

Effect of Porosity and Magnetic Field Dependent Viscosity on Revolving Ferrofluid Flow in the Presence of a Stationary Disk

Anupam Bhandari¹ and Vipin Kumar²

Abstract: The purpose of this paper is to study the flow characteristics of a ferrofluid revolving through a porous medium with a magnetic-field-dependent viscosity in the presence of a stationary disk. A Finite Difference Method is employed to discretize the set of nonlinear coupled differential equations involved in the problem. The discretized nonlinear equations, in turn, are solved by a Newton method (using MATLAB) taking the initial guess with the help of a PDE Solver. Results displayed in graphical form are used to assess the effect of the variable viscosity and porosity parameters on the velocity components. The displacement thickness of the boundary layer is also calculated for different values of these parameters by the Simpson's three-eight's rule of numerical integration. Further, the skin friction coefficients in the tangential and radial direction are determined.

Keywords: MFD viscosity, porosity, boundary layer, ferrofluid, finite difference method, displacement thickness, shear stress.

1 Introduction

Ferromagnetic fluids (ferrofluids) are colloidal liquids made of nanoscale ferromagnetic, or ferrimagnetic, particles suspended in a carrier fluid. The hydro-dynamics of such fluids are a (challenging) subject of interest for several reasons ranging from fundamental fluid mechanics to a variety of applications in engineering. After their first stable synthesis in the early 1960s, development of these suspensions in carrier liquid proved the high potential for new technological applications, thereby opening a new field of research, generally referred to as "ferrohydrodynamics". One of the many fascinating features of the ferrofluids is the prospect of influencing flow by a magnetic field and vice-versa [Engel et al. (2003)].

¹ Department of Mathematics, College of Engineering Studies University of Petroleum and Energy Studies, Dehradun, India. E-mail: pankaj.anupam6@gmail.com

² Department of Mathematics, College of Engineering Studies University of Petroleum and Energy Studies, Dehradun, India. E-mail: deshwal.vipin@gmail.com

As outlined above, ferrofluids do not exist in nature; they are synthesized fluids. The principal type is the “colloidal” ferrofluid, a suspension of finely divided particles in a certain medium which settles out slowly. Such ferrofluids are composed of small (3-15nm) particles of solid magnetite coated with a molecular layer of a dispersant and suspended in a liquid carrier. Thermal agitation keeps the particles suspended because of Brownian motion and coating prevents the particles from sticking to each other [Rosensweig (1985)]. Detailed accounts of magneto viscous effects in ferrofluids have been given in a monograph by Odenbach (2002).

A typical ferrofluid contains 10^{23} particles per cubic meter.

A study of flow within the boundary layer and its effect on the general flow around the body, in detail, are given in Schlichting (1960). A revolving flow of an ordinary viscous fluid in the presence of a stationary disk is also explained in Schlichting (1960) by using similarity transformation. The pioneering study of ordinary viscous fluid flow due to the infinite rotating disk was carried by Von Karman (1921). He introduced the famous transformation which reduces the governing equations into nonlinear differential equations in dimensionless form. Cochran(1934) obtained asymptotic solutions for the steady hydrodynamic problem formulated by Von Karman. Benton (1966) improved Cochran’s solutions, and solved the unsteady case. Bar-Yoseph et al. (1981) studied the effect of inertia on flow between a rotating and stationary disk in tilted position with respect to the axis of rotation. Rashidi et al. (2011) have studied the MHD convective and slip flow due to a rotating disk. Attia (1998) investigated the unsteady MHD flow near a rotating porous disk with uniform suction or injection. The steady flow of ordinary viscous fluid due to a rotating disk with uniform high suction was studied by Mithal (1961). Attia (2004) discussed about flow due to an infinite rotating disk in the presence of an axial uniform magnetic field by taking Hall effect into consideration. The swirling flow of a viscoelastic fluid with rotating disk was studied by Itoh et al. (2006).

Nanjundappa et al. (2010) studied Benard-Marangoni ferroconvection in a ferrofluid layer in the presence of a uniform vertical magnetic field with magnetic field dependent (MFD) viscosity. Ram et al. (2010, 2013) solved the nonlinear differential equations under Neuringer-Rosensweig model for ferrofluid flow by using power series approximations and discