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Effect of Suspended Particles on the Onset of Thermal Convection in a Compressible Viscoelastic Fluid in a Darcy-Brinkman Porous Medium

G. C. Rana[1], **and R. C. Thakur**[2]

Abstract: In this paper, the effect of suspended particles on thermal convection in a compressible viscoelastic fluid hosted in a porous medium is considered. For the porous medium, the Brinkman model is employed with the Rivlin-Ericksen approach used in parallel to describe the rheological behaviour of the viscoelastic fluid. By applying a normal mode analysis method, a dispersion relation is derived and solved analytically. It is observed that the medium permeability, suspended particles, gravity field and viscoelasticity introduce oscillatory modes. For stationary convection, it is found that the Darcy-Brinkman number has a stabilizing effect whereas the suspended particles and medium permeability have a destabilizing influence on the system.

Keywords: Darcy-Brinkman porous medium, Rivlin-Ericksen fluid, Suspended particles, Thermal convection, Viscosity, Viscoelasticity.

1 Introduction

Over the last few decades, considerable interest has been devoted to the study of thermal instabilities in porous media because this subject has various applications in different fields (geophysics, food processing and nuclear reactors just to cite a fiew).

Experimental studies have been devoted to the case of Newtonian fluid since the beginning of the past century (e.g. Chandra (1938) and many others). A number of theoretical and numerical analyses are also available (see the reviews by Chandrasekhar (1981); Lappa (2004, 2005, 2007a,b, 2013) and many others).

For the case of convective flow in a porous medium, it is worth citing Lapwood

[1] Department of Mathematics, NSCBM Govt. P. G. College, Hamirpur-177 005, Himachal Pradesh, INDIA

[2] Department of Mathematics, Government P. G. College Dhaliara-177103, Himachal Pradesh, INDIA

(1948) who used the linearized stability theory. The Rayleigh instability of a thermal boundary layer in a porous medium was considered by Wooding (1960).

Scanlon and Segel (1973) were among the first authors to investigate the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by the particles. The suspended particles were thus found to destabilize the layer. There are many elastico-viscous fluids that cannot be characterized by Maxwell's constitutive relations or Oldroyd's constitutive relations. One such class of fluids is Rivlin-Ericksen elastico-viscous fluid having relevance in chemical technology and industry. Rivlin and Ericksen (1955) proposed a theoretical model for such an elastico-viscous fluid.

The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium has been given by Vafai and Hadim (2000), Ingham and Pop (1981) and Nield and Bejan (2006). More recent studies have been developed by Choukairy and Bennacer (2012); Hamimid, Guellal, Amroune, and Zeraibi (2012); Al-Ajmi and Mosaad (2012); Ram and Bhandari (2012); Labed, Bennamoun, and Fohr (2012) (see also the references in these works)

Kuznetsov and Nield (2010) have studied thermal instability in a porous medium layer saturated by a nanofluid: Brinkman model. In this context it is also worth mentioning Corcione (2011).

Sharma and Rana (2001) studied thermal instability of an incompressible Walters' (Model B') elastico-viscous in the presence of variable gravity field and rotation in porous medium. When the fluids are compressible, the equations governing the system become quite complicated. To simplify them, Boussinesq tried to justify the approximation for compressible fluids for density variations arising principally from thermal effects. Spiegal and Veronis (1960) simplified the set of equations governing the flow of compressible fluids under the following assumptions:

(a) The vertical dimension of the fluid is much less than any scale height, as defined by Spiegal and Veronis (1960);

(b) The motion-induced perturbations in density and pressure do not exceed, in order of magnitude, their total static variations.

Under the above approximations, Spiegal and Veronis (1960) showed that the equations governing convection in a compressible fluid are formally equivalent to those for an incompressible fluid if the static temperature gradient is replaced by its excess over the adiabatic one and C_v is replaced by C_p; where C_v and C_p are the specific heats at constant volume and constant pressure, respectively, which is important in ground water hydrology, chemical engineering, modern technology and industries. Recently, Rana (2011) studied the onset of convection in Rivlin-Ericksen fluid in a Darcy-Brinkman porous medium heated from below whereas Rana and Kango (2011) studied the stability of incompressible Rivlin-Ericksen superposed fluid under rotation in porous medium.

