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Flow Characteristics of Revolving Ferrofluid with Variable Viscosity in a Porous Medium in the Presence of Stationary Disk

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Abstract: The present problem is formulated by considering the dynamics of a ferromagnetic fluid of variable viscosity permeating a porous medium in a rotating system in the presence of a stationary boundary. The fluid at large distance from such a boundary (disk) is assumed to rotate at a given uniform angular velocity. The viscosity of the fluid is assumed to depend on the intensity of the applied magnetic field. The governing nonlinear partial differential equations are transformed into a set of coupled nonlinear ordinary differential equations resorting to a similarity transformation. The resulting system of equations is solved numerically by applying a shooting iteration technique combined with a forth-order Runge-Kutta method.

Keywords: Ferrofluid, boundary layer, MFD viscosity, stationary disk

Nomenclature

$\overrightarrow{B}$	Magnetic induction (T)
B	Magnitude of $\overrightarrow{B}$ (T)
$\overrightarrow{H}$	Magnetic field intensity
$\overrightarrow{M}$	Magnetization $\left(\frac{A}{m}\right)$
p	Fluid pressure $\left(\frac{kg}{ms^2}\right)$
$\overrightarrow{V}$	Velocity of ferrofluid $\left(\frac{m}{s}\right)$
ν	Kinematic viscosity $\left(\frac{m^2}{s}\right)$
ν_1	Kinematic variable viscosity $\left(\frac{m^2}{s}\right)$

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μ	Reference viscosity of ferrofluid $\left(\frac{kg}{m\,s}\right)$
μ_0	Magnetic permeability of free space $\left(\frac{H}{m}\right)$
ρ	Fluid density $\left(\frac{kg}{m^3}\right)$
χ	The magnetic susceptibility
∇	Gradient operator $\left(m^{-1}\right)$
$\overrightarrow{\delta}$	Linear measure of the viscosity variations with the applied magnetic field $\left(T^{-1}\right)$
k	MFD viscosity parameter
α	Dimensionless axial distance
ω	Angular velocity $\left(\frac{rad}{s}\right)$
v_r	Radial velocity $\left(\frac{m}{s}\right)$
v_θ	Tangential velocity $\left(\frac{rad}{s}\right)$
v_z	Axial velocity $\left(\frac{m}{s}\right)$
K	Darcy (medium) permeability
β	Medium permeability parameter

1 Introduction

Ferromagnetic fluids (ferrofluids) are colloidal liquids made of nanoscale ferromagnetic, or ferrimagnetic, particles suspended in a carrier fluid. The hydro-dynamics of such fluids are a (challenging) subject of interest for several reasons ranging from fundamental fluid mechanics to a variety of applications in engineering. After their first stable synthesis in the early 1960s, development of these suspensions in carrier liquid proved the high potential for new technological applications, thereby opening a new field of research, generally referred to as "ferrohydrodynamics".

As outlined above, ferrofluids do not exist in nature; they are synthesized fluids. The principal type is the "colloidal" ferrofluid, a suspension of finely divided particles in a certain medium which settles out slowly. Such ferrofluids are composed of small (3-15nm) particles of solid magnetite coated with a molecular layer of a dispersant and suspended in a liquid carrier. Thermal agitation keeps the particles suspended because of Brownian motion and coating prevents the particles from sticking to each other.

A typical ferrofluid contains 10^{23} particles per cubic meter.

One of the many fascinating features of ferrofluids is the possibility of influencing the flow by a magnetic field and vice versa (Feynman, Leighton, Sands (1963) and Shliomis (2004)). Ferrofluids are widely used in sealing of hard disk drives and rotating x-ray tubes. In particular, sealing of the rotating shafts is the most known application of a magnetic fluid.

The major application for ferrofluids used in synergy with electrical fields is the control of heat in loudspeakers, which makes their life longer and increases acoustical power without any change in the geometrical shape of the speaker system.

Magnetic fluid are also used in the contrast medium in X-ray examinations and for positioning tamponade for retinal detachment repair in eye surgery. Therefore, ferrofluids play an important role in bio-medical applications as well.

In the presence of an uniform magnetic field, the magnetization characteristics depend on particle spin, but do not depend on fluid velocity. Convection of ferromagnetic fluids is gaining much importance due to such peculiar physical properties.

The dynamics of fluids in "rotating systems" has also attracted much attention as a fundamental problem in fluid-dynamics.

