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Effects of Non-Newtonian Micropolar Fluids on the Dynamic Characteristics of Wide Tapered-Land Slider Bearings

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Abstract: We investigate the influence of non-Newtonian micropolar fluids on the dynamic characteristics of wide tapered-land slider bearings. The study is carried out on the basis of the micro-continuum theory originally developed by Eringen (1966). Analytical expressions for the linear dynamic coefficients are provided and compared with earlier results in the literature. In particular, direct comparison with the Newtonian fluid-lubricated tapered-land bearings by Lin et al. (2006) indicates that the use of non-Newtonian micropolar fluids can lead to a significant increase in the values of stiffness and damping coefficients. Such improvements are found to be even more pronounced for larger values of the non-Newtonian parameters. Moreover, comparison with the non-Newtonian micropolar fluid-lubricated bearings with an inclined plane film by Naduvinamani and Marali (2007), leads to the conclusion that tapered-land bearings with large geometric parameters have higher dynamic stiffness coefficients. Furthermore, such bearings can provide better damping characteristics with respect to the case of inclined-plane bearings.

Keywords: micropolar fluids, stiffness coefficients, damping coefficients

1 Introduction

Slider bearings are generally designed to support axial thrust in rotor bearing systems. By using a Newtonian fluid as the lubricant, the steady state characteristics of slider bearings have been investigated for different film shapes, such as the study of Hamrock (1994) and Lin (2001). Taking into account the effects of squeezing action, the dynamic characteristics of a tapered-land slider bearing

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are further analyzed by Lin *et al.* (2006). Expressions for dynamic stiffness and damping coefficients varying with bearing parameters are derived though a small perturbation method. Owing to the development of modern engineering, the use of non-Newtonian fluids as lubricants to improve the lubrication performance of bearing systems is becoming of great interest. Common lubricants displaying non-Newtonian natures are the bio-fluids, liquid crystals, polymer-thickened oils, and oils mixed with additives. In order to describe the flow behavior of these kinds of non-Newtonian fluids, a micro-continuum theory of micropolar fluids has been generated by Eringen (1964, 1966). This micropolar fluid model can describe the inertial characteristics of the substructure particles and allow the presence of the local rotational inertia, the body couples and the couple stresses. In addition, it can also be applied to describing the flow behavior of animal bloods, colloidal fluids, and oils with certain additives. Based on the Eringen's theory of micropolar fluids, Bayada and Lukaszewicz (1996) derived an analogue of the classical Reynolds equation of the lubrication theory. Through the procedure of asymptotic analysis, Bayada *et al.* (2005) derived a generalized micropolar Reynolds equation. Performance characteristics of slider bearing are obtained. Many investigators have also applied the Eringen's theory of micropolar fluids to study the non-Newtonian effects on the bearing characteristics, for example, the squeeze film bearings by Nigam *et al.* (1982), Al-Fadhalah and Elsharkawy (2008), Ashraf *et al.* (2009), Lin *et al.* (2010); the journal bearings by Zaheeruddin and Isa (1978) and Das *et al.* (2005); and the slider bearings by Isa and Zaheeruddin (1978), Agrawal and Bhat (1980), Naduvinamani and Marali (2007) and Lin *et al.* (2012). According to their results, the non-Newtonian effects of micropolar fluids upon the bearing performances are not negligible. In order to provide more information for engineers in bearing selection, a further investigation is motivated on the non-Newtonian dynamic characteristics of slider bearings with a tapered-land film profile.

On the basis of the micro-continuum theory generated by Eringen (1966), the influences of non-Newtonian micropolar fluids on the dynamic characteristics of wide tapered-land slider bearings are investigated in this study. Applying the technique of linear theory, analytical solutions for dynamic coefficients will be derived. Comparing with the case of a Newtonian lubricant, the dynamic characteristics of tapered-land slider bearings are provided and discussed through the variation of non-Newtonian parameters. For engineering applications, the comparison of the dynamic characteristics varying with the shoulder parameter for different film shapes of slider bearings is also provided.

2 Description of the problem

Figure 1 shows the physical configuration of a wide tapered-land slider bearing lubricated with a non-Newtonian micropolar fluid including the effects of squeeze motion $\partial h/\partial t$. The film thickness can be expressed as:

$$h(x,t) = h_\alpha(x) + h_m(t) \tag{1}$$

where $h_\alpha(x)$ is the film shape function of the inclined part for the bearing, and $h_m(t)$ is the minimum film thickness.

$$h_\alpha(x) = \begin{cases} d \cdot [1 - x/(\alpha A)], & 0 \le x \le \alpha A \\ 0, & \alpha A \le x \le A \end{cases} \tag{2}$$

The symbol d is the shoulder height denoting the steady inlet-outlet thickness difference, and α is the geometric parameter denoting the ratio between the inclined-part length and the total length of the bearing.

$$d = h_{10} - h_{m0} \tag{3}$$

$$\alpha = (\alpha A)/A \tag{4}$$

where the subscript "0" denotes the steady state.

