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On the KP Equation with Hysteresis

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Abstract: The Kadomtsev-Petviashvili (KP) equation describes the evolution of nonlinear, long waves of small amplitude with slow dependence on the transverse coordinate. The KP equation coupled with the generalized play operator is studied in this paper in order to explain the dilatonic behavior of the soliton interaction and the generation of huge waves in shallow waters. Hirota bilinear method and results from a nonlinear semigroup theory are applied to simulate the resonant soliton interactions.

Keywords: KP equation, chaotic hysteresis, Hirota bilinear method, dilaton, nonlinear semigroup theory.

1 Introduction

It is well known that the hysteresis, bifurcations and chaos arise naturally in many areas of science and engineering. Numerous techniques have been presented in the literature to analyze and control nonlinear chaotic dynamical systems by using state feedback controllers, sometimes based on classical control theoretic design methodologies [Chen and Yu (2003); Chen, Hill andYu (2003)].

A system exhibits chaotic hysteresis if it simultaneously exhibits chaos and hysteresis. Since hysteresis causes loss of energy in a system, it seems to have a damping effect against chaotic oscillations in the system. Furthermore, hysteresis plays more complicated role in the system behavior by increasing its nonlinearity. Since the hysteresis involves the persistence of a state after the causal is removed, it involves multiple equilibria for given sets of control conditions. Such systems generally exhibit sudden jumps from one equilibrium state to another. If chaos appears either prior to or just after such jumps, or is persistent throughout each of the various equilibrium states, then the system is said to exhibit chaotic hysteresis.

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The main idea of exploiting the chaos in control applications was originally introduced by Ott, Grebogi and Yorke (1990). Stoten and di Bernardo (1997ab, 2006) have developed a general theory in order to take into account some properties of chaos, namely boundedness and topological transitivity. An interesting procedure which employs the close-return method to identify and stabilize the unstable period orbits is due to Pereira-Pinto, Ferreira and Savi (2004) based on the Ott-Grebogi-Yorke method.

Chaotic hysteresis is a phenomenon in which the chaotic response of a physical system to an external influence depends not only on the present magnitude of the influence but also on the previous history of the system. Hysteresis operators are characterized by two main properties - memory effect and rate independence. Krasnoselskiı and Pokrovskiı (1989) have studied the concept of hysteresis operator, acting in spaces of time dependent functions. Further researches were developed in a series of pornographies dedicated to the hysteresis in connection with PDEs and applicative problems [Brokate and Sprekels (1996); Krećí (1997); Visintin (1995)]. A useful survey can be found in Visintin (2002, 2006). Nonlinear semigroup theory in a Hilbert space was developed by Kōmura (1967) and extended to Banach spaces by Crandal and Liggett (1971) and Barbu (1976). Nonlinear semigroup theory represents a widely used tool for solving nonlinear PDEs. Surveys of basic relevant results from a nonlinear semigroup theory are formulated in a Banach space by Kopfová (2007) and Visintin (1993).

Several models of mechanical hysteresis may be represented via analogical models, namely the rheological models in mechanics, circuital models in electromagnetism, by arranging elementary components in series and/or in parallel [Bertotti and Mayergoyz (2006); Mayergoyz (2003); Bertotti (1998)]. These models consist of a family of elements, which can be interpreted as representing the mesoscopic structure of a composite material. Therefore, the procedure known as homogenization may be applied to provide an averaged representation of the system [Visintin (2008)].

In this paper, the generalized play operator is analyzed in connection with KP equation. The main purpose of this paper is the analysis the chaotic hysteresis in the KP equation. The mechanism of generation of huge waves in shallow water such as Tsunami, may be explaines by means of the resonance interaction of solitons. Recently, the coupling between the hysteresis operators and some evolution equations have been properly applied to solving practical engineering problems [Mosnegutu and Chiroiu (2010); Preda, Ionescu, Chiroiu and Sireteanu (2010); Gliozzi, Munteanu, Sireteanu and Chiroiu (2010)] The KP equation describes the evolution of 2D shallow water waves of small amplitude with slow dependence on the transverse coordinate [Segur and Finkel (1985), Hammack *et al.* (1989, 1995)].

