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Heat Transfer in FHD Boundary Layer Flow with Temperature Dependent Viscosity over a Rotating Disk

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Abstract: The present study is carried out to examine the effects of temperature dependent variable viscosity on the three dimensional steady axi-symmetric Ferrohydrodynamic (FHD) boundary layer flow of an incompressible electrically non-conducting magnetic fluid in the presence of a rotating disk. The disk is subjected to an externally applied magnetic field and is maintained at a uniform temperature. The nonlinear coupled partial differential equations governing the boundary layer flow are non dimensionalized using similarity transformations and are reduced to a system of coupled ordinary differential equations. To study the effects of temperature dependent viscosity on velocity profiles and temperature distribution within the generated boundary layer, solution of the problem is obtained by employing Finite Difference and Shooting methods, subsequently. Beside the flow profiles, skin friction coefficients, rate of heat transfer at the wall and the boundary layer displacement thickness are also calculated. All of the obtained results are validated, and discussed quantitatively and through graphs giving their physical interpretations.

Keywords: Ferrofluids, magnetic field, boundary layer flow, temperature dependent viscosity, finite difference method.

1 Introduction

Ferrohydrodymanics (FHD) is the branch of fluid dynamics investigating the flow behavior of magnetic fluids in the presence of magnetic field. An extensive research work has been going on in the area of fluid dynamics of non-conducting magnetic fluid since it was first discovered in 1960's. Because of the potential industrial

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applications of ferrofluids, since last five decades, the investigation on them fascinated the researchers and engineers, vigorously. Rosensweig (1985) has given an authoritative introduction to the research on magnetic liquids in his monograph and studied the effect of magnetization resulting in interesting information. However, general information on magneto-viscous effects in ferrofluids has been given in Odenbach (2002). Verma and Ram (1993) studied the flow of magnetic fluids through a helical pipe using space coordinates. Numerous applications of ferrofluids have been proposed in engineering, physical and medical sciences in Berkowsky et al. (1993). Hennenberg et al. (2007) examined the coupling between Cowley-Rosensweig and stationary Marangoni instabilies in a deformable ferrofluid layer. Ferrofluids are widely used in sealing of hard disc drives, rotating X-ray tubes and underwater robotic vehicle under engineering applications (Kim et al. 2010). The major applications of ferrofluid in electrical field is that controlling of heat in loudspeakers. Control on heating makes the life of sound speakers longer and increases the acoustical power without any change in its geometrical shape. Magnetic fluids are used in the contrast medium in X-ray examinations and for positioning tamponade for retinal detachment repair in eye surgery. Ferrofluids can be used to deliver certain drugs to specific area of human body and in treatment of cancer by heating the tumor soaked in ferrofluid by the way of alternating magnetic field. Therefore, ferrofluids play an important role in the field of bio-medical science also.

In fluid dynamics, much attention has been paid through research in literature on the fluid flow over a rotating disk. In fact, rotating disk flows of conducting and non-conducting fluids are not only of theoretical interest, but they are also of practical significance in many areas, such as rotating machinery, computer storage devices, air cleaning machines, medical equipments, gas turbine rotors and specially aerodynamics applications. The pioneering study of ordinary viscous fluid flow due to the infinite rotating disk was carried by Karman (1921). He introduced the famous transformation, which reduces the governing partial differential equations into ordinary differential equations. His rotating disk problem is extended to the case of flow started impulsively from rest, and also the steady state is solved to a higher degree of accuracy than previously done by a simple analytical method which neglects the resembling difficulties in Cochran (1934). Benton (1966) improved Cochran's solutions and also, solved the unsteady case. Chauhan and Agrawal (2010) studied the MHD flow in a parallel-plate channel partially filled with a porous medium in a rotating system including Hall current. Ram et al. (2010) solved the non-linear partial differential equations under Neuringer - Rosensweig model by using power series approximations and discussed the effect of magnetic field-dependent viscosity on velocity components and pressure profile. Shahmohamadi and Rashidi (2011) used variation iteration method to solve steady three dimensional problem of condensation film on inclined rotating disk. Further, disk driven ferrofluid flow saturating the porous medium is investigated by Ram and Kumar (2012). Effect of rotation on a ferrofluid flow over a rotating disk is studied by Ram and Sharma (2012). Swirling flow of co/-counter rotating disks in a cylindrical enclosure with vertical temperature graident is examined by Mahfoud and Bessaih (2012). Hayat et al. (2012) investigated the axi-symmetric flow of MHD third grade fluid between two permeable disks by solving the involved differential equations using homotopy analysis method. Ram and Bhandari (2012) examined the revolving ferrofluid flow saturating a porous medium over a stationary disk.