The interest for investigations of non-Newtonian fluids is also motivated by a wide range of engineering applications which include ground pollutions by chemicals which are non-Newtonian like lubricants and polymers and in the treatment of sewage sludge in drying beds. Recently, polymers are being used in agriculture, communications appliances and in bio medical applications. Examples of these applications are filtration processes, packed bed reactors, insulation system, ceramic processing, enhanced oil recovery, chromatography etc. Viscoelastic materials have the properties of both viscous and elastic materials, and are modeled by combining elements that represents both characteristics. There are several models of interest to quantify the related behaviour. Rivlin-Ericksen fluid model is one of such model. Stress relaxation describes how viscoelastic materials relieve stress under constant strain.

With the importance in various applications mentioned above, our main aim in the present paper is to study the effect of suspended particles on thermal convection for a compressible Rivlin-Ericksen elastico-viscous fluid in a Darcy-Brinkman porous medium. To the best of our knowledge, this problem has not been investigated so far.

2 Mathematical model and perturbation equations

Here we consider an infinite, horizontal, compressible Rivlin-Ericksen elastico-viscous fluid of depth d, bounded by the planes $z = 0$ and $z = d$ in an isotropic and homogeneous medium of porosity ε and permeability k_1, which is acted upon by gravity $g(0,0,-g)$ as shown in figure 1. This layer is heated from below such that a steady adverse temperature gradient $\beta = \left(\left|\frac{dT}{dz}\right|\right)$ is maintained. The character of equilibrium of this initial static state is determined by supposing that the system is slightly disturbed and then following its further evolution.

Let ρ, v, $v^{'}$, p, ε, T, α and v, denote respectively, the density, kinematic viscosity, kinematic viscoelasticity, pressure, medium porosity, temperature, thermal coefficient of expansion and velocity of the fluid.

The equations expressing the conservation of momentum, mass and temperature for compressible Rivlin-Ericksen elastico-viscous fluid in a Brinkman porous medium [Chandrasekhar (1981); Scanlon and Segel (1973); Sharma and Rana (2001); Rana

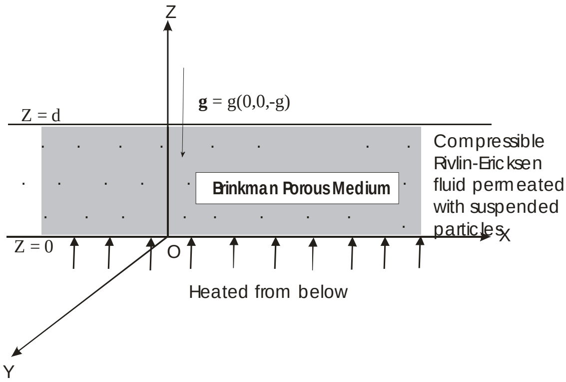

Figure 1: Schematic Sketch of Physical Situation

(2011)] are

$$\frac{\rho}{\varepsilon}\left[\frac{\partial v}{\partial t}+\frac{1}{\varepsilon}(v.\nabla)v\right]=-\nabla p+g\rho-\frac{1}{k_1}\left(\mu+\mu'\frac{\partial}{\partial t}\right)\vec{v}+\tilde{\mu}\nabla^2 v+\frac{K'N}{\varepsilon}(v_d-v), \quad (1)$$