The development of the KP equation [Kadomtsev and Petviashvili (1970)] happened almost simultaneously with the inverse scattering transform [Ablowitz and Segur (1981); Novikov, Manakov, Pitaevski and Zakharov (1984)]. This method for the solution of the initial-value problem for nonlinear PDE was originally developed for equations in one spatial dimension, such as the KdV equation [Munteanu and Donescu (2004)].

The constructing of explicit N-fold Darboux transformations and their Vandermonde-like determinants' representations of the two known soliton equations based on their Lax pairs for KP equation has been addressed by Huanga and Zhang (2008), whilst Dai and Geng (2000); Tiong, Ong and Mukheta (2006) have applied successfully the Hirota bilinear method to simulate the resonant soliton interactions.

The 1-soliton solution of the KP equation with power law nonlinearity using the solitary wave ansatz is obtained by Biswas and Ranasinghe (2009). They have computed an exact soliton solution and a couple of conserved quantities. In addition, the topological 1-soliton solution and the identification of the parameter domain for these solitons to exist are reported by Biswas and Ranasinghe (2010).

In objective of this paper is to analyse the KP equation with hysteresis in order to explain the dilatonic behavior of the soliton interactions and the generation of huge waves in shallow waters. Results from a nonlinear semigroup theory are applied to obtain the existence and uniqueness for KP equation with hysteresis.

2 Hysteresis operators

In this section, some well known results on the standard results of the nonlinear semigroup theory and the hysteresis operators are revised.

Definition 1 [Kopfová (2007)]: Let B be a Banach space, A a nonlinear and multi-valued hysteresis operator $A : D(A) \subset B \to B$ is accretive if

$$\forall u_i \in D(A), \quad \forall v_i \in A(u_i), \quad i = 1,2,$$

$$||u_1 - u_2||_B \le ||u_1 - u_2 + \lambda(v_1 - v_2)||_B, \quad \forall\lambda > 0. \tag{2.1}$$

Definition 2: Let B be a Banach space, the hysteresis operator A is called m-accretive if $Rg(I + \lambda A) = B$, $\forall\lambda > 0$.

Suppose that the derivative in the evolution equation can be approximated by a backward-difference quotient of step size $h > 0$ and f by a step functions f_k^h. We have

$$f_k^h \in \frac{u_k^h - u_{k-1}^h}{h} + A(u_k^h), \quad k = 1,2,..., \quad u_0^h = u_0, \tag{2.2}$$

$$u_k^h(t) = u_k^h \text{ for } kh \leq t < (k+1)h. \tag{2.3}$$

The scheme (2,2) is uniquely solved recursively and the Crandall-Liggett theorem holds:

Theorem 1[Crandall and Liggett (1971)]: If A is m-accretive, $f \in L^1(0,T,B)$ and $u_0 \in \bar{D}(A)$and$f^h \to f$ in $L^1(0,T,B)$, then $u^h \to u$ uniformly as $h \to 0$ and $u \in C(0,T,B)$.

Theorem 2: If A is m-accretive, $f \in L^1(0,T,B)$ and $u_0 \in \bar{D}(A)$, then the Cauchy problem

$$f \in u_{,t} + A(u(t)), \quad u(0) = u_0. \tag{2.4}$$

has one and only one integral solution u. For $f = 0$, we have $u = S(t)u_0$, where $S(t)$ is the nonlinear semigroup of contractions generated by A. If f has bounded variation in $[0,T]$ and $u_0 \in D(A)$, then the integral solution is Lipschitz continuous.

Definition 3: The function u is an integral solution of Eq.(2.4) in the sense of Benilan if (i) $u : [0.T] \to B$ is continuous; (ii) $u(t) \in \bar{D}(A)$ for any $t \in [0,T]$; and (iii)$u(0) = u_0$ and

$$||u(t_2 - v)||_B^2 \leq ||u(t_1 - v)||_B^2 + \\ + 2\int_{t_1}^{t_2} \lim_{\lambda \to 0} \frac{||u(\tau - v) + \lambda(f(\tau) - z))||_B^2 - ||u(\tau - v)||_B^2}{2\lambda} \mathrm{d}\tau. \tag{2.5}$$