In all these studies, the viscous property of the fluid was assumed to be independent of temperature. But significant variation may be recorded in this physical property due to change in temperature. So to characterize a realistic flow behavior, it becomes necessary to consider this variation in viscosity. In history, many researchers have considered the effect of this variation in vicosity in their studies. Ramanathan and Muchikel (2006) studied the effect of temperature dependent viscosity on ferroconvective instability in a porous medium. Hooman and Gurgenci (2008) used the exponential viscosity temperature relation to discuss the effects of temperature dependent viscosity on the forced convection of a liquid. The effects of depth and temperature dependent viscosity and Hall current on an unsteady flow of an electrically conducting fluid due to rotating disk have been investigated by Maleque (2010). Yuan et al. (2009) developed a method to study the influence of temperature dependent viscosity on biodiesel fuels. Duangthongsuk and Wongwises (2009) measured the thermo-physical properties, temperature dependent viscosity and thermal conductivity for the water based nanofluids with an idea to use these properties to enhance the rate of heat transfer. The behavior of stably stratified turbulent channel flow with temperature dependent fluid properties is studied by Zonta et al. (2012). Vajravelu et al. (2013) examined the effects of temperature dependent viscosity on flow and heat transfer in nanofluid over a flat surface.

The study of heat transfer in boundary layer flows is of great significance in various engineering applications such as drag reduction, transpiration, the design of thrust bearings and radial diffusers etc. Attia (2006) studied the flow and heat transfer of a conducting non-Newtonian fluid above a rotating disk with consideration of ion slip. The disk driven steady flow and heat transfer of the power-law fluid is examined by Ming et al. (2011). Rashidi et al. (2012) employed the Homotopy Analysis method to obtain the analytical approximate solutions of fluid flow in porous medium and heat transfer. Flow in a square cavity with heat tranfer influenced by the porous layer is investigated by Hamimid et al. (2012). Ram and Kumar (2013) investigated the heat transfer in ferrofluid boundary layer over a stretchable rotating disk.

In the present work, we have considered the forced convection heat transfer in the boundary layer flow of ferrofluid due to a rotating disk with temperature dependent variable viscosity. The non linear partial differential equations governing the fluid flow are non-dimensionalised using similarity transformations and then solved using the Finite difference and Shooting Methods subsequently in MATLAB environment. All the results are discussed with their graphical representations under section "*Discussion of Results*". This problem concerning the effects of temperature dependent viscosity on ferrofluids boundary layer over a heated rotating disk, to the best of our knowledge, has not been investigated yet.

2 Mathematical Formulation of the Problem

Here, an impermeable electrically non-conducting disk of infinite radius is placed at $z = 0$. The disk is considered to be rotating with a constant angular velocity ω about the z-axis, the axis of rotation and normal to plane of the disk. Also the space $z > 0$ is filled with the viscous incompressible electrically non-conducting ferrofluid. Cylinderical polar coordinates (r, ϕ, z) are used to represent the problem mathematically. The velocity of fluid is $\overrightarrow{q}$ with u, v and w as the radial, tangential and axial components, respectively. The disk is maintained at a uniform temperature T_w, and the fluid, far away from the disk is at constant termperature T_∞. The flow field is subjected to an externally applied magnetic field $\overrightarrow{H}$, and the magnetization $\overrightarrow{M}$ is aligned with the field. The viscosity of the ferrofluid is taken to be temperature dependent variable viscosity and the thermal conductivity as constant. The rotation of the disk within a ferrofluid at rest gives rise to the boundary layer on the surface of disk. Due to viscosity of the ferrofluid and no slip condition, the fluid layer just in contact with the surface of disk also rotates with the same angular velocity as that of the disk.

The constitutive equations for steady and axi-symmetric ferrofluid flow with tempeatrure dependent viscosity are given as follows:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\begin{aligned} &\rho\left(u\frac{\partial u}{\partial r} - \frac{v^2}{r} + w\frac{\partial u}{\partial z}\right) \\ &= -\frac{\partial p}{\partial r} + \mu_0\left|\overrightarrow{M}\right|\frac{\partial}{\partial r}\left|\overrightarrow{H}\right| + \frac{\partial}{\partial r}\left(\mu_T\frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial r}\left(\mu_T\frac{u}{r}\right) + \frac{\partial}{\partial z}\left(\mu_T\frac{\partial u}{\partial z}\right) \end{aligned} \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial r} + \frac{uv}{r} + w\frac{\partial v}{\partial z}\right) = \frac{\partial}{\partial r}\left(\mu_T\frac{\partial v}{\partial r}\right) + \frac{\partial}{\partial r}\left(\mu_T\frac{v}{r}\right) + \frac{\partial}{\partial z}\left(\mu_T\frac{\partial v}{\partial z}\right) \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)$$
$$=-\frac{\partial p}{\partial z}+\mu_0\left|\overrightarrow{M}\right|\frac{\partial}{\partial z}\left|\overrightarrow{H}\right|+\frac{\partial}{\partial r}\left(\mu_T\frac{\partial w}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}(\mu_T w)+\frac{\partial}{\partial z}\left(\mu_T\frac{\partial w}{\partial z}\right) \tag{4}$$

$$k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial z^2}\right)=\rho C_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right) \tag{5}$$

The boundary conditions for the flow are

$$\left.\begin{array}{l} u=0,\ v=r\omega,\ w=0,\ T=T_w\ \ at\ z=0 \\ u\to 0,\ v\to 0,\ T\to T_\infty\ \&\ w \text{ tends to some finite neagative value as } z\to\infty \end{array}\right\} \tag{6}$$

In above equations, ρ is the fluid density, p is the reduced fluid pressure, μ_0 is free space magnetic permeability, μ_T is temperature dependent viscosity, C_p is the specific heat at constant pressure and k is thermal conductivity.