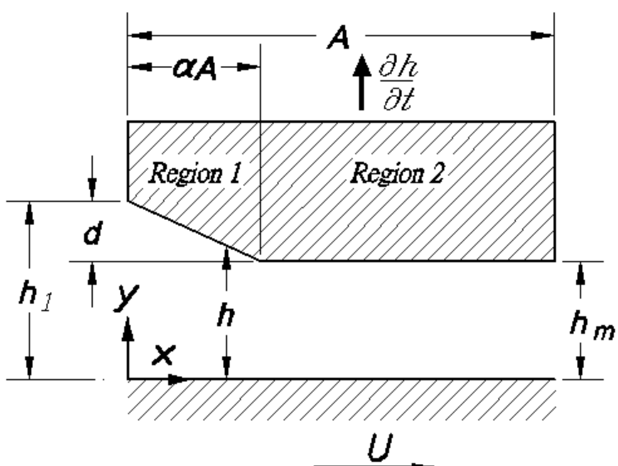

Figure 1: Geometry of a tapered-land slider bearing lubricated with a micropolar fluid.

3 Analysis

The lubricant for the wide bearing is taken to be an incompressible micropolar fluid. It is assumed that (a) the hydrodynamic thin-film lubrication theory is applicable; (b) the film thickness h is small as compared to the length of the slider bearing A; (c) the body forces and the body couples are absent; (d) the inertia terms are small as compared to the viscous terms; and (e) the variation of pressure across the film thickness is negligible, $\partial p/\partial y = 0$. According to the micro-continuum theory of micropolar fluids of Eringen (1966), the field equations expressed in two dimensional rectangular coordinates can be written as:

Linear momentum: $$\frac{1}{2}(2\mu+\chi)\frac{\partial^2 u}{\partial y^2}+\chi\frac{\partial \upsilon_z}{\partial y}=\frac{\partial p}{\partial x} \tag{5}$$

ngular momentum: $$\gamma\frac{\partial^2 \upsilon_z}{\partial y^2}-\chi\frac{\partial u}{\partial y}-2\chi\upsilon_z=0 \tag{6}$$

Continuity equation: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

where p is the fluid film pressure, u and v are the velocity components in the $x-$ and $y-$directions, v_z is the micro-rotational velocity component in the $z-$direction, μ is the classical viscosity coefficient, and γ and χ are the spin gradient viscosity coefficient and the vortex viscosity coefficient of micropolar fluids, respectively. As the value of the spin gradient viscosity coefficient is equal to zero, the effects of non-Newtonian micropolar fluids vanish; and the momentum equations (5) and (6) reduce to the classical momentum equation.

The boundary conditions for the velocity components and the micro-rotational velocity component are

$$u=U,\quad v=0,\quad \upsilon_z=0,\quad \text{at}\quad y=0 \tag{8}$$

$$u=0,\quad v=\frac{\partial h}{\partial t},\quad \upsilon_z=0,\quad \text{at}\quad y=h \tag{9}$$

The zero boundary conditions for $v=0$ at $y=0$ and for $u=0$ at $y=h$ satisfy the non-slip conditions. The zero boundary conditions for $\upsilon_z=0$ at $y=0$ and $y=h$ describe that the micro-rotational velocity of micropolar fluids vanishes at the solid boundaries of bearing surfaces.

Equations (5) and (6) are solved simultaneously by applying the corresponding velocity boundary conditions. The procedure is summarized as follows: (a) Taking the partial derivative of equation (6) with y yields an equation including the term $\partial^2 u/\partial y^2$; (b) Using equation (5), the term $\partial^2 u/\partial y^2$ can be eliminated; (c) Then a third order equation containing the terms $\partial^3 \upsilon_z/\partial y^3$ and $\partial \upsilon_z/\partial y$ can solved; (d) Substituting the expression of υ_z into equation (6), one can derive the expression of u. After an arrangement, the expressions of velocity components can be obtained.