Let us consider that the state of the system is characterized by two scalar variables, the input function $u(t)$ and the output function $w(t)$, confined to a set $L \subset R^2$. $\forall t \in [0,T]$. The function $w(t)$ depends on the previous evolution of $u(t)$ (memory effect) and on the initial statew_0, such as

$$w(t) = A(u,w_0)(t), \quad \forall t \in [0,T], \quad (u(0),w_0) \in L, \quad A(u,w_0)(0) = w_0, \tag{2.6}$$

where $A(u,w_0)$ is a hysteresis operator defined in a Banach space of time-dependent functions for any fixed w_0. This operator is causal and memory rate-dependent. That means $\forall (u_1,w_0),(u_2,w_0)$ with $u_1 = u_2$ in $[0,T]$, then $A(u_1,w_0)(t) = A(u_2,w_0)(t)$, and respectively, the operator is not invariant to any increasing diffeomorphism $\varphi : [0,T] \to [0,T]$, i.e. $A(u \circ \varphi, w_0) \neq A(u,w_0) \circ \varphi$, $\forall t \in [0,T]$.

The generalized play operator $w := A(u,w_0) : R^+ \to R$ is defined in the sense of Visintin (Fig.2.1) as follows. Let $u(t)$ be any continuous, piecewise linear function onR^+, linear on $[t_{i-1},t_i]$, $i = 1,2,....$

We define $w(t) = A(u,w_0)(t)$ by

$$w(t) = \min\{\gamma_l(u(0)), \max\{\gamma_r(u(0)), w_0\}\} \text{ for } t = 0 \text{ and } w_0 \in R, \tag{2.7}$$

$w(t) = \min\{\gamma_l(u(t_i)), \max\{\gamma_r(u(t_i)), w(t_{i-1})\}\}$ for $t \in (t_{i-1}, t_i)$, $i = 1, 2, \ldots,$

where $\gamma_l, \gamma_r : R \to R$ are maximal monotone, possible multivalued functions with

$$\inf \gamma_r(u) \leq \sup \gamma_l(u), \quad \forall u \in R. \tag{2.8}$$

Note that $w(0) = w_0$ only if $\gamma_r(u(0)) \leq w_0 \leq \gamma_l(u(0))$. The classical play operator can be obtained from the general play operator by choosing

$$\gamma_l(u) = u + r, \ \ \gamma_r(u) = u - r, \tag{2.9}$$

with $r \geq 0$ a parameter, $u(t)$ a continuous input function on $[0, T]$ and $w_{r0} \in [-r, r]$ an initial state. Fig.2.2 presents the play operator with threshold r.

If γ_l, γ_r are continuous, Visintin (1995) has proved that for any continuous piecewise linear functions u_1, u_2 on R^+, any continuous function $f : \mathrm{R} \to \mathrm{R}$ and any constant $M > 0$, for which $|f|_M(h)$ is its local modulus of continuity, we have

$$\begin{aligned} &\max\nolimits_{[t_1,t_2]} |A(u_1, w_{10}) - A(u_2, w_{20})| \\ &\leq \max\left\{A(u_1, w_{10})(t_1) - A(u_2, w_{20})(t_2), m_M\left(\max\nolimits_{[t_1,t_2]} |u_1 - u_2|\right)\right\}, \end{aligned} \tag{2.10}$$

$$\forall [t_1, t_2] \subset [0, T], \quad T \in R^+,$$

where

$$M := \max\{|u_i(t)| : t \in [0, T], \ i = 1, 2\}.$$

Therefore $A(u, w_0)$ has a unique continuous extension

$$A(u, w_0) : C(\mathrm{R}^+) \times \mathrm{R} \to C(\mathrm{R}^+). \tag{2.11}$$

The inequality (2.10) holds also for this extended operator, which is then uniformly continuous on bounded sets. If γ_r and γ_l are Lipschitz-continuous, then the operator $A(u, w_0)$ is also Lipschitz continuous and transforms $(u, v) \in W^{1,p}(0, T) \times \mathrm{R}$ into the unique function $w \in W^{1,p}(0, T)$ for any$p > 0$.