For viscous fluids, Hooman and Gurgenci (2008) has suggested an exponential viscosity-temperature relation as $\mu_T=\mu_\infty\exp(-b\theta)$, where $\theta=(T-T_\infty)/(T_w-T_\infty)$ is the dimensionless temperature, and T_w & T_∞ are wall temperature and temperature of free fluid stream, respectively.

This viscosity-temperature relation, on expansion, takes the form

$$\mu_T=\mu_\infty(1-b\theta) \tag{7}$$

where b, a non negative real number, is the viscosity variation parameter.

The flow of the ferrofluid rotating due to rotation of an infinite disk is in equilibrium under the influence of centrifugal force which is balanced by the resultant of a radial pressure gradient and radial component of the magnetic body force. So, the boundary layer approximation to (2) is as follows:

$$-\frac{\partial p}{\partial r}+\mu_0\left|\overrightarrow{M}\right|\frac{\partial}{\partial r}\left|\overrightarrow{H}\right|=-r\omega^2 \tag{8}$$

On considering negligible variation in the magnetic field in z-direction and using the similarity transformations [Karman (1921)]

$$\begin{array}{l} u=r\omega E(\alpha),\ v=r\omega F(\alpha),\ w=\sqrt{\upsilon_\infty\omega}\,G(\alpha), \\ p=\rho\omega\upsilon_\infty P(\alpha),\ T-T_\infty=\Delta T\,\theta(\alpha) \end{array} \tag{9}$$

where $\Delta T=T_w-T_\infty$, $\alpha=z\sqrt{\frac{\omega}{\upsilon_\infty}}$ and υ_∞ is the reference kinematic viscosity in the set of equations (1) – (5), we get a system of non-linear differential equations in dimensionless variables $E,F,G,$ and θ as:

$$G'+2E=0 \tag{10}$$

$$(1-b\theta)E''-bE'\theta'-E^2+F^2-GE'-1=0 \tag{11}$$

$$(1-b\theta)F''-bF'\theta'-2EF-GF'=0 \tag{12}$$

$$P'-(1-b\theta)G''+bG'\theta'+GG'=0 \tag{13}$$

$$\theta''-\Pr\, G\theta'=0 \tag{14}$$

where $\Pr = \frac{\mu_\infty C_p}{k}$ is the Prandtl number.

And, the boundary conditions (6) reduce to dimensionless boundary conditions as:

$$\left.\begin{array}{l} E(0)=0,\ F(0)=1,\ G(0)=0,\ P(0)=0,\ \theta(0)=1 \\ E\to 0,\ F\to 0,\ G\to -c,\ P\to 0 \ and\ \theta\to 0\ as\ \alpha\to\infty,\ (c>0) \end{array}\right] \tag{15}$$

3 Numerical Solution

The system of nonlinear coupled differential equations (10) – (14) along with the boundary conditions (15) is solved numerically in the semi infinite domain $[0, \infty)$ leaving the equation (13), as the dimensionless fluid pressure P can be found directly from it, once the dimensionless vertical component of velocity and temperature is made known. We adopted the second order numerical scheme which combines the features of Finite difference method and Shooting method (Ariel 1992; Ram and Kumar 2012) where the centeral differences used in dicretization ensure the accuracy of the method. For finiding numerical solution, the semi infinite integration domain $\alpha \in [0, \infty)$ is replaced by finite domain $\alpha \in [0, \infty)$. It is to be noted that if this numerical infinity α_∞ is not taken large enough, the numerical solution will not only depend upon the parameters b and Pr, but also on this α_∞. A finite value large enough is taken for this numerical infinity α_∞which ensures that solution is independent of this value and closely approximate the asymptotic boundary conditions. The value $\alpha_\infty = 7.5$ is found suitable to simulate $\alpha = \infty$ for all considered values of physical parameters.