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho v)=0, \quad (2)$$

$$\rho C_f\left(\frac{\partial}{\partial t}+v.\nabla\right)T+mNC_{pt}\left[\varepsilon\frac{\partial}{\partial t}+v_d.\nabla\right]T=k_T\nabla^2 T, \quad (3)$$

where $v_d(\bar{x},t)$ and $N(\bar{x},t)$ denote the velocity and number density of the particles respectively, $K'=6\phi\rho v\eta$, where η is particle radius, is the Stokes drag coefficient, $v_d=(l,r,s)$ and $\bar{x}=(x,y,z)$, C_f, C_{pt} and k_T denote, respectively, the heat capacity of the pure fluid, heat capacity of particles and 'effective thermal conductivity' of pure fluid.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are

$$mN\left[\frac{\partial v_d}{\partial t}+\frac{1}{\varepsilon}(v_d.\nabla)v_d\right]=K'N(v-v_d), \quad (4)$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla(Nv_d)=0. \quad (5)$$

The presence of particles adds an extra force term proportional to the velocity difference between particles and fluid and appears in the equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles (4). The buoyancy force on the particles is neglected. Interparticles reactions are not considered either since we assume that the distance between the particles are quite large compared with their diameters. These assumptions have been used in writing the equations of motion (4) for the particles.

The state variables pressure, density and temperature are expressed in the form [Spiegal and Veronis (1960)]

$$f(x,y,z,t) = f_m + f_0(z) + f^{'}(x,y,z,t), \tag{6}$$

where f_m denotes for constant space distribution f, f_0 is the variation in the absence of motion, and $f^{'}(x,y,z,t)$, is the fluctuation resulting from motion. The basic state of the system is

$$p = p(z), \rho = \rho(z), T = T(z), v = (0,0,0) \tag{7}$$

where

$$p(z) = p_m - g\int_0^z (\rho_m + \rho_0)\,dz, \rho(z) = \rho_m[1 - \alpha_m(T - T_0) + K_m(p - p_m)],$$

$$T = -\beta z + T_0, \alpha_m = -\left(\frac{1}{\rho}\frac{\partial p}{\partial t}\right)_m, K_m = \left(\frac{1}{\rho}\frac{\partial \rho}{\partial p}\right)_m.$$

Here p_m and ρ_m denote a constant space distribution of p and ρ while T_0 and ρ_0 denote temperature and density of the fluid at the lower boundary.

The equation of state is

$$\rho = \rho_m[1 - \alpha(T - T_0)], \tag{8}$$

where α is the coefficient of thermal expansion, as the density variations arise mainly due to temperature variations. Following the assumptions given by Spiegal and Veronis (1960) and the results for compressible fluid, the flow equations are found to be the same as that of incompressible fluid except that the static temperature gradient β is replaced by the excess over the adiabatic $\beta - g/c_p$, c_p being specific heat of the fluid at constant pressure.

Let $v(u,v,w)$, $v_d(l,r,s)$, N_0, θ, δ_p and δ_ρ denote, respectively, the perturbations in fluid velocity $v(0,0,0)$, particle velocity $v_d(0,0,0)$, suspended particles number density N, temperature T, pressure p and density ρ.

The change in density δ_ρ caused by perturbation θ in temperature is given by

$$\delta_\rho = -\alpha\rho_m\theta. \tag{9}$$

The linearized perturbation equations governing the motion of fluid are

$$\frac{1}{\varepsilon}\frac{\partial v}{\partial t} = -\frac{1}{\rho_m}\nabla\delta_p - g\alpha\theta - \frac{1}{k_1}\left(v + v^{'}\frac{\partial}{\partial t}\right)\vec{v} + \frac{\tilde{\mu}}{\rho_m}\nabla^2 v + \frac{K^{'}N}{\rho_m\varepsilon}(v_d - v), \tag{10}$$

$$\nabla.v = 0, \tag{11}$$

$$(E + b\varepsilon)\frac{\partial\theta}{\partial t} = \left(\beta - \frac{g}{c_p}\right)(w + bs) + \kappa\nabla^2\theta, \tag{12}$$

$$mN_0\frac{\partial v_d}{\partial t} = K^{'}N(v - v_d), \tag{13}$$

$$\varepsilon\frac{\partial N}{\partial t} + \nabla.(Nv_d) = 0, \tag{14}$$

where $v = \frac{\mu}{\rho_m}$, $v^{'} = \frac{u^{'}}{\rho_m}$, $\kappa = \frac{k_T}{\rho_m C_f}$, $\frac{g}{C_f}$ and w stand for kinematic viscocity, kinematic viscoelasticity, thermal diffusivity, adiabatic gradient and vertical fluid velocity, respectively.
Also $b = \frac{mN_0 C_{pt}}{\rho_m C_f}$ and w, s are the vertical fluid and particles velocity and

$$E = \varepsilon + (1 - \varepsilon)\left(\frac{\rho_s C_s}{\rho_m C_f}\right),$$

which is constant, κ is the thermal diffusivity and $\tilde{\mu}$ is effective viscosity of porous medium; ρ_s, C_s; ρ_m, C_f denote the density and heat capacity of solid (porous) matrix and fluid respectively.

3 The dispersion relation

Following the normal mode analyses, we assume that the perturbation quantities have x, y and t dependence of the form

$$[w,\theta] = [W(z),\Theta(z)]\exp(ilx + imy + nt), \tag{15}$$

where l and m are the wave numbers in the x and y directions, $k = (l^2 + m^2)^{1/2}$ is the resultant wave number and n is the frequency of the harmonic disturbance, which is, in general, a complex constant. Using equation (15), Equations (10)–(14) after a little algebra, can be written in non-dimensional form as

$$\left[1 + \left(\frac{P_l}{\varepsilon} + \frac{MP_l}{\varepsilon(1 + \tau_1\sigma)} + F\right)\sigma - D_A(D^2 - a^2)\right](D^2 - a^2)W + \frac{g\alpha a^2 d^2 P_l\Theta}{v} = 0, \tag{16}$$

$$(D^2-a^2-E^{'}p_1\sigma)\Theta=-\left(\frac{\beta d^2}{\kappa}\right)\left(\frac{G-1}{G}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \tag{17}$$

where we have put $a=kd$, $\sigma=\frac{nd^2}{\nu}$, $E^{'}=E+b\varepsilon$, $\tau=\frac{m}{K'}$, $\tau_1=\frac{\tau\nu}{d^2}$, $M=\frac{mN_0}{\rho_0}$, $B=b+1$, $P_l=\frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1=\frac{\nu}{\kappa}$, is the thermal Prandtl number, $F=\frac{\nu^{'}}{d^2}$, is the dimensionless kinematic viscoelasticity, $D_A=\frac{\tilde{\mu}k_1}{\mu d^2}$, is the Darcy-Brinkman number and $D^*=d\frac{d}{dz}$ and the superscript $*$ is suppressed. Eliminating Θ between equations (16) and (17), we obtain

$$\left\{\left[1+\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1\sigma)}+F\right)\sigma-D_A(D^2-a^2)\right](D^2-a^2)(D^2-a^2-Ep_1\sigma)\right\}W-$$
$$Ra^2P_l\left(\frac{G-1}{G}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W=0, \tag{18}$$

where $R=\frac{g\alpha\beta d^4}{\nu\kappa}$, is the thermal Rayleigh number.

Here we assume that the temperature at the boundaries is kept fixed, the fluid layer is confined between two boundaries and adjoining medium is electrically non-conducting. The boundary conditions appropriate to the problem are [Chandrasekhar (1981)]

$$W=D^2W=D^4W=\theta=0 \text{ at } z=0 \text{ and } 1. \tag{19}$$

The case of two free boundaries, though a little artificial is the most appropriate for stellar atmospheres. Using the boundary conditions (19), we can show that all the even order derivatives of W must vanish for $z=0$ and $z=1$ and hence the proper solution of W characterizing the lowest mode is

$$W=W_0\sin\pi z;\quad W_0 \text{ is a constant.} \tag{20}$$

Substituting equation (20) in (18), we get

$$R_1xP\left(\frac{G-1}{G}\right)=\left[1+\left(\frac{P}{\varepsilon}+\frac{MP}{\varepsilon(1+\tau_1 i\sigma_1)}+\tau^2F\right)i\sigma_1+D_{A_1}(1+x)\right]$$
$$(1+x+E^{'}p_1 i\sigma_1)\left(\frac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right), \tag{21}$$

where we have put

$$R_1=\frac{R}{\pi^4},D_A=\frac{D_A}{\pi^2},x=\frac{a^2}{\pi^2},i\sigma=\frac{\sigma}{\pi^2},P=\pi^2P_l.$$

Equation (21) is required dispersion relation accounting for the onset of thermal convection in compressible Rivlin-Ericksen elastico-viscous fluid permeated with suspended particles in a Darcy-Brinkman porous medium.