The generalized play operator can be also equivalently defined as a solution in the Sobolev space $W^{1,1}(0, T)$, $w \in W^{1,1}(0, T)$ of a variational inclusion of the type

$$w_{,t} \in \phi(u, w) \text{ in } (0, T), \quad w(0) = w_0, \tag{2.12}$$

where

$$\phi(u, w) = \begin{cases} \{\infty\} & \text{if } w < \inf \gamma_r(u), \\ [0, +\infty] & \text{if } w \in \gamma_r(u) \backslash \gamma_l(u), \\ \{0\} & \text{if } \sup \gamma_r(u) < w < \inf \gamma_l(u), \\ [-\infty, 0] & \text{if } w \in \gamma_l(u) \backslash \gamma_r(u), \\ \{-\infty\} & \text{if } w > \sup \gamma_r(u), \\ [-\infty, +\infty] & \text{if } w \in \gamma_l(u) \cap \gamma_r(u). \end{cases} \tag{2.13}$$

The hysteresis relationship with the PDEs can be written as [Kopfová (2007)]

$$w(x,t) = [A(u(x,...),w_0(x))](t) \text{ in } Q = \Omega \times [0,T] \tag{2.14}$$

where Ω is a bounded subset of R^n. The generalized play operator discussed in this paper is dissipative, in the sense that $||(\lambda I - A)x|| \geq \lambda ||x||$ for $\forall \lambda > 0$, where I is the identity mapping. The norm in $W^{1,1}(0,T)$ is defined as

$$||f||_{k,p} = \left(\sum_{i=0}^{k} ||f^{(i)}||_p^p \right)^{1/p} = \left(\sum_{i=0}^{k} \int^{f^{(i)}} |^p \mathrm{d}t \right)^{1/p} .$$

We present here one example of PDE with hysteresis [Kopfová(2007)]

$$(u+w)_{,t} - \Delta u = f \text{ in } Q, \quad Q = \Omega \times [0,T], \tag{2.15}$$

related to a generalized play operator $w(t) = A(u,w_0)(t)$ defined by (2.7), is formally equivalent to [Visintin (1995)]

$$u_{,t} + \xi - \Delta u = f,\ w,\ \xi \in \phi(u,w) \text{ in } Q, \tag{2.16}$$

where ϕ is defined by (2.13) and comma represents the differentiation with respect to the specified variable. The Cauchy problem for Eqs.(2.16) is coupled with homogeneous Dirichlet boundary conditions as

$$F \in U_{,t} + A_1 U \text{ in } Q, \quad U(0) = U_0 \text{ in } \Omega, \tag{2.17}$$

where

$$U = (u,w)^T, \quad F = (f,0)^T, \quad A_1 U = (\xi - \Delta u, -\xi)^T, \quad \xi \in \phi(U) \cap R, \tag{2.18}$$

$$D(A_1) = \left\{ U = (u,w)^T; \inf \gamma_r(u) \leq w \leq \sup \gamma_l(u) \text{ a.e. on } \Omega,\ U \in L^1(\Omega, R^2) \right.$$
$$\left. u \in W_0^{1,1}(\Omega),\ -\Delta u \in L^1(\Omega) \right\}. \tag{2.19}$$

3 The KP equation with hysteresis

The equation KP is the 2D version of the Korteweg-de Vries (KdV) equation. The hysteresis version of KP equation is obtained by coupling the classical KP equation with the generalized play operator $w(t) = A(u,w_0)(t)$ defined by (2.7) as

$$((u+w)_{,t} + 6uu_{,x} + u_{,xxx})_{,x} - 3u_{,yy} = 0 \text{ in } Q = [-\infty, \infty] \times [0,T],$$