Now discretizing the nonlinear coupled differential equations (10) – (12) and (14) by approximating the first and second order differential coefficients with central differences for the meshes defined by $\alpha_i = ih$ $(i = 1, 2, ..., n)$, h being the mesh size, we get

$$G_{i+1}=G_i-h(E_i+E_{i+1}) \tag{16}$$

$$\begin{aligned} &\frac{1}{h^2}(1-b\theta_i)(E_{i+1}-2E_i+E_{i-1})-\frac{b}{4h^2}(E_{i+1}-E_{i-1})(\theta_{i+1}-\theta_{i-1}) \\ &-E_i^2+F_i^2-\frac{1}{2h}G_i(E_{i+1}-E_{i-1})-1=0 \end{aligned} \tag{17}$$

$$\frac{1}{h^2}(1-b\theta_i)(F_{i+1}-2F_i+F_{i-1})-\frac{b}{4h^2}(F_{i+1}-F_{i-1})(\theta_{i+1}-\theta_{i-1})$$
$$-2E_iF_i-\frac{1}{2h}G_i(F_{i+1}-F_{i-1})=0 \tag{18}$$

$$\frac{1}{h^2}(\theta_{i+1}-2\theta_i+\theta_{i-1})-\frac{\Pr}{2h}G_i(\theta_{i+1}-\theta_{i-1})=0 \tag{19}$$

In the process of discretization, equations (11), (12) & (14) are written at i^{th} mesh point by approximating the first and second derivatives with central differences centered at i^{th} mesh point, while equation (10) is written at $(i+1/2)^{th}$ mesh point by approximating first derivative with the difference quotient at i^{th} and $(i+1)^{th}$ mesh points, and right hand side is approximated by the respective averages at the same two mesh points.

Now in order to start recursion in equations (17) – (19) which are three term recurrence relations in variables $E,\ F\,\&\,\theta$, besides the values of $E_0,\ F_0\,\&\,\theta_0$, the values of $E_1,\ F_1\,\&\,\theta_1$ are also required. These values can be obtained by Taylor's series expansion of $E,\ F\,\&\,\theta$ near $\alpha=0$ and assuming

$$E'(0)=l_1,\ F'(0)=l_2,\ \theta'(0)=l_3 \tag{20}$$

Thus, we have
$E_1=E(0)+hE'(0)+\frac{h^2}{2}E''(0)+\cdots,\ F_1=F(0)+hF'(0)+\frac{h^2}{2}F''(0)+\cdots$ and $\theta_1=\theta(0)+h\theta'(0)+\frac{h^2}{2}\theta''(0)+\cdots$.

Here the values of $E(0)$, $F(0)$ & $\theta(0)$ are given in equation (15) and the values of $E''(0),\ F''(0)$ & $\theta''(0)$ can be obtained directly from equations (11), (12) & (14) with the help of equation (20). After getting the values of $E_1,\ F_1\,\&\,\theta_1$, the integration can be performed as: first of all, G_1 is obtained from equation (16), and then this value of G_1 can be used in equations (17) – (19) to get the values of $E_2,\ F_2$ & θ_2, respectively. In the next cycle, G_2 is obtained from equation (16), and using the value of G_2 in equations (17) – (19), we get the values of $E_3,\ F_3$ & θ_3, respectively. This order of computation is followed in the subsequent cycles until the values of $E,\ F,\ G\,\&\,\theta$ are obtained at all of the mesh points.

Here, we need to satisfy the asymptotic boundary conditions (15), for which the values of $l_1,\ l_2\,\&l_3$ are obtained by shooting method along with Runge-Kutta method of fourth order, so as to fulfill the boundary conditions at $\alpha=\alpha_\infty$. In this process, the guesses on $E'(0)$, $F'(0)$ & $\theta'(0)$ can be improved by some suitable zero finding algorithm like variation of Secant method, Newton's method etc. But, these methods are quite sensitive to initial guess and require a lot more iterations. So, we have used the method given by Broyden (1965, 2000) as zero finding algorithms, in which Richardson's extrapolation method is used to hike the order of accuracy to $o(h^4)$.

Along with the solution of coupled differential equations, boundary layer displacement thickness, skin friction coefficients and the rate of heat transfer at the surface of rotating disk are also calculated. The viscous property of fluid layer adjacent to the plate sets up a stress which opposes the revolution of the fluid. Newtonian formulae are used to calculate the radial stress (τ_r) and the tangential shear stress (τ_t) as:

$$\tau_r = \left[\mu(T)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\right]_{z=0} = \mu_\infty(1-b)R_e^{1/2}\omega E'(0)$$

$$\tau_t = \left[\mu(T)\left(\frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}\right)\right]_{z=0} = \mu_\infty(1-b)R_e^{1/2}\omega F'(0)$$

So, the radial and tangential skin frictions are, respectively given by

$$(1-b)^{-1}R_e^{1/2}C_{f_r} = E'(0) \tag{21}$$

$$(1-b)^{-1}R_e^{1/2}C_{f_t} = F'(0) \tag{22}$$

where C_{f_r} and C_{f_t} are the coefficients of radial and tangential skin friction, respectively, and $R_e = {r^2\omega}/{\upsilon_\infty}$ is the local rotational Reynolds number.