Several boundary value problems, in detail, have been discussed in Schlichting (1960). The pioneering study of ordinary viscous fluid flow due to an infinite rotating disk was carried by Von Karman. He introduced the famous transformation which reduces the governing equations to non linear differential equations in dimensionless form. Karman's (1921) rotating disk problem was extended to flows started impulsively from rest in Cochran's (1934). Cochran (1934) obtained asymptotic solutions for the steady hydrodynamic problem formulated by Von Karman. Benton (1966) improved Cochran's solutions, and solved the unsteady case. The effect of uniform high suction on the steady flow of non-Newtonian fluid due to a rotating disk was considered by Mithal (1961). Attia (2004) discussed about flow due to an infinite disk rotating in the presence of an axial uniform magnetic field by taking the Hall effect into consideration.

Rosensweig (1985) has given an authoritative introduction to the research on magnetic liquids in his monograph. A detail account of magneto viscous effects in ferrofluids has been given in the subsequent monograph by Odenbach (2002).

In general, magnetization is a function of magnetic field, temperature and density of the fluid. This leads to convection of a ferrofluid in the presence of a magnetic field gradient. Sunil, Diva, Sharma (2005) studied the effect of magnetic-field-dependent (MFD) viscosity on thermosolutal convection in a ferromagnetic fluid saturating a porous medium. Nanjundappa, Shivakumara, Arunkumar (2010) studied Marangoni-Bénard convection in a ferrofluid layer in the presence of a uniform vertical magnetic field with MFD viscosity (see also Hennenberg et al., 2007). Sekar, Vaidyanathan, Ramanathan (1993) examined the ferroconvection in a densely packed porous medium assumed to be bounded by stress free boundaries and heated from below.

Ram, Bhandari, Sharma (2010, 2011) studied the effect of MFD viscosity and porosity on the characteristics of a revolving ferrofluid flow due to an infinite rotating disk. Attia (1998) investigated the flow near a porous disk in the presence of an applied uniform magnetic field. Frusteri, Osalusi (2007) examined the laminar convective and slip flow of an electrically conducting Newtonian fluid with variable properties over a rotating porous disk. Attia (2009) studied the steady flow of an incompressible viscous fluid above an infinite rotating disk in a porous medium with heat transfer and also discussed the effect of the medium porosity on the velocity and temperature distribution. An analysis of the effect of MFD viscosity on thermal convection in a ferromagnetic fluid in a porous medium is due to Sunil, Bharti, Sharma, Sharma (2004). Other relevant studies about flow in porous media are due to Bataller (2010), Al- Ajmi and Mosaad (2012), Hamimid, Guellal, Amroune and Zeraibi (2012), Choukairy and Bennacer (2012), Labed, Bennamoun and Fohr (2012).

In the present paper, the effects of medium porosity and MFD viscosity are investigated in the presence of a stationary disk assuming that the fluid angular velocity (ω) is uniform at a large distance from the plate.

We take cylindrical coordinates r, θ, z where z-axis is normal to the disk and this axis is considered as the axis of rotation. The boundary layer equations together with boundary conditions are solved numerically. This specific problem (a revolving ferrofluid with variable viscosity in a porous medium in the presence of stationary disk), to the best of our knowledge, has not been investigated yet.

2 Mathematical formulation of the problem

The problem is considered with the following assumptions:

a. The magnetization $\overrightarrow{M}$ is parallel to the applied magnetic field $\overrightarrow{H}$.

b. The fluid and ferrous particles have the same velocity.

c. The fluid and the disk are both electrically non-conducting.

d. The magnetic field affects only viscosity, not other properties.

e. Thermal effects are not taken into consideration.

f. The flow is steady, axi-symmetric and incompressible.

The constitutive equations of motion are as follows

$$\rho\left[\frac{\partial\overrightarrow{V}}{\partial t}+\left(\overrightarrow{V}.\nabla\right)\overrightarrow{V}\right]=-\nabla p+\mu_0\overrightarrow{M}.\nabla\overrightarrow{H}+\mu\left(1+\overrightarrow{\delta}.\overrightarrow{B}\right)\nabla^2\overrightarrow{V}-\frac{\mu}{K}\overrightarrow{V} \qquad (1)$$

$$\nabla.\overrightarrow{V}=0,\quad \nabla\times\overrightarrow{H}=\overrightarrow{0},\quad \nabla.\left(\overrightarrow{H}+4\pi\overrightarrow{M}\right)=0 \qquad (2)$$

With $\overrightarrow{M}=\chi\overrightarrow{H}$, $\overrightarrow{M}\times\overrightarrow{H}=\overrightarrow{0}$, $\overrightarrow{B}=\mu_0\left(\overrightarrow{H}+\overrightarrow{M}\right)$.