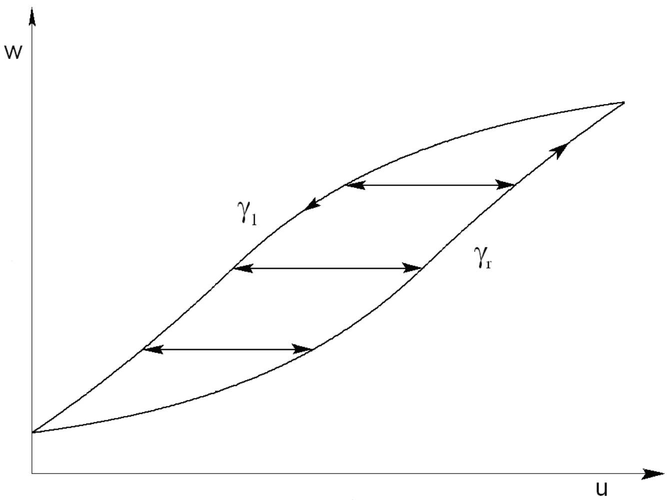

Figure 2.1: The generalized play operator.

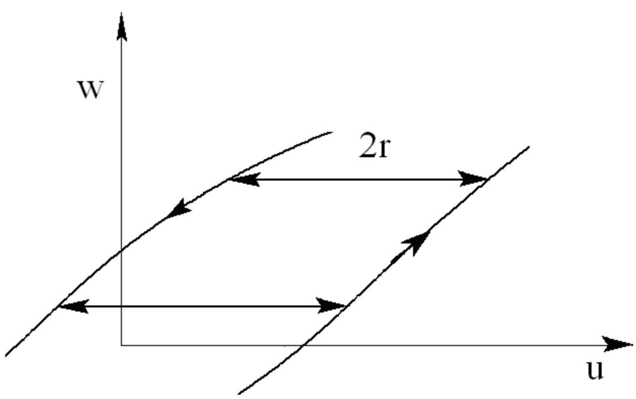

Figure 2.2: The play operator with threshold r.

$$w(x,t) = A(u(x,t), w_0), \quad w_{,t} \in \phi(u,w), \quad w(x,0) = w_0(x) \text{ in } (0,T), \tag{3.1}$$

where $u(x,y,t)$ is a scalar function (the displacement), x and y are respectively the longitudinal and transverse spatial coordinates, and $\phi(u,w)$ is defined by (2.13). The hysteresis relation (2.7) is also valid

To solve (3.1) let us consider the traveling wave solutions

$$u(x,y,t) = u(\eta), \quad w(x,y,t) = w(\eta), \quad \eta = kx + my + \omega t, \tag{3.2}$$

where k,m are wave numbers, ω is the frequency determined by dispersion relation $k\omega = 3m^2 - k^4$, and η is the phase variable. For $w = 0$, the one-soliton solution of KP equation is a traveling wave

$$u(x,y,t) = \frac{1}{2}a^2 \mathrm{sech}^2\left(\frac{1}{2}a\left(x - by - \frac{\omega t}{a} - x_0\right)\right), \tag{3.3}$$

where a,b,x_0 are arbitrary parameters and ω depends on a and b.

In 1971, Hirota showed that certain evolution equations can be reduced to bilinear differential equations. He introduces a dependent-variable transformation (Munteanu and Donescu (2004))

$$u(x,t) = -2\frac{\partial^2}{\partial x^2}\ln f(x,t), \tag{3.4}$$

where f has the property

$f, f_x, f_{xx} \to 0$, as $|x| \to \infty$.

For example, in the case of the KdV equation $u_t - 6uu_x + u_{xxx} = 0$,

substituting (3.4) into the KdV equation we obtain

$$ff_{xt} - f_x f_t + ff_{xxxx} - 4f_x f_{xxx} + 3f_{xx}^2 = 0.$$

This equation can be reduced to a bilinear form

$$D_t^m D_x^n : V \times V \to V,$$
$$D_t^m D_x^n(a,b)(x,t) = \left(\tfrac{\partial}{\partial t} - \tfrac{\partial}{\partial t'}\right)^m \left(\tfrac{\partial}{\partial x} - \tfrac{\partial}{\partial x'}\right)^n a(x,t)b(x',t')\Big|_{\substack{x=x' \\ t=t'}},$$

where m, n are positive integers, V is a functions space, in particular

$$V = \{f : \mathrm{R}\times\mathrm{R} \to \mathrm{R}, f \in \mathrm{C}^n(\mathrm{R}) \times \mathrm{C}^m(\mathrm{R})\},$$

and a, b two arbitrary functions in V.