$$u = U + \frac{y^2}{2\mu}\frac{\partial p}{\partial x} + \frac{C^2}{m}\left\{a \cdot \sinh\left[my\left(1 - \frac{1}{C^2}\right)\right] - b \cdot \cosh\left(my - 1\right)\right\} \tag{10}$$

$$\upsilon_z = \frac{h}{2\mu}\frac{\partial p}{\partial x}\left\{\frac{y}{h} - \frac{\sinh(my)}{\sinh(mh)} + \frac{1}{2}\left[\tanh(0.5my) - \tanh(0.5mh)\right] \cdot \sinh(my)\right\} \tag{11}$$

where

$$a = \frac{h}{2\mu}\frac{\partial p}{\partial x} + \frac{U}{h - 2C^2 \tanh(0.5mh)/m} \tag{12}$$

$$b = \frac{h}{2\mu}\frac{\partial p}{\partial x}\coth(0.5mh) + \frac{U}{h\coth(0.5mh) - 2C^2/m} \tag{13}$$

$$m = \frac{C}{l} \tag{14}$$

The symbol l can be regarded as the characteristic material length of micropolar fluids, and C is defined as the non-dimensional coupling parameter relating the vortex viscosity coefficient to the classical viscosity.

$$l = (\gamma/4\mu)^{1/2} \tag{15}$$

$$C = [\chi/(2\mu + \chi)]^{1/2} \tag{16}$$

Substitute the velocity component into the integrated continuity equation (7) across the film thickness and use the corresponding boundary conditions.

$$\int_{y=0}^{h} \frac{\partial u}{\partial x} dy = -\int_{y=0}^{h} \frac{\partial v}{\partial y} dy \tag{17}$$

One can derive the non-Newtonian dynamic Reynolds equation of the tapered-land slider bearing lubricated with a micropolar-fluid.

$$\frac{\partial}{\partial x}\left\{\left[h^3 + 12l^2 h - 6Clh^2 \coth(0.5Ch/l)\right]\frac{\partial p}{\partial x}\right\} = 6\mu U\frac{\partial h}{\partial x} + 12\mu\frac{\partial h}{\partial t} \tag{18}$$

In addition, the volume flow rate in the $x-$direction can also be obtained after integrating the velocity component u.

$$Q = \frac{1}{12\mu}\left\{6\mu UBh - \left[h^3 + 12l^2 h - 6Clh^2 \coth(0.5Ch/l)\right] B\frac{\partial p}{\partial x}\right\} \tag{19}$$

where B denotes the width of the bearing. In order to conveniently analyze the bearing characteristics, the non-dimensional variables and parameters are introduced as follows.

$$x^* = \frac{x}{A}, \quad h^* = \frac{h}{h_{m0}}, \quad h_m^* = \frac{h_m}{h_{m0}}, \quad t^* = \frac{Ut}{A}, \tag{20}$$

$$p^* = \frac{ph_{m0}^2}{\mu UA}, \quad Q^* = \frac{Q}{Uh_{m0}B}, \quad \delta = \frac{d}{h_{m0}}, \quad I = \frac{l}{h_{m0}} \tag{21}$$

As a result, the non-dimensional Reynolds equation and the volume flow rate can be written as:

$$\frac{\partial}{\partial x^*}\left\{\phi(h^*,I,C)\frac{\partial p^*}{\partial x^*}\right\} = 6\frac{\partial h^*}{\partial x^*} + 12V^* \tag{22}$$

$$Q^* = \frac{1}{12}\left\{6h^* - \phi(h^*,I,C)\frac{\partial p^*}{\partial x^*}\right\} \tag{23}$$

where

$$\phi(h^*,I,C) = h^{*3} + 12I^2h^* - 6CIh^{*2}\coth(0.5Ch^*/I) \tag{24}$$

$$h^*(x^*,t^*) = h_\alpha^*(x^*) + h_m^*(t^*) \tag{25}$$

Region 1:

$$h_\alpha^*(x^*) = \delta \cdot (1 - x^*/\alpha), \qquad 0 \le x^* \le \alpha \tag{26}$$

Region 2:

$$h_\alpha^*(x^*) = 0, \qquad \alpha \le x^* \le 1 \tag{27}$$

The symbol $V^* = dh_m^*/dt^*$ displays the non-dimensional squeezing velocity in the vertical direction, δ is the shoulder parameter describing the ratio between the shoulder height and the steady outlet film thickness, and I is the interacting parameter defining the ratio between the characteristic material length and the steady outlet film thickness. The non-dimensional Reynolds equation (22) agrees with the derivation of Lin *et al.* (2012) for a parabolic-film slider bearings with a micropolar fluid model. In addition, when the interacting parameter Ior and the coupling parameter C approach zero, equations (22)-(27) reduce to the case of a Newtonian fluid-lubricated tapered-land bearing by Lin *et al.* (2006).