Assuming that the overall system (geometry, flow and applied fields) is rotationally symmetric, the variables are considered independent of the angular coordinates (see, e.g., Mahfoud and Bessaih, 2012).

Equations (1) and (2) can be written in cylindrical coordinates as:

$$\frac{\partial v_r}{\partial r}+\frac{v_r}{r}+\frac{\partial v_z}{\partial z}=0 \tag{3}$$

$$v_r\frac{\partial v_r}{\partial r}+v_z\frac{\partial v_r}{\partial z}-\frac{v_\theta^2}{r}+\frac{\mu}{\rho K}v_r=$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu}{\rho}\left|\overrightarrow{M}\right|\frac{\partial}{\partial r}\left|\overrightarrow{H}\right|+\nu_1\left[\frac{\partial^2 v_r}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_r}{r}\right)+\frac{\partial^2 v_r}{\partial z^2}\right] \tag{4}$$

$$v_r\frac{\partial v_\theta}{\partial r}+v_z\frac{\partial v_\theta}{\partial z}+\frac{v_r v_\theta}{r}+\frac{\mu}{\rho K}v_\theta=\nu_1\left[\frac{\partial^2 v_\theta}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right)+\frac{\partial^2 v_\theta}{\partial z^2}\right] \tag{5}$$

$$v_r\frac{\partial v_z}{\partial r}+v_z\frac{\partial v_z}{\partial z}+\frac{\mu}{\rho K}v_z=$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left|\overrightarrow{M}\right|\frac{\partial}{\partial z}\left|\overrightarrow{H}\right|+\nu_1\left[\frac{\partial^2 v_z}{\partial r^2}+\frac{1}{r}\frac{\partial v_z}{\partial r}+\frac{\partial^2 v_z}{\partial z^2}\right] \tag{6}$$

Where $\nu_1=\frac{\mu\left(1+\overrightarrow{\delta}\,.\,\overrightarrow{B}\right)}{\rho}$, $\nu=\frac{\mu}{\rho}$ and

$$\overrightarrow{\delta}.\,\overrightarrow{B}=\overrightarrow{\delta}\,.\mu_0\left(\overrightarrow{H}+\chi\overrightarrow{H}\right)=\mu_0\,(1+\chi)\,\overrightarrow{\delta}\,.\overrightarrow{H}=\mu_0\,(1+\chi)\,(\delta_1 H_r+\delta_3 H_z)$$

The approximate initial and boundary conditions for revolving flow of ferrofluid in the presence of stationary disk with constant angular velocity ω are given by

$$\text{at } z=0;\ v_r=0,\ v_\theta=0,\ v_z=0 \text{ and at } z=\infty;\ v_r=0,\ v_\theta=r\omega \tag{7}$$

Here, v_z does not vanish at $z=\infty$, but tends a finite value.

Using the boundary layer approximation $\frac{1}{\rho}\frac{\partial p}{\partial r}-\frac{\mu_0}{\rho}\left|\overrightarrow{M}\right|\frac{\partial}{\partial r}\left|\overrightarrow{H}\right|=r\omega^2$ and similarity transformations, $v_r=r\omega E(\alpha)$, $v_\theta=r\omega F(\alpha)$, $v_z=\sqrt{\nu\omega}\,G(\alpha)$; where $\alpha=z\sqrt{\frac{\omega}{\nu}}$, the system reduces to a set of non-linear coupled differential equations in the form of E, F and G as follows:

$$kE''-GE'-E^2+F^2-\beta E-1=0 \tag{8}$$

$$kF''-GF'-2EF-\beta F=0 \tag{9}$$

$$G' + 2E = 0 \tag{10}$$

$$E(0) = 0, \quad F(0) = 0, \quad G(0) = 0; \quad E(\infty) = 0, \quad F(\infty) = 1 \tag{11}$$

Where the expressions $\beta = \frac{\nu}{K\omega}$ and $k = \frac{\nu_1}{\nu}$, represents porosity parameter and MFD viscosity parameter, respectively. In particular the latter parameter in explicit form reads

$$k = \frac{\nu_1}{\nu} = 1 + \overrightarrow{\delta}.\overrightarrow{B} = 1 + \mu_0 (1 + \chi) (\delta_1 H_r + \delta_3 H_z)$$

3 Problem solution

Equations (8)-(10) are nonlinear and coupled differential equations. Now, we apply Shooting Method for numerical solution. First, we reduce the above differential equations to the first order differential equations by using the following transformations:

$$E(\alpha) = y_1(\alpha), \quad E'(\alpha) = y_2(\alpha), \quad F(\alpha) = y_3(\alpha), \quad F'(\alpha) = y_4(\alpha),$$

$$G(\alpha) = y_5(\alpha)$$

Let $y_2(0) = a$ and $y_4(0) = b$, we get an initial value problem 1. We will find a and b later.

Initial value problem 1

$$\frac{dy_1}{d\alpha} = y_2; \quad y_1(0) = 0 \tag{12}$$

$$\frac{dy_2}{d\alpha} = \frac{1}{k}\left(y_2 y_5 + y_1^2 - y_3^2 + \beta y_1 + 1\right); \quad y_2(0) = a \tag{13}$$

$$\frac{dy_3}{d\alpha} = y_4; \quad y_3(0) = 0 \tag{14}$$

$$\frac{dy_4}{d\alpha} = \frac{1}{k}(2y_1 y_3 + y_5 y_4 + \beta y_3); \quad y_4(0) = b \tag{15}$$

$$\frac{dy_5}{d\alpha} = -2y_1; \quad y_5(0) = 0 \tag{16}$$

Differentiating (12)-(16) partially with respect to a as:
$Y_i = \frac{\partial y_i}{\partial a}$ for $i = 1, 2, 3, 4, 5$
We get the initial value problem 2 as:

Initial value problem 2

$$\frac{dY_1}{d\alpha} = Y_2; \quad Y_1(0) = 0 \tag{17}$$

$$\frac{dY_2}{d\alpha} = \frac{1}{k}\left(Y_5\,y_2 + y_5\,Y_2 + 2\,y_1\,Y_1 - 2\,y_3\,Y_3 + \beta\,Y_1\right); \quad Y_2\left(0\right) = 1 \tag{18}$$

$$\frac{dY_3}{d\alpha} = Y_4; \quad Y_3\left(0\right) = 0 \tag{19}$$

$$\frac{dY_4}{d\alpha} = \frac{1}{k}\left(2\,Y_1\,y_3 + 2\,y_1\,Y_3 + Y_5\,y_4 + y_5\,Y_4 + \beta\,Y_3\right); \quad Y_4\left(0\right) = 0 \tag{20}$$

$$\frac{dY_5}{d\alpha} = -2\,Y_1; \quad Y_5\left(0\right) = 0 \tag{21}$$

Again, differentiating (12)-(16) partially with respect to b such as:
$Z_i = \frac{\partial y_i}{\partial b}$ for $i = 1, 2, 3, 4, 5$
We get the initial value problem 3 as:

Initial value problem 3

$$\frac{dZ_1}{d\alpha} = Z_2; \quad Z_1\left(0\right) = 0 \tag{22}$$

$$\frac{dZ_2}{d\alpha} = \frac{1}{k}\left(Z_5\,y_2 + y_5\,Z_2 + 2\,y_1\,Z_1 - 2\,y_3\,Z_3 + \beta\,Z_1\right); \quad Z_2\left(0\right) = 0 \tag{23}$$

$$\frac{dZ_3}{d\alpha} = Z_4; \quad Z_3\left(0\right) = 0 \tag{24}$$

$$\frac{dZ_4}{d\alpha} = \frac{1}{k}\left(2\,Z_1\,y_3 + 2\,y_1\,Z_3 + Z_5\,y_4 + y_5\,Z_4 + \beta\,Z_3\right); \quad Z_4\left(0\right) = 1 \tag{25}$$

$$\frac{dZ_5}{d\alpha} = -2\,Z_1; \quad Z_5\left(0\right) = 0 \tag{26}$$

Let

$$f_1\left(\infty; a_n, b_n\right) = y_1\left(\infty; a_n, b_n\right) - y_1\left(\infty\right) = y_1\left(\infty; a_n, b_n\right) - 0 = y_1\left(\infty; a_n, b_n\right) \tag{27}$$

and

$$f_2\left(\infty; a_n, b_n\right) = y_3\left(\infty; a_n, b_n\right) - y_3\left(\infty\right) = y_3\left(\infty; a_n, b_n\right) - 1 \tag{28}$$