4 Oscillatory modes

Here, we examine the possibility of oscillatory modes, if any, in compressible Rivlin-Ericksen elastico-viscous fluid due to the presence of suspended particles, viscoelasticity, medium permeability and gravity field. Multiplying equation (18) by W^*, the complex conjugate of W, integrating over the range of z and making use of equations (18) with the help of boundary conditions (19), we obtain

$$\left[1+\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1\sigma}+F\right)\sigma\right]I_1-D_AI_2-$$
$$\frac{\alpha a^2 g\kappa P_l}{\nu\beta}\left(\frac{1+\tau_1\sigma_*}{B+\tau_1\sigma^*}\right)\left(\frac{G}{G-1}\right)(I_3+Ep_1\sigma^*I_4)=0, \tag{22}$$

where

$$I_1=\int_0^1(|DW|^2+a^2|W|^2)\,dz$$
$$I_2=\int_0^1(|D^2W|^2+a^4|W|^2+2a^2|DW|^2)\,dz$$
$$I_3=\int_0^1(|D\Theta|^2+a^2|\Theta|^2)\,dz$$
$$I_4=\int_0^1(|\Theta|^2)\,dz,$$

The integral part I_1-I_4 are all positive definite. Putting $\sigma=i\sigma_i$ in equation (22), where σ_i is real and equating the imaginary parts, we obtain

$$\left[\left(\frac{P_l}{\varepsilon}+\frac{MP_l}{\varepsilon(1+\tau_1^2\sigma_i^2)}+F\right)I_1+\frac{\alpha a^2 g\kappa P_l}{\nu\beta}\left(\frac{G}{G-1}\right)\right.$$
$$\left.\left\{\left(\frac{\tau_1(B-1)}{B^2+\tau_1^2\sigma_i^2}\right)(I_3+E'p_1I_4)\right\}\right]\sigma_i=0, \tag{23}$$

Equation (23) implies that $\sigma_i=0$ or $\sigma_i\neq 0$ which mean that modes may be non-oscillatory or oscillatory. The oscillatory modes introduced due to presence of viscosity, viscoelasticity, suspended particles and medium permeability, which were non-existent in their absence.

5 The Stationary Convection

For stationary convection, putting $\sigma=0$ in equations (21), we obtain

$$R_1=\frac{(1+x)^2}{xPB}\left(\frac{G}{G-1}\right)[1+(1+x)D_{A_1}] \tag{24}$$

Equation (24) expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters G, B, D_{A_1}, P and compressible Rivlin-Ericksen elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ.

Let the non-dimensional number G accounting for compressibility effect is kept as fixed, then we get

$$\bar{R}_c = \left(\frac{G}{G-1}\right) R_c, \tag{25}$$

where $\bar{R}_c$ and R_c denote, respectively, the critical number in the presence and absence of compressibility. Thus, the effect of compressibility is to postpone the instability on the onset of thermal instability. The cases $G = 1$ and $G < 1$ correspond to infinite and negative values of Rayleigh numbers due to compressibility which are not relevant to the present study.

To study the effects of suspended particles, Darcy number and medium permeability, we examine the behavior of $\frac{\partial R_1}{\partial B}$, $\frac{\partial R_1}{D_{A_1}}$ and $\frac{\partial R_1}{\partial P}$ analytically.