Also the rate of heat transfer from the surface of the disk to the ferrofluid is calculated by using the Fourier's law given as:

$$q = -\left(k\frac{\partial T}{\partial z}\right)_{z=0} = -k\Delta T\sqrt{\frac{\omega}{\upsilon_\infty}}\theta'(0)$$

So, the Nusselt number (Nu) is given by

$$R_e^{1/2}\mathrm{Nu} = -\theta'(0) \tag{23}$$

The skin friction coefficients and the rate of heat transfer from surface of the disk are presented in Tab.1.

The boundary layer displacement thickness is calculated [Ref. Benton 1966] as

$$d = \frac{1}{r\omega}\int_0^\infty v\,dz = \int_0^\infty F(\alpha)\,d\alpha \tag{24}$$

After all above computation, we got the boundary layer thickness presented in Tab.2.

Table 1: Skin friction coefficients and Rate of heat transfer.

	For Pr = 1			**For Pr = 7**		
	$E'(0)$	$-F'(0)$	$-\theta'(0)$	$E'(0)$	$-F'(0)$	$-\theta'(0)$
***b* = 0**	0.51234	0.61542	0.32756	0.50981	0.62124	0.32102
***b* = 0.25**	0.55178	0.66387	0.37846	0.54887	0.66952	0.37314
***b* = 0.5**	0.58549	0.69221	0.41322	0.58023	0.69894	0.41112
***b* = 1**	0.60125	0.71827	0.44225	0.59427	0.72123	0.44028

Table 2: Boundary Layer Displacement Thickness (d)

	***b* = 0**	***b* = 0.25**	***b* = 0.50**	***b* = 1**
For Pr = 1	2.11357	1.86966	1.60206	1.31982
For Pr = 7	2.19234	1.92326	1.65126	1.35128

4 Discussion of Results

The present problem involves a number of parameters influencing the ferrofluid flow due to a rotating disk. On the basis of the variation in these parameters, a number of results have been drawn, of these derived results a brief summary is presented here. The viscosity variation parameter b ranges from 0 to 1, where $0< b <1$ is the case is when rate of change in viscosity is less than that of temperature and $b = 1$ is the case when rate of changes are equal for both. And following, Anderson and Valnes (1998), the values of the Prandtl number has been taken Pr=1 and 7, which represents the cases of weak and strong convective heat transfer, respectively, at the surface of the disk. On taking $b = 0$ (the case when viscosity of ferrofluid is constant *i.e.* independent of variation in temperature) and removing the magnetic body force term $\mu_0(\overrightarrow{M}\cdot\nabla\overrightarrow{H})$ from the momentum equation, the problem reduces to the case of an ordinary viscous fluid flow due to rotating disk (Schilichting 1960), numerical results so obtained here are in quite agreement with them, which validates the present numerical scheme.

For the Prandtl number Pr = 1 (case of weak convective heat transfer), the effects of viscosity variation parameter b on the radial component of the velocity are demonstrated in Fig. 1. It is observed that radial velocity increases in the beginning and after reaching its peak point in each case it starts decreasing and finally tends to zero. On increasing the values of viscosity variation parameter, radial velocity increases near the surface of the disk, but towards the end it decreases. Also, the rate of its convergence is more for larger value of the viscosity variation parameter. The peak value of the radial velocity is minimum while the rate of convergence to its limiting value (*i.e.* zero) is maximum for $b = 1$.

Fig. 2 represents the radial velocity profiles for the variation in the Prandtl number at a fixed value of the viscosity variation parameter $b = 0.5$. It is noticed that for increase in the value of the Prandtl number, the radial velocity remains unaffected near the wall, but at distant points the radial velocity gets decreased. Also on increasing the Prandtl number, radial velocity converges to zero at a comparatively faster rate.

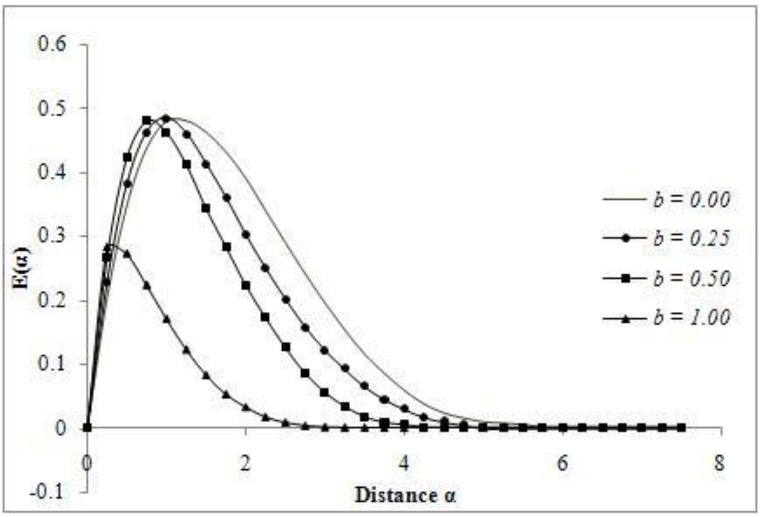

Figure 1: Effect of viscosity variation parameter b on radial velocity for $Pr = 1$.