Now, we can find a & b as follows:

$$\begin{aligned}\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} &= \begin{bmatrix} a_n \\ b_n \end{bmatrix} - \begin{bmatrix} \frac{\partial f_1}{\partial a} & \frac{\partial f_1}{\partial b} \\ \frac{\partial f_2}{\partial a} & \frac{\partial f_2}{\partial b} \end{bmatrix}^{-1}_{a_n, b_n} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}_{a_n, b_n} \\ &= \begin{bmatrix} a_n \\ b_n \end{bmatrix} - \begin{bmatrix} \frac{\partial y_1}{\partial a} & \frac{\partial y_1}{\partial b} \\ \frac{\partial y_3}{\partial a} & \frac{\partial y_3}{\partial b} \end{bmatrix}^{-1}_{a_n, b_n} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}_{a_n, b_n}\end{aligned} \tag{29}$$

The initial conditions, a & b, are calculated from (29) after iteration.

4 Discussion

The problem considered here involves a number of parameters, on the basis of which a wide range of numerical results have been derived. Of these results, a small section is presented here for brevity.

In the present problem, the fluid at large distance from the stationary disk at $z = 0$ rotates with a constant angular velocity (ω). The fluid particles which rotate at a large distance from the wall are in equilibrium due to centrifugal force balanced by the radial pressure gradient and radial magnetization force gradient.

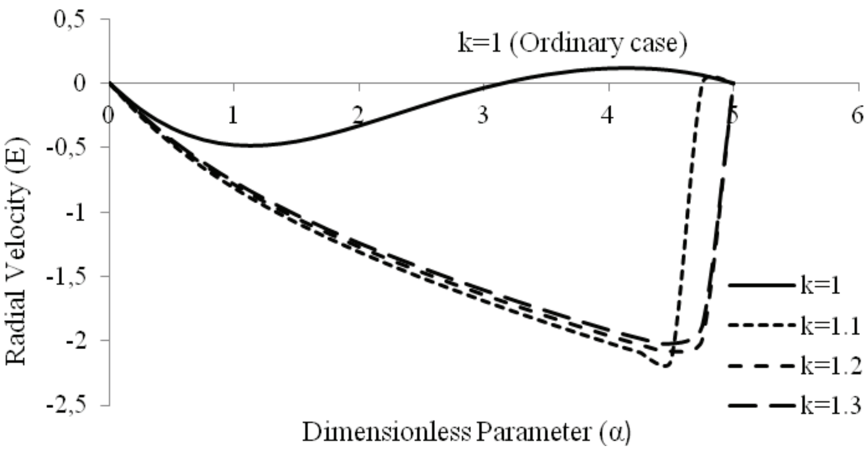

Figure 1: Radial velocity profile for various values of k at $\beta = 1$.

Figures 1, 4, 7 show the radial velocity profile for different values of MFD viscosity parameter k at $\beta = 1, 2, 3$ respectively. A negative value of the radial velocity indicates that the flow is directed radially inward whereas a positive value of the radial velocity represents flow directed radially outward. The radial velocity is directed radially inward but for increasing values of β it is less negative in comparison to $\beta = 1$. However, for $k = 1$(no MFD viscosity) and $\beta = 0$ (no medium permeability), the problem reduces to the classical ordinary case of viscous incompressible flow in the presence of a stationary disk (see Schlichting (1960)). In such a case, the radial velocity is negative for small values of α because near the wall the particles flow radially inward so that the peripheral velocity of the fluid particle near the wall is reduced thus decreasing the centrifugal force. The flow behavior of magnetic fluid, in the range $\alpha = 0$ to $\alpha = 3$, is radially inward and for larger values of α, it is in the opposite direction.