We also apply the Hirota bilinear method (1971) to obtaining the n-soliton solutions of Eq.(3.1). The function f has the form

$$f = \delta_{ij} + \frac{a_i}{l_i + n_j} \exp(\eta_i), \tag{3.5}$$

$$\eta_i = k_i x + m_i y + \omega_i t, \quad k_i = l_i + n_i, \quad m_i = n_i^2 - l_i^2, \quad k_i \omega_i = 3m_i^2 - k_i^4, \quad i, j = 1, 2. \tag{3.6}$$

The function (3.5) can be written as

$$f = 1 + \varepsilon_1 \exp(\eta_1) + \varepsilon_2 \exp(\eta_2) + A_{12} \varepsilon_1 \varepsilon_2 \exp(\eta_1 + \eta_2), \tag{3.7}$$

where

$$A_{12} = \frac{(n_1 - n_2)(l_1 - l_2)}{(n_1 + n_2)(l_1 + l_2)}.$$

Eq. (3.1) is formally equivalent to

$$(u_{,t} + 6uu_{,x} + u_{,xxx} + \xi)_{,x} - 3u_{,yy} = 0, \quad w_, \quad \xi \in \phi(u, w) \text{ in } Q. \tag{3.8}$$

Substituting (3.2) into (3.8), we obtain the equations

$$k^4 u'''' + (k\omega - 3m^2) u'' + 6k^2 (uu')' + \omega \xi' = 0, \quad \omega w' - \xi = 0, \quad \xi \in \phi(u, w). \tag{3.9}$$

The solutions of Eqs.(3.9) can be written as

$$u(x, y, t) = 2 \frac{\partial^2}{\partial x^2} \ln f, \quad w = \sum_{i=0}^{M} p_i \phi^i, \tag{3.10}$$

with p_i are unknown constants. The function ϕ satisfies the Riccati equation $\phi' = \alpha \phi^2 + \beta$, with α, β constants. The function $f$of Eq.(3.9)$_1$ has the form

$$f = 1 + \varepsilon_1 \exp(\eta_1) + \varepsilon_2 \exp(\eta_2) + A_{12} \varepsilon_1 \varepsilon_2 \exp(\eta_1 + \eta_2) + \ldots\ldots A_{ij} \varepsilon_i \varepsilon_j \exp(\eta_i + \eta_j), \tag{3.11}$$

$i, j = 1, 2, 3, \ldots$

By equating the coefficients of like powers of ϕ^i, $i = 1,2,3,...$ to zero, a system of algebraic equations in $q_0, q_1, q_2, ...$ and k, m, ω is obtained. From this system of equations we have

$$p_0 = \frac{3m^2 - k(\omega + 8k^3\alpha\beta)}{6k^2}, \quad p_1 = 0, \quad p_2 = -2k^2\alpha^2. \tag{3.12}$$

There is an additional property of KP equation that is important in understanding the chaotic hysteresis. We refer to the solitonic resonance. The interaction between solitons exhibit both, the resonance and hysteresis. Interaction with resonance will occur when the values of A_{ij} are very close to zero $A_{ij} \approx 0$. Therefore, the values ofω_i, m_i, $i, j = 1,2,3,...$ will determine the resonance structure of the soliton interaction.

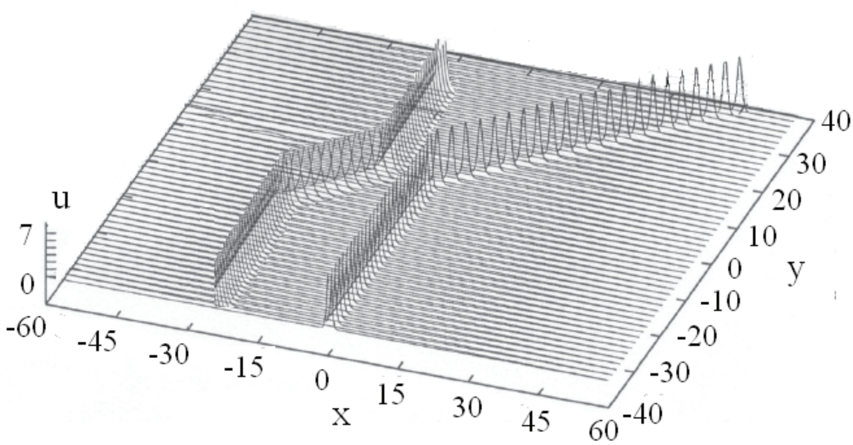

Figure 3.1: Scheme of interaction ($A_{12} \approx 0$ and $A_{23} \approx 0$).

If we fix the values of m_i, then ω_i will determine the value of A_{ij}. For example, the structure of interactions $A_{12} \approx 0$, $A_{23} \approx 0$, and also $A_{12} \approx 0$, $A_{13} \approx 0$, $A_{34} \approx 0$, are shown in Figs. 3.1 and 3.2 respectively, for waves traveling in the same direction at $t = 30$. Here, the amplitude of the solitons seems to be constant.

It is well known that the opposite of dissipation is amplification. The amplification of waves arises from an influx of energy in the medium, where the energy is pumping from a source or defect to wave motion, or due to interphase damage for fibre composite, or due to separations at the interface between the fiber and matrix [Smojver and Soric (2007); Zhang and Xia (2005)]. The mechanism, called the dilaton

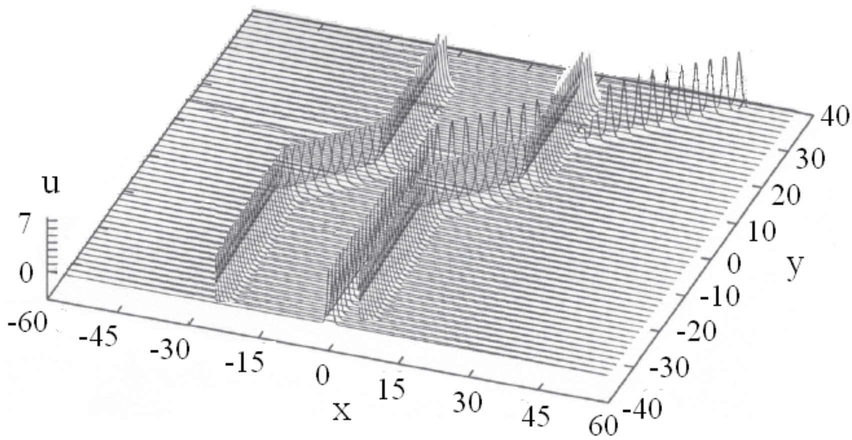

Figure 3.2: Scheme of interaction ($A_{12} \approx 0$, $A_{13} \approx 0$ and $A_{34} \approx 0$).

mechanism, has been proposed for explaining the possible amplification of nonlinear seismic waves [Engelbrecht and Khamidullin (1988), Engelbrecht (1991)]. Zhurkov (1983) and Petrov (1983) have introduced the dilaton concept to explain the fracture of solids.

The cases when $A_{ij} \approx 0$, $i, j \geq 3$ exhibit a veritable dilatonic interaction. The amplitudes dramatically increase, as shown in Fig.3.3 for $A_{33} \approx 0$ and $A_{34} \approx 0$. The dilaton is a fluctuation of internal energy of a medium with loosened bonds between its structural elements. The dilaton is able to absorb energy from the surrounding medium, and when the accumulated energy in it has reached its critical value, the dilaton breaks up releasing the stored energy, causing the amplification of waves. This mechanism is controlled by the intensity of the propagating wave. Low-intensity waves give a part of their energy away to the dilatons, and high-intensity waves cause the dilatons to break up [Munteanu and Donescu (2004)].

The crests continuously move and amplify in time by multisoliton resonant interaction mechanism, explaining the generation of huge waves as in shallow water or in the ocean basin (as in Fig. 3.4 for $A_{33} \approx 0$ and $A_{34} \approx 0$).