From equation (24), we get

$$\frac{\partial R_1}{\partial B} = -\frac{(1+x)^2}{xPB^2}\left(\frac{G}{G-1}\right)[1+(1+x)D_{A_1}], \tag{26}$$

which is negative. Hence, suspended particles have destabilizing effect on the thermal convection in Rivlin-Ericksen elastico-viscous fluid in a Brinkman porous medium. This destabilizing effect is an agreement of the earlier work of Scanlon and Segel (1973).

From equation (24), we get

$$\frac{\partial R_1}{\partial D_{A_1}} = \frac{(1+x)^3}{xPB}\left(\frac{G}{G-1}\right), \tag{27}$$

which is positive implying thereby the stabilizing effect of Darcy number on the system. This stabilizing effect is an agreement with the earlier work of Rana (2011).

It is evident from equation (24) that

$$\frac{\partial R_1}{\partial P} = -\frac{(1+x)^2}{xP^2B}\left(\frac{G}{G-1}\right)[1+(1+x)D_{A_1}]. \tag{28}$$

From equation (28), we observe that medium permeability has destabilizing effect on the on the system. This destabilizing effect is an agreement of the earlier work of Scanlon and Segel (1973); Sharma and Rana (2001); Rana (2011); Rana and Kango (2011).

The dispersion relation (24) is analyzed numerically. Graphs have been plotted by giving some numerical values to the parameters, to depict the stability characteristics.

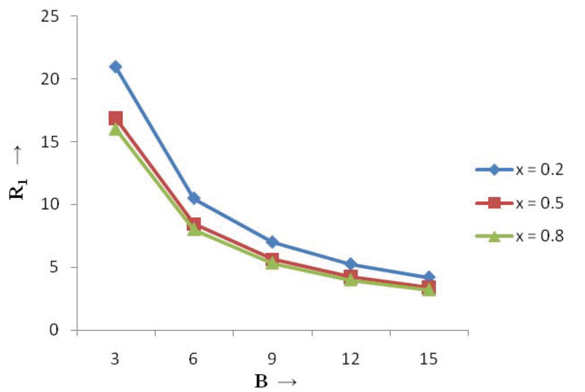

Figure 2: Variation of Rayleigh number R_1 with suspended particles B for $G = 5$, $P = 2$ and $D_{A_1} = 10$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$

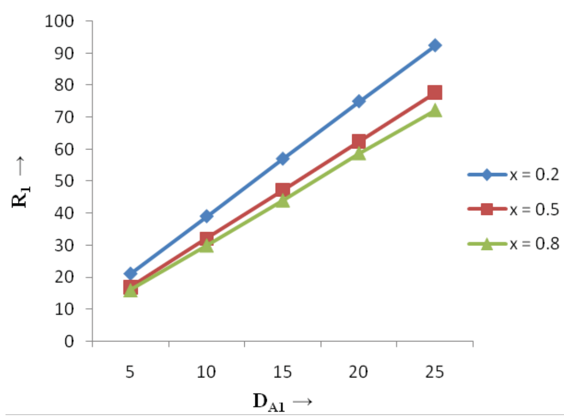

Figure 3: Variation of Rayleigh number R_1 with Darcy-Brinkman number D_{A_1} for $G = 5$, $P = 2$ and $B = 3$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

In fig. 2, Rayleigh number R_1 is plotted against suspended particles B for $G = 5$, $P = 2$ and $D_{A_1} = 10$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$. This shows that suspended particles has a destabilizing effect on the thermal convection in Rivlin-Ericksen elastico-viscous fluid in a Darcy-Brinkman porous medium. **In**

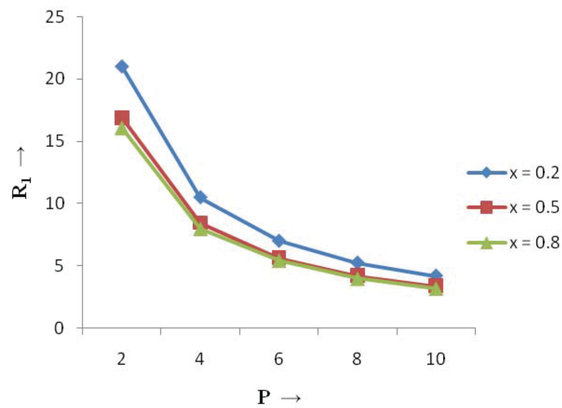

Figure 4: Variation of Rayleigh number R_1 with medium permeability P for $G = 5$, $B = 3$ and $D_{A_1} = 10$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$.