Figures 2, 5, 8 represent the tangential velocity profile. In figure 2, for $\beta = 1$,

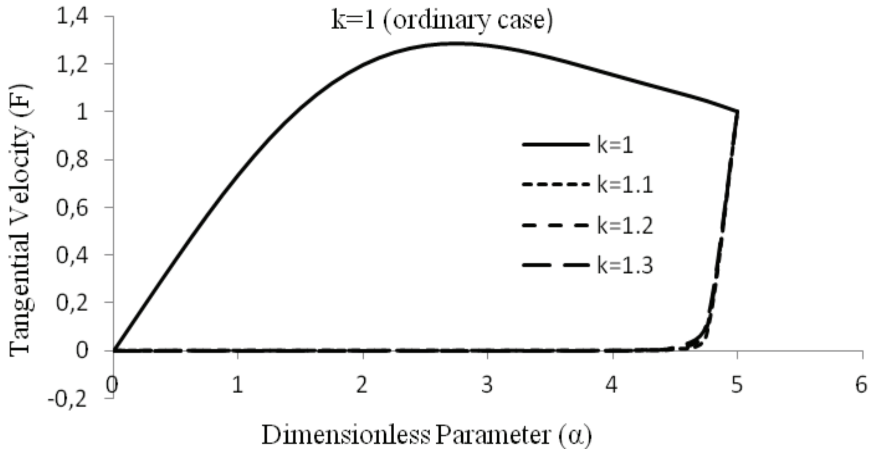

Figure 2: Tangential velocity profile for various values of k at $\beta = 1$.

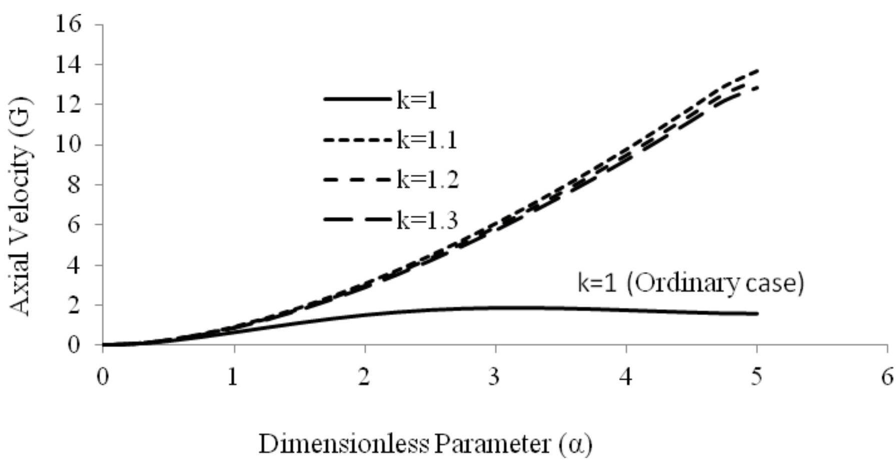

Figure 3: Axial velocity profile for various values of k at $\beta = 1$.

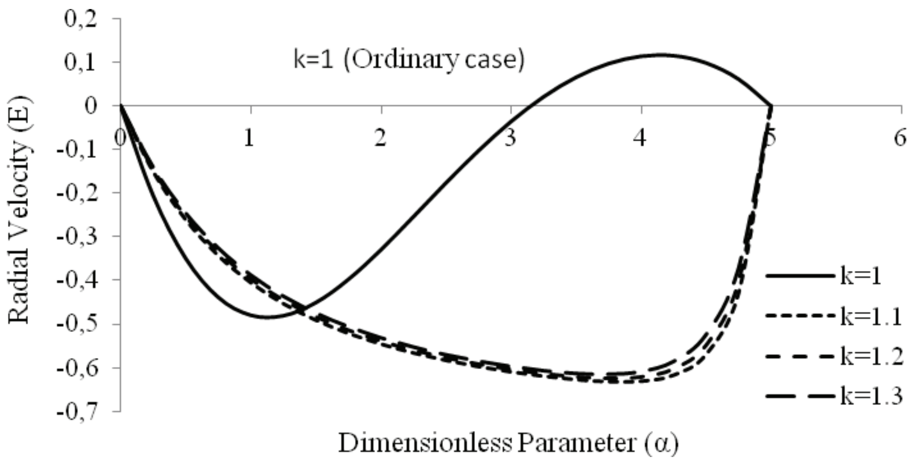

Figure 4: Radial velocity profile for various values of k at $\beta = 2$.