The central result in analyzing the chaotic behavior of the KP equation with hysteresis is presented in Fig.3.5. This is a specific response to short multisoliton resonant signals. We see that the input variable drift, the output variable relaxation and non-closure of hysteretic loops are present. The input variable drift appears when cycled between two unequal output variable, the output variable relaxation

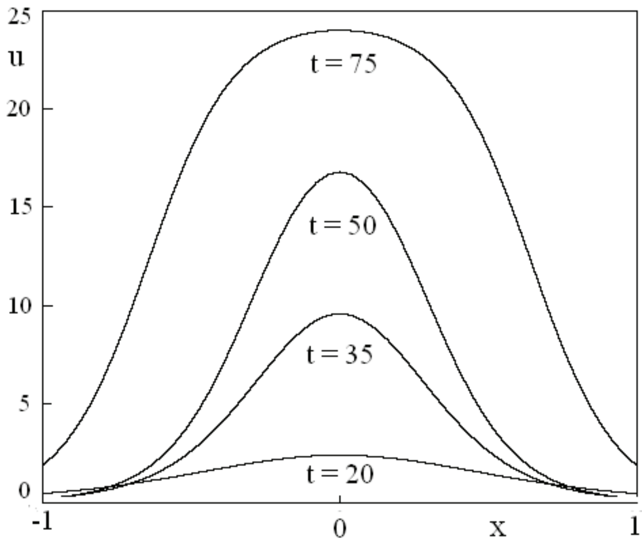

Figure 3.3: Scheme of explosive amplification of the amplitude for $A_{33} \approx 0$ and $A_{34} \approx 0$.

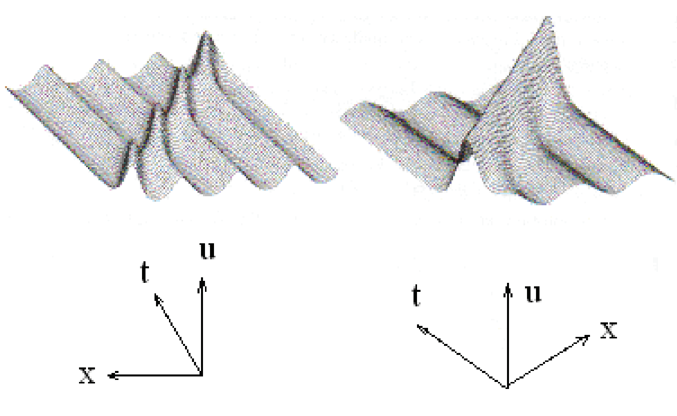

Figure 3.4: Amplification of the amplitude for $A_{33} \approx 0$ and $A_{34} \approx 0$.

appears when cycled between two unequal input variables [Preda, Ionescu, Chiroiu and Sireteanu (2010)]. The singularity which creates non-analyticity in the constitutive behavior at a particular reversal point related to the history of the system is a hallmark of many engineering systems.

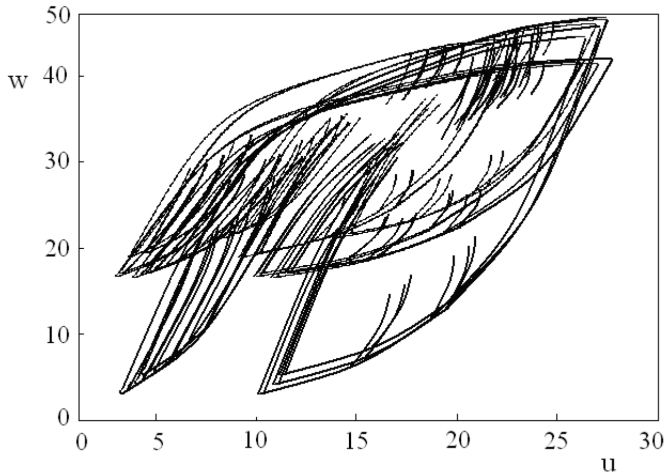

Figure 3.5: Chaotic hysteresis characterized by drift, relaxation and non-closure of the loops.

4 Concluding remarks

In this paper, the KP equation coupled with the generalized play operator is studied in order to explain the dilatonic behavior of the soliton interaction and the generation of huge waves in shallow waters or in the ocean basin. Hirota bilinear method and results from a nonlinear semigroup theory are applied to simulate the resonant soliton interaction. Equation exhibits the chaotic aspects such as drift, relaxation and non-closure of the loops.

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