4 Dynamic Stiffness and Damping Characteristics

The boundary conditions and the continuity conditions for the film pressure and volume flow rate are:

$$p_1^*(x^* = 0) = 0 \tag{28}$$

$$p_1^*(x^* = \alpha) = p_2^*(x^* = \alpha) \tag{29}$$

$$Q_1^*(x^* = \alpha) = Q_2^*(x^* = \alpha) \tag{30}$$

$$p_2^*(x^* = 1) = 0 \tag{31}$$

Integrating the non-dimensional Reynolds equation (17) and applying the above conditions, the dynamic film pressure can be obtained.

$$p^* = p_1^* + p_2^* \tag{32}$$

$$p_1^* = 6 \cdot f_1(x^*, h_m^*) + 12V^* \cdot f_2(x^*, h_m^*) + c(h_m^*, V^*) \cdot f_3(x^*, h_m^*), \quad 0 \le x^* \le \alpha \tag{33}$$

$$p_2^* = 12V^* \cdot f_4(x^*, h_m^*) + c(h_m^*, V^*) \cdot f_5(x^*, h_m^*), \quad \alpha \le x^* \le 1 \tag{34}$$

where the associated functions $f_1, \ldots, f_5$ and the integrating function c are defined in **Appendix A**. Integrating the dynamic film pressure yields the dynamic film force.

$$F = \int_{x=0}^{A} p \cdot B dx = \int_{x=0}^{\alpha A} p_1 \cdot B dx + \int_{x=\alpha A}^{A} p_2 \cdot B dx \tag{35}$$

In terms of a non-dimensional form, one has

$$F^* = \frac{F h_{m0}^2}{\mu U A^2 B} = \int_{x^*=0}^{1} p^* dx^* = \int_{x^*=0}^{\alpha} p_1^* dx^* + \int_{x^*=\alpha}^{1} p_2^* dx^* \tag{36}$$

After performing the integrations, the dynamic film force is obtained.

$$\begin{aligned} F^*(h_m^*, V^*) = {} & 6 \cdot F_1(h_m^*) + 12V^* \cdot [F_2(h_m^*) + F_4(h_m^*)] \\ & + c(h_m^*, V^*) \cdot [F_3(h_m^*) + F_5(h_m^*)] \end{aligned} \tag{37}$$

where the functions $F_1, \ldots, F_5$ are shown in **Appendix A**. The film force F^* varies with the outlet film thickness h_m^* and the squeezing velocity V^*. According to the linear theory, the dynamic stiffness and damping coefficients are calculated from the partial derivatives of F^* with respect to h_m^* and V^* respectively, and then take the results under the steady state "0": $h_{m0}^* = (h_m^*)_0 = const$ and $V^* = 0$. As a result:

$$K^* = \frac{K h_{m0}^3}{\mu U A^2 B} = -\left(\frac{\partial F^*}{\partial h_m^*}\right)_0 \tag{38}$$

$$D^* = \frac{Dh_{m0}^3}{\mu A^3 B} = -\left(\frac{\partial F^*}{\partial V^*}\right)_0 \tag{39}$$

As a result, one can obtain

$$K^* = -6\cdot\left(\frac{\partial F_1}{\partial h_m^*}\right)_0 - [(F_3)_0 + (F_5)_0]\cdot\left(\frac{\partial c}{\partial h_m^*}\right)_0 - (c)_0\cdot\left[\left(\frac{\partial F_3}{\partial h_m^*}\right)_0 + \left(\frac{\partial F_5}{\partial h_m^*}\right)_0\right] \tag{40}$$

$$D^* = -12\cdot[(F_2)_0 + (F_4)_0] - [(F_3)_0 + (F_5)_0]\cdot\left(\frac{\partial c}{\partial V^*}\right)_0 \tag{41}$$

where the associated functions are described in **Appendix A**.

5 Results and Discussion

According to the above analysis, and the tapered-land bearing characteristics are influenced by the shoulder parameter defined in equation (21): $\delta = d/h_{m0}$, the geometric parameter defined in equation (4): $\alpha = \alpha A/A$, the interacting parameter defined in equation (21): $I = l/h_{m0}$, and the coupling parameter defined in equation (16): $C = [\chi/(2\mu+\chi)]^{1/2}$.

(1) For the values of $\delta \neq 0$, α, I=0 or C=0: the present study reduces to the Newtonian fluid-lubricated tapered-land bearing case by Lin *et al.* (2006).