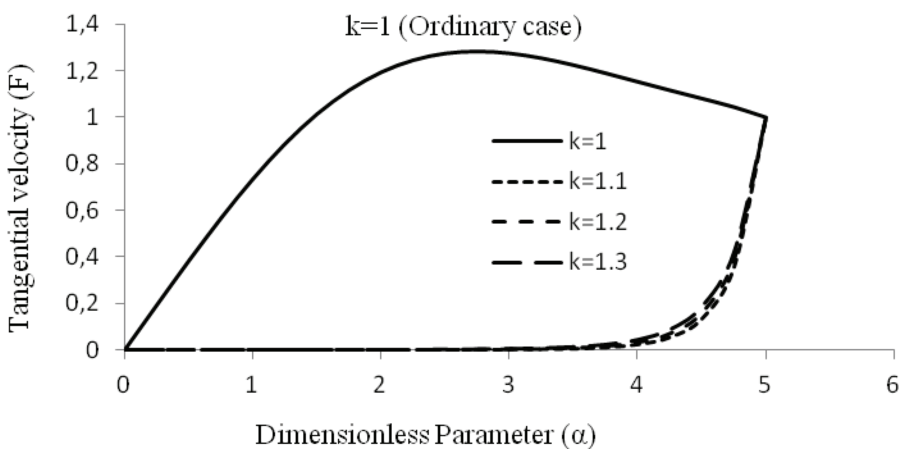

Figure 5: Tangential velocity profile for various values of k at $\beta = 2$.

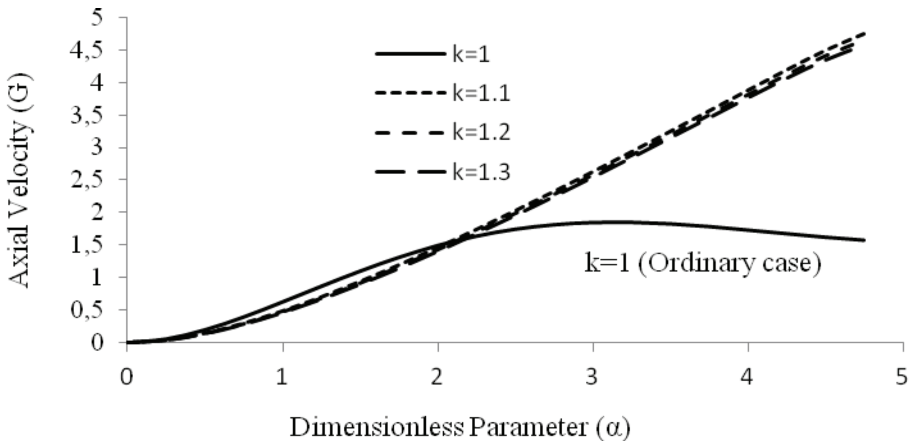

Figure 6: Axial velocity profile for various values of k at $\beta = 2$.

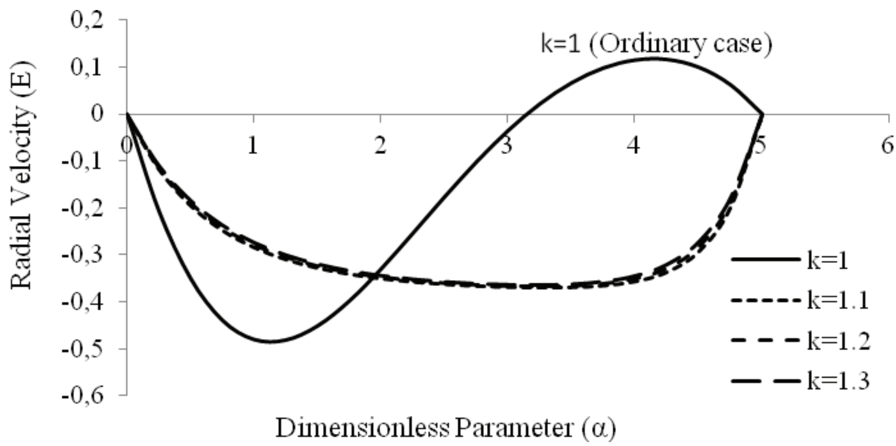

Figure 7: Radial velocity profile for various values of k at $\beta = 3$.