Fig. 3, Rayleigh number R_1 is plotted against with Darcy-Brinkman number D_{A_1} for $G = 5$, $P = 2$, $B = 3$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$. This shows that Darcy-Brinkman number has a stabilizing effect on the thermal convection in Rivlin-Ericksen elastico-viscous fluid permeated with suspended particles in a Darcy-Brinkman porous medium.

In fig. 4, Rayleigh number R_1 is plotted against medium permeability P for $G = 5$, $D_{A_1} = 10$ and $B = 3$ for fixed wave numbers $x = 0.2$, $x = 0.5$ and $x = 0.8$. This shows that medium permeability has a destabilizing effect on the thermal convection in Rivlin-Ericksen elastico-viscous fluid permeated with suspended particles in a Darcy-Brinkman porous medium.

6 Conclusion

The effect of suspended particles on thermal convection in a compressible Rivlin-Ericksen Walters' (Model B') elastico-viscous fluid heated from below in a Darcy-Brinkman porous medium has been investigated. A dispersion relation, including the effects of suspended particles, Darcy-Brinkman number, medium permeability and viscoelasticity on thermal convection has been derived. From the analysis, the following main conclusions have been derived:

(i) For the case of stationary convection, the compressible Rivlin-Ericksen elastico-viscous fluid behaves like an ordinary Newtonian fluid in the limit as the elastico-viscous parameter F vanishes with σ.

(ii) Expressions for $\frac{\partial R_1}{\partial B}$, $\frac{\partial R_1}{D_{A_1}}$ and $\frac{\partial R_1}{\partial P}$ have been examined analytically and it has been found that the Darcy-Brinkman number has a stabilizing effect whereas the suspended particles and medium permeability have a destabilizing effect on the system.

(iii) Oscillatory modes are introduced due to presence of viscoelasticity, suspended particles, gravity and medium permeability.

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Appendix A: Nomenclature

d	Depth of fluid layer, m
F	Dimensionless kinematic viscoelasticity
P_l	Dimensionless medium permeability
g	Gravitational acceleration, ms^{-2}
n	Growth rate of the disturbance, s^{-1}
C_{pt}	Heat capacity of particle, $Jkg^{-1}K^{-1}$
C_f	Heat capacity of fluid $Jkg^{-1}K^{-1}$
G	Dimensionless compressibility
m	Mass of suspended particle, kg
D_A	Darcy-Brinkman number
p	Pressure, Pa
C_p	Specific heat of the fluid at constant pressure, $Jkg^{-1}K^{-1}$
C_{pt}	Heat capacity of particle, $Jkg^{-1}K^{-1}$
K	Stokes drag coefficient
N	Suspended particle number density, m^{-3}
p_1	Thermal Prandtl number
v	Velocity of fluid, ms^{-1}
v_d	Velocity of suspended particles, ms^{-1}
k	Wave number of disturbance, m^{-1}

T	Temperature, K

Greek Symbols

β	Adverse temperature gradient, Km^{-1}
$\tilde{\mu}$	Effective viscosity of the porous medium, $kgm^{-1}s^{-1}$
ρ	Fluid density, kgm^{-3}
μ	Fluid viscosity, $kgm^{-1}s^{-1}$
$\mu^{'}$	Fluid viscoelasticity, $kgm^{-1}s^{-1}$
ν	Kinematic viscosity, m^2s^{-1}
$\nu^{'}$	Kinematic viscoelasticity, m^2s^{-1}
δ	Perturbation in respective physical quantity
θ	Perturbation in temperature, K
η	Radius of suspended particles, m
κ	Thermal diffusitivity, m^2s^{-1}
α	Thermal coefficient of expansion, K^{-1}