(2) For the values of $\delta \neq 0$, α=1, $I \neq 0$ and $C \neq 0$: the present study reduces to the case of inclined-plane bearings lubricated with a micropolar fluid by Naduvinamani and Marali (2007).

Figure 2 illustrates the variation of the dynamic stiffness coefficient K^* with with the geometric parameter α for different values of the interacting parameter I and the coupling parameter C under δ=1 and h_{m0}^*=0.5. The stiffness coefficient increases gradually with the geometric parameter until a critical value is obtained and thereafter decreases with the geometric parameter. Comparing with the case of a Newtonian lubricant, the influences of micropolar fluids (I=0.1, C=0.5) provide an increase the bearing stiffness. Increasing values of the interacting parameter and the coupling parameter (I=0.5, C=0.5; I=0.5, C=0.8) increases the non-Newtonian effects on the value of the stiffness coefficient.

Figure 3 describes the variation of the dynamic damping coefficient D^* with the geometric parameter α for different values of the interacting parameter I and the coupling parameter C under δ=1 and h_{m0}^*=0.5. It is shown that the damping coefficient decreases with increasing values of the geometric parameter. Comparing with the Newtonian-lubricant case, the effects of non-Newtonian micropolar fluids (I=0.1,

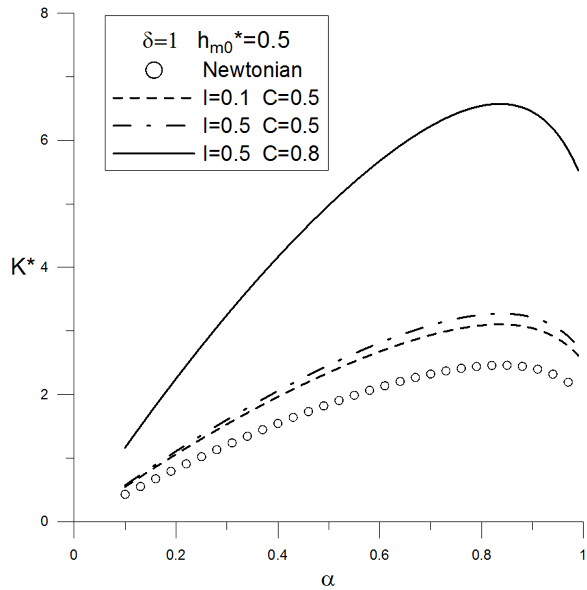

Figure 2: Variation of the dynamic stiffness coefficient K^* with α for different I and C.

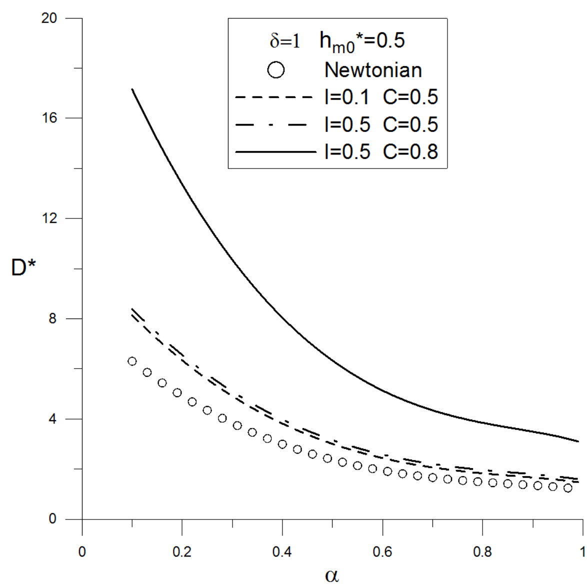

Figure 3: Variation of the dynamic damping coefficient D^* with α for different I and C.

C=0.5; I=0.5, C=0.5) yield higher values of the damping coefficient. The improvements in damping characteristics are more emphasized especially for a larger value of the interacting parameter and the coupling parameter (I=0.5, C=0.8).

Figure 4 shows the variation of the stiffness coefficient K^* and the damping coefficient D^* with the coupling parameter C under δ=1, and α=0.82. Under the film thickness h^*_{m0}=0.5, the effects of the interacting parameter (I=0.5) are observed to result in higher values of the stiffness and damping coefficients for the bearing with larger values of the coupling parameter C. Furthermore, decreasing the film thickness down to h^*_{m0}=0.4 provides larger increments of the stiffness and damping coefficients arising from the non-Newtonian influences of micropolar fluids.