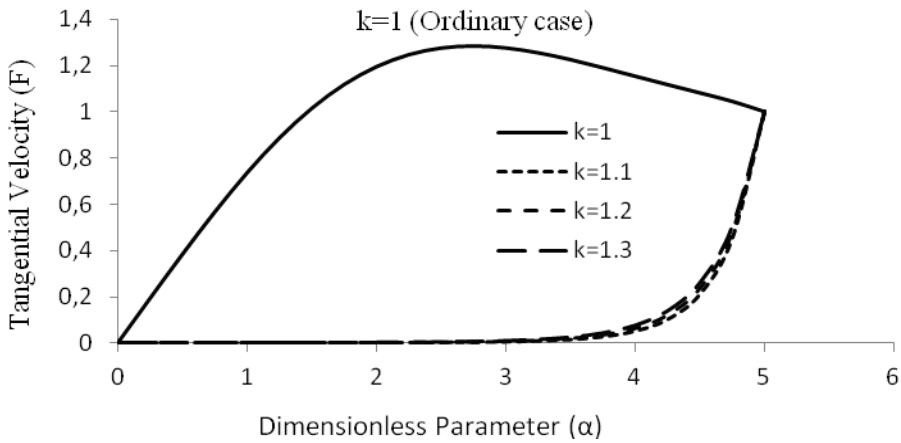

Figure 8: Tangential velocity profile for various values of k at $\beta == 3$

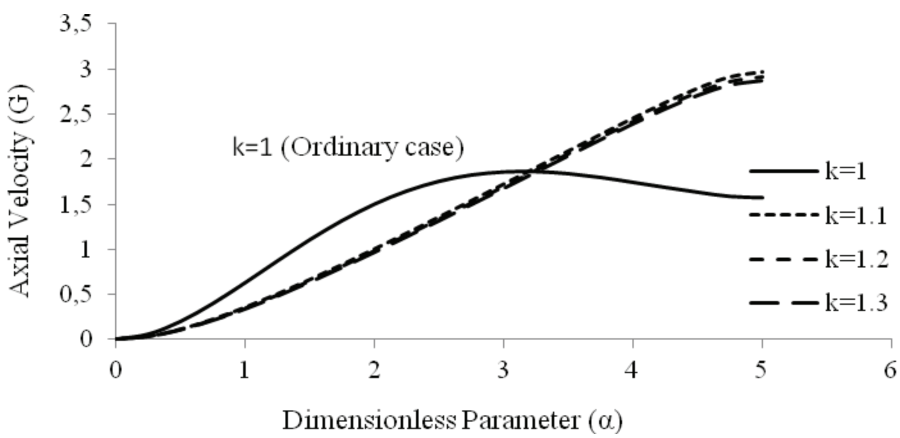

Figure 9: Axial velocity profile for various values of k at $\beta = 3$.

the circumferential velocity becomes very small in the range $\alpha = 0$ to $\alpha = 4.5$; however in figure 5, this regime is attained in the range $\alpha = 0$ to $\alpha = 3.8$ and finally, in figure 8, it is in the range $\alpha = 0$ to $\alpha = 3.5$. It is evident from figures that increasing values of parameter β favors the rotational motion of the ferrofluid. In the ordinary case, tangential velocity increases rapidly for increasing values of α, and at a large distance from the plate, the fluid rotates with constant angular velocityω.

Figure 3, 6, 9 indicate the axial velocity profile. For $\beta = 1$, the magnitude of the axial velocity is very large as the fluid flows radially in the negative direction, resulting in high axial flows perpendicular to the ground as indicated in figure 3. For increasing values of β, the axial velocity decreases but it is larger than in the ordinary case. It is also observed that the axial velocity tends to a finite value at infinite. In a nut shell, these results indicate that in a porous medium, axial motion is larger in comparison to ordinary case.

5 Conclusions

The present study has background application in areas such as rotating machinery, lubrication, oceanography, computer storage devices, viscometry and crystal growth process. The results indicate that the flow resulting from the revolution of a ferrofluid about an axis perpendicular to a stationary disk may be controlled by properly tuning some parameters.

Flow characteristics depend essentially on the porosity of the medium and change significantly when a variable viscosity is considered in comparison to the classical case in the literature. It is also remarkable that at a large distance from the stationary disk, the axial velocity component gets a maximum value, whereas it does not depend much on the distance from the rotational axis.

Acknowledgement: Authors thankfully acknowledge the help provided by the reviewers in improving the quality of the work.

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