Fig. 3 visualizes the effects of change in viscosity variation parameter b on tangential velocity profiles for Pr = 1. It is clearly seen that tangential velocity decreases smoothly form 1 to 0 with increasing α distance from the disk in each case. Also, it decreases on increasing the viscosity variation parameter. However, this decrease in tangential velocity is smaller near the surface of the disk as compared to the decrease towards the end. Also for a larger value of the variation parameter b, tangential velocity decreases and converges to zero at a faster rate.

The effects of change in the Prandtl number on the tangential velocity for a specified value of the viscosity parameter $b = 0.5$ are shown in Fig. 4. An increase in the value of the Prandtl number (i.e. strong convective heat transfer at the surface of the disk) decreases the tangential velocity, and this decrease is comparatively more towards the converging point.

The effects of change in the viscosity variation parameter on the axial velocity for

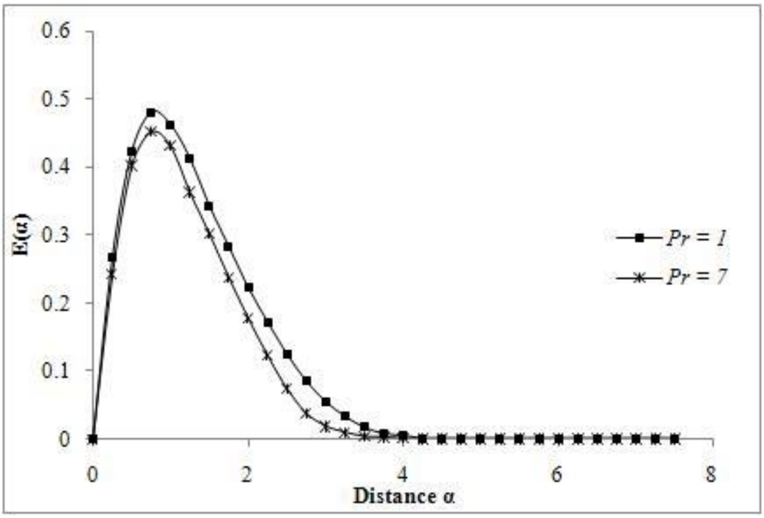

Figure 2: Effect of the Prandtl number on radial velocity for viscosity variation parameter $b = 0.5$.

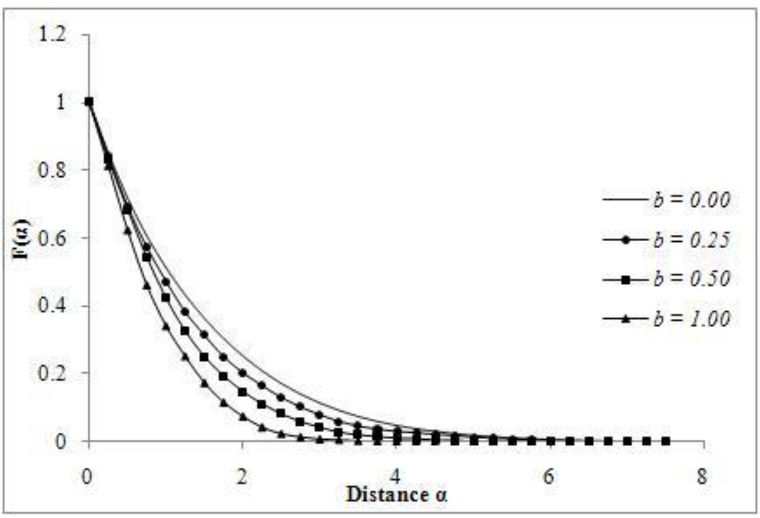

Figure 3: Effect of viscosity variation parameter b on tangential velocity for Pr = 1.

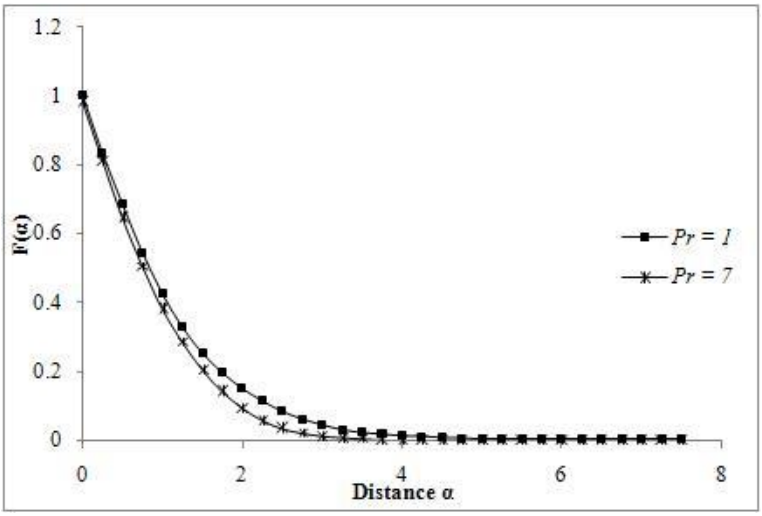

Figure 4: Effect of the Prandtl number on tangential velocity for viscosity variation parameter b = 0.5.