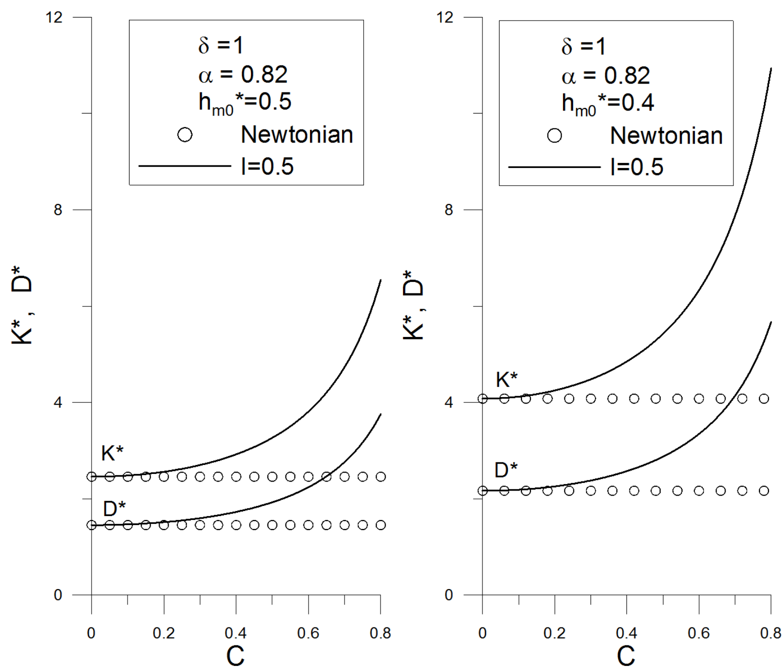

Figure 4: Variation of the dynamic stiffness and damping coefficients with C.

Figure 5 describes the variation the stiffness coefficient K^* and the damping coefficient D^* with the interacting parameter I under δ=1, and α=0.82. Under the film thickness h^*_{m0}=0.5, the effects of the coupling parameter (C=0.5) are seen to provide an apparent increase in the stiffness and damping coefficients as compared to the case of a Newtonian lubricant. Decreasing the film thickness down to h^*_{m0}=0.4, the influences of non-Newtonian micropolar fluids on the dynamic coefficients are more pronounced.

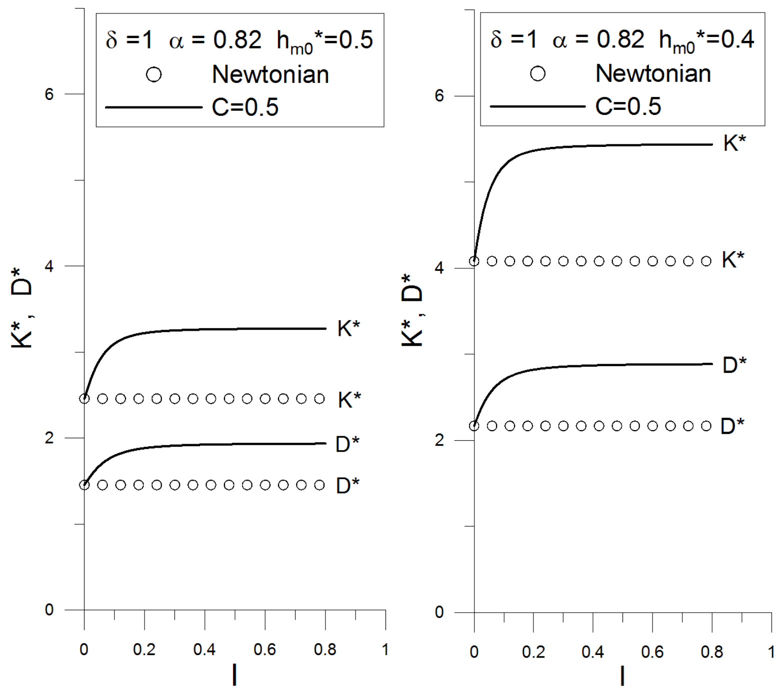

Figure 5: Variation of the dynamic stiffness and damping coefficients with I.

Recently, the dynamic performances of inclined-plane slider bearings with micropolar fluids have been contributed by Naduvinamani and Marali (2007). Using their derivations and using the same non-dimensional definitions of the present study, bearing characteristics can be evaluated. Figures 6 presents the comparison of the dynamic stiffness coefficient K^* with the inclined-plane slider bearing under C=0.5, I=0.5 and h^*_{m0}=0.5. It is observed that the values of the stiffness coefficient K^* for tapered-land bearings with α=0.55 are close to those of the inclined-plane bearings. However, the tapered-land bearings with $\alpha > 0.55$ result in higher values of the dynamic stiffness coefficient. Figures 7 describes the comparison of the dynamic damping coefficient D^* with the inclined-plane slider bearing under C=0.5, I=0.5 and h^*_{m0}=0.5. It is shown that the tapered-land bearings provide better damping characteristics even with small values of the geometric parameter α as compared to the inclined-plane bearings.

