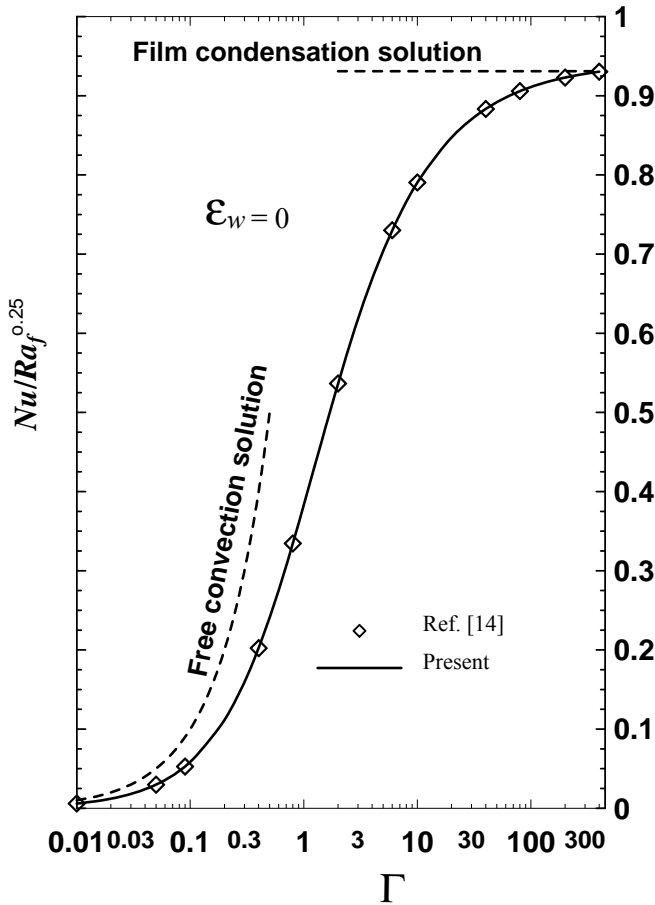


Figure 5: Mean Nusselt number as a function of  $\Gamma$  - parameter; for  $\varepsilon_w \rightarrow 0$ .

- The heat transfer effectiveness of the film condensation relative to the natural convection is higher for a higher conjugation parameter  $\Gamma$ .
- Increasing the wall thermal parameter  $\varepsilon_w$  reduces the heat transfer performance of both heat transfer modes.
- Mean Nusselt numbers increase with  $\Gamma$ -parameter and decrease with  $\varepsilon_w$ -parameter.

For the thin walls of negligible thermal resistance, the solution reduces to that of natural convection on an isothermal vertical surface embedded in a porous medium when  $\Gamma \rightarrow 0$ , while reduces to that of laminar film condensation on isothermal

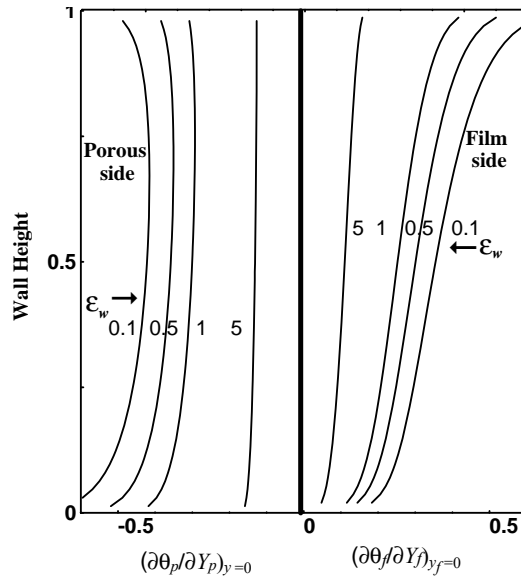


Figure 6: Fluid temperature gradient at both wall sides as a function of  $\epsilon_w$ . for  $\Gamma = 1$ .

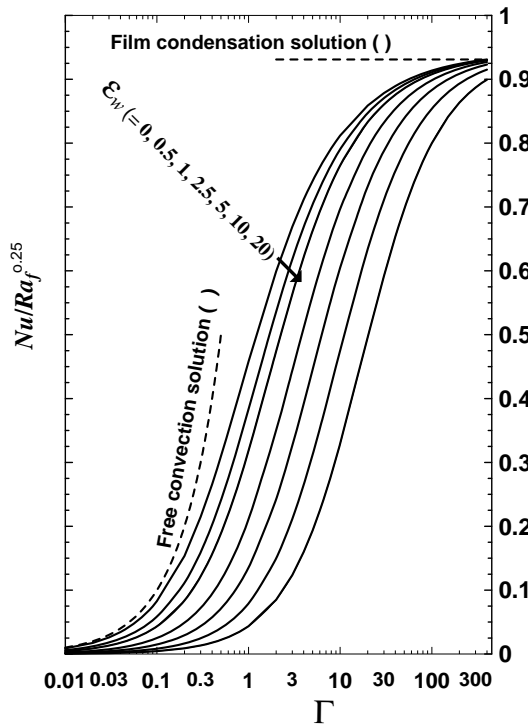


Figure 7: Dependence of mean overall Nusselt number on  $\epsilon_w$  and  $\Gamma$  parameters.

vertical surfaces as  $\Gamma \rightarrow \infty$ . These two extreme solutions confirm validity of the proposed model.

The model verifies previous model [Mosaad (1999)] developed for the simplified problem case of the thin walls of negligible thermal resistance.

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