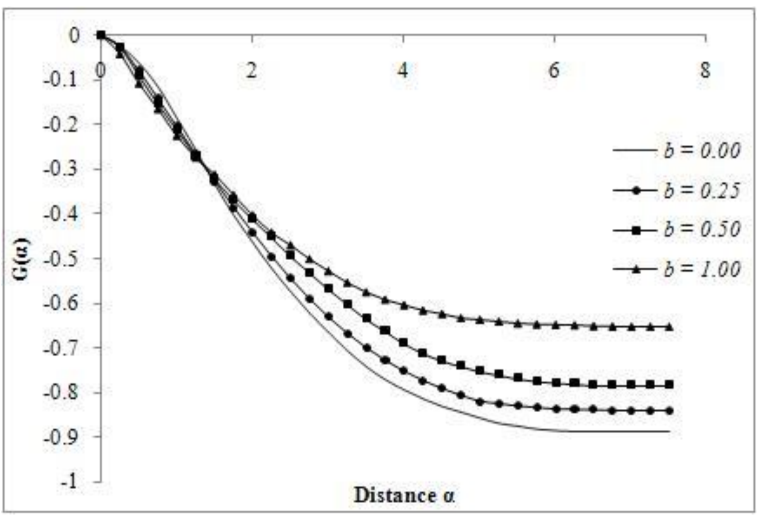

Figure 5: Effect of viscosity variation parameter b on axial velocity for Pr = 1.

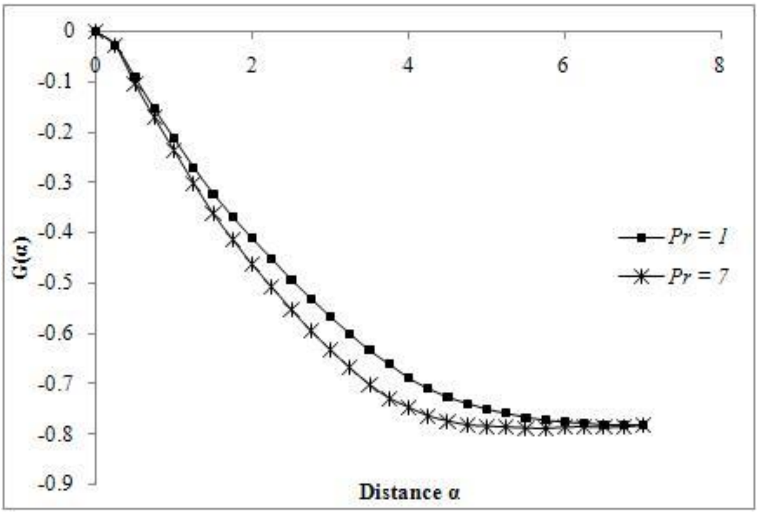

Figure 6: Effect of the Prandtl number on axial velocity for viscosity variation parameter $b = 0.5$.

the Prandtl number Pr = 1, are presented in Fig. 5. As discussed under *Mathematical Formulation of the Problem*, to compensate the radially outward flow of ferrofluid near the surface of disk, fluid flows in axially downward direction and finally converges to a finite negative value i.e. the axial velocity remains negative in the generated boundary layer. Thus, the axial velocity is negative for each value of the viscosity variation parameter. However, on increasing the value of viscosity variation parameter, the axial velocity decreases near the disk (approximately for $\alpha = 1.5$), but towards the end it increases. In the figure velocity curves seems to cross at a single point, however it is worth to mention that they are crossing at different points in a closer range. Also for $b = 1$, axial velocity is minimum near the disk while towards the end it is maximum. Fig. 6 shows the effects of change in the value of the Prandtl number on axial velocity for a fixed value of the viscosity variation parameter $b = 0.5$. Like radial and tangential component of velocity, the axial component also decreases on increasing the Prandtl number.

Fig. 7 reflects the effects of the variation in viscosity parameter on the temperature profiles for the Prandtl number Pr = 1. It is noticed that temperature decreases form one to zero for each value of the viscosity variation parameter and the rate of decrease in temperature near the surface of the disk is more in comparison to that towards the end. Also, temperature distribution within the boundary layer decreases on increasing the value of the viscosity variation parameter i.e. thermal