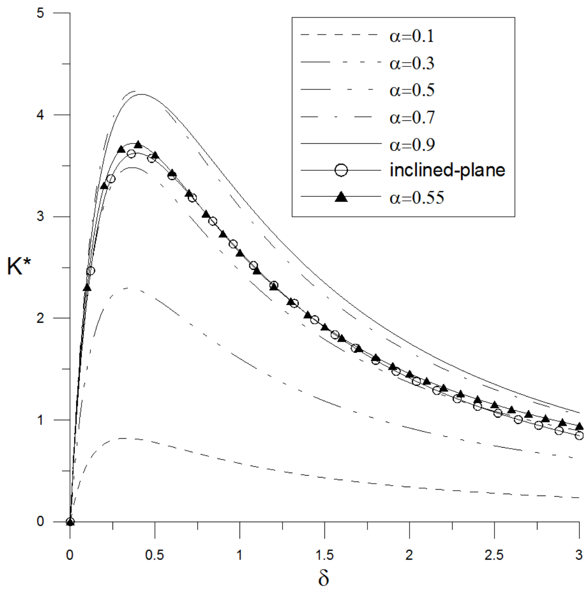

Figure 6: Comparison of the dynamic stiffness coefficient K^* with the inclined-plane slider bearing under C=0.5, I=0.5 and h^*_{m0}=0.5.

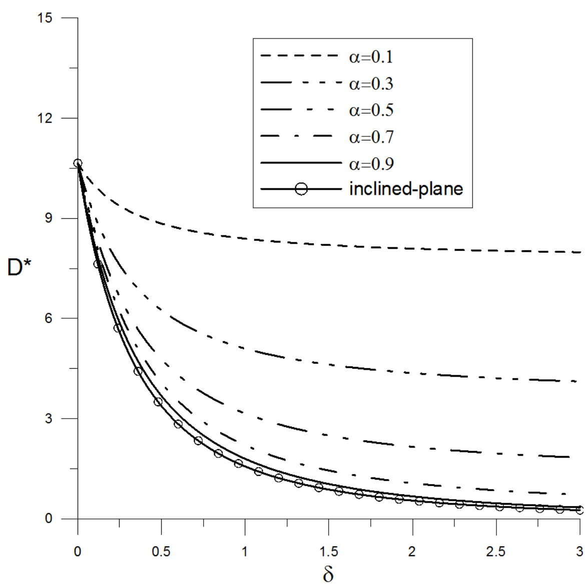

Figure 7: Comparison of the dynamic damping coefficient D^* with the inclined-plane slider bearing under C=0.5, I=0.5 and h^*_{m0}=0.5.

6 Conclusions

Based on the micro-continuum theory of Eringen (1966), the non-Newtonian effects of micropolar fluids on the dynamic characteristics of wide tapered-land slider bearings have been investigated. By applying the linear theory, analytical expressions for the dynamic coefficients have been derived. Comparing with the Newtonian fluid-lubricated tapered-land bearings by Lin *et al.* (2006), the non-Newtonian micropolar lubricants provide an increase in the values of stiffness and damping coefficients. The improvements on the bearing dynamic characteristics are further emphasized for larger interacting parameters and coupling parameters. Comparing with the non-Newtonian micropolar fluid-lubricated bearings with an inclined plane film by Naduvinamani and Marali (2007) the tapered-land bearings with larger geometric parameters result in higher values of the dynamic stiffness coefficient. In addition, the tapered-land bearings provide better damping characteristics even with small values of the geometric parameter as compared to the inclined-plane bearings.