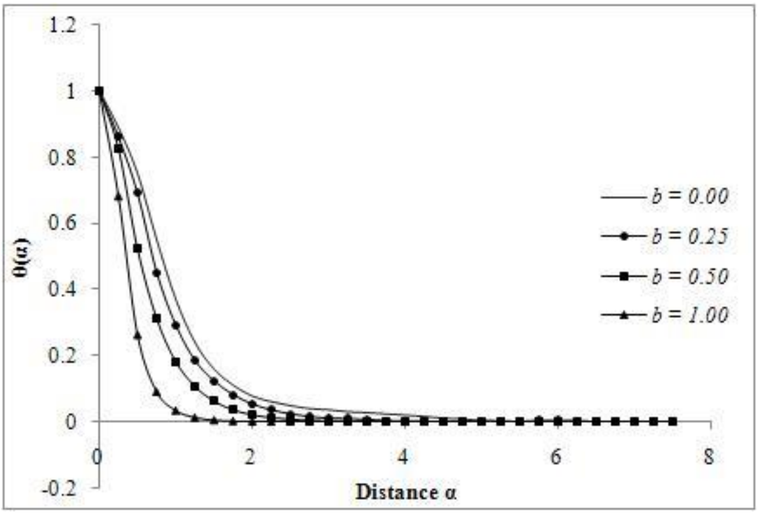

Figure 7: Effect of viscosity variation parameter *b* on temperature distribution for Pr = 1.

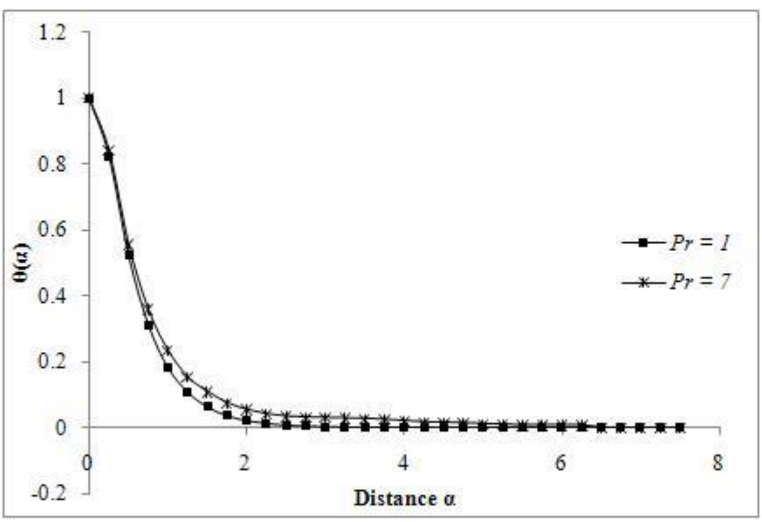

Figure 8: Effect of the Prandtl number on temperature distribution for viscosity variation parameter $b = 0.5$.

boundary layer generated over the disk becomes relatively thinner for large values of the parameter *b*. The effects of change in the Prandtl number on temparature distribution are presented in Fig. 8. Here, for an increase in the value of the Prandtl number the temperature remains unaffected in the beginning but towards the end it increases significantly *i.e.* an increase in the Prandtl number thickens the thermal boundary layer.

Tab. 1 shows the effects of viscosity variation parameter *b* on radial, tangential skin frictions and rate of forced convective heat transfer on the surface of the disk for specific values of the Prandtl number Pr =1 & 7 representing the weak and strong convection, respectively. It is noticed that both of the skin frictions and rate of heat transfer get increased on increasing the variation parameter *b*. Also tangential skin friction increases on increasing the Prandtl number, whereas radial skin friction and rate of heat transfer behave conversly i.e. these get decreased for each value of the parameter *b*. Further, the effects of changes in the parameter *b* and the Prandtl number on the boundary layer displacement thickness are presented in Tab. 2. The displacement thickness decreases on increasing the viscosity variation parameter b, while shows opposite trend in case of incrase in the Prandtl number *i.e.*the thickness increases.

5 Concluding Remarks

The FHD boundary layer flow due to a heated rotating disk with temperature dependent viscosity has been investigated. The followings are the concluding remarks:

1. On increasing the value of the viscosity variation parameter *b*, radial velocity increases in the beginnings and towards the end it decreases whereas the axial velocity behaves conversely, tangential velocity and temperature just decreases through the whole region.

2. On increasing the Prandtl number (i.e. for strong convective heat transfer), each of the three components of velocity viz. radial, tangential and axial component decreases while the temperature increases within the boundary layer.

3. The thermal boundary layer becomes thinner on increasing the viscosity variation parameter, while an increase in the Prandtl number thickens it.

4. The boundary layer displacement thickness decreases on increasing the viscosity variation parameter, and behaves conversly for the Prandtl number.

5. Skin frictions and rate of heat transfers increases on increasing the variation parameter. And, tangential skin friction also increases with an increase in the

Prandtl number, whereas radial skin friction and rate of heat transfer behve conversly.

In a nut shell, dependence of fluid viscosity on temperature is a significant factor affecting the boundary layer flow over a heated disk and helps to present a more realistic nature of flow and various flow characteristics. Thus, the present investigation is a theoretical motivation and provides the basis to scientists and engineers for experimental work on magnetic fluids.

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