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Appendix A

The associated functions involved for p^*, F^*, K^* and D^*

$$f_1(x^*,h_m^*) = \int_{x^*=0}^{x^*} \frac{\delta\cdot(1-x^*/\alpha)}{\phi(h^*,I,C)}dx^* \tag{A1}$$

$$f_2(x^*,h_m^*) = \int_{x^*=0}^{x^*} \frac{x^*}{\phi(h^*,I,C)}dx^* \tag{A2}$$

$$f_3(x^*,h_m^*) = \int_{0}^{x^*} \frac{1}{\phi(h^*,I,C)}dx^* \tag{A3}$$

$$f_4(x^*,h_m^*) = \int_{x^*=1}^{x^*} \frac{x^*}{\phi(h^*,I,C)}dx^* \tag{A4}$$

$$f_5(x^*,h_m^*) = \int_{x^*=1}^{x^*} \frac{1}{\phi(h^*,I,C)} dx^* \tag{A5}$$

$$F_1(h_m^*) = \int_{x^*=0}^{\alpha} \int_{x^*=0}^{x^*} \frac{\delta \cdot (1 - x^*/\alpha)}{\phi(h^*,I,C)} dx^* dx^* \tag{A6}$$

$$F_2(h_m^*) = \int_{x^*=0}^{\alpha} \int_{x^*=0}^{x^*} \frac{x^*}{\phi(h^*,I,C)} dx^* dx^* \tag{A7}$$

$$F_3(h_m^*) = \int_{x^*=0}^{\alpha} \int_{0}^{x^*} \frac{1}{\phi(h^*,I,C)} dx^* dx^* \tag{A8}$$

$$F_4(h_m^*) = \int_{x^*=\alpha}^{1} \int_{x^*=1}^{x^*} \frac{x^*}{\phi(h^*,I,C)} dx^* dx^* \tag{A9}$$

$$F_5(h_m^*) = \int_{x^*=\alpha}^{1} \int_{x^*=1}^{x^*} \frac{1}{\phi(h^*,I,C)} dx^* dx^* \tag{A10}$$

$$c(h_m^*,V^*) = \frac{12V^* \cdot [f_4(\alpha,h_m^*) - f_2(\alpha,h_m^*)] - 6 \cdot f_1(\alpha,h_m^*)}{f_3(\alpha,h_m^*) - f_5(\alpha,h_m^*)} \tag{A11}$$

$$\frac{\partial f_{1\alpha}}{\partial h_m^*} = -\int_{x^*=0}^{\alpha} \delta \cdot (1 - x^*/\alpha) \cdot \phi^{-2} \frac{\partial \phi}{\partial h_m^*} dx^* \tag{A12}$$

$$\frac{\partial f_{3\alpha}}{\partial h_m^*} = -\int_{x^*=0}^{\alpha} \phi^{-2} \frac{\partial \phi}{\partial h_m^*} dx^* \tag{A13}$$

$$\frac{\partial f_{5\alpha}}{\partial h_m^*} = -\int_{x^*=1}^{\alpha} \phi^{-2} \frac{\partial \phi}{\partial h_m^*} dx^* \tag{A14}$$

$$\frac{\partial F_1}{\partial h_m^*} = -\int_{x^*=0}^{\alpha} \int_{x^*=0}^{x^*} \delta \cdot (1 - x^*/\alpha) \cdot \phi^{-2} \cdot \frac{\partial \phi}{\partial h_m^*} dx^* dx^* \tag{A15}$$

$$\frac{\partial F_3}{\partial h_m^*} = -\int_{x^*=0}^{\alpha} \int_{0}^{x^*} \phi^{-2} \cdot \frac{\partial \phi}{\partial h_m^*} dx^* dx^* \tag{A16}$$

$$\frac{\partial F_5}{\partial h_m^*} = -\int_{x^*=\alpha}^{1} \int_{x^*=1}^{x^*} \phi^{-2} \cdot \frac{\partial \phi}{\partial h_m^*} dx^* dx^* \tag{A17}$$

$$\left(\frac{\partial c}{\partial h_m^*}\right)_0 = \left\{ \frac{-6}{(f_{3\alpha} - f_{5\alpha})^2} \cdot \left[(f_{3\alpha} - f_{5\alpha}) \cdot \frac{\partial f_{1\alpha}}{\partial h_m^*} - f_{1\alpha} \cdot \left(\frac{\partial f_{3\alpha}}{\partial h_m^*} - \frac{\partial f_{5\alpha}}{\partial h_m^*} \right) \right] \right\}_0 \tag{A18}$$

$$\left(\frac{\partial c}{\partial V^*}\right)_0 = \frac{12 \cdot [-(f_{2\alpha})_0 + (f_{4\alpha})_0]}{(f_{3\alpha})_0 - (f_{5\alpha})_0} \tag{A19}$$

$$\frac{\partial \phi}{\partial h_m^*} = 12I^2 - 12CIh^* \coth\left(\frac{Ch^*}{2I}\right) + 3h^{*2} \left[1 + C^2 \operatorname{csc} h^2 \left(\frac{Ch^*}{2I}\right)\right] \tag{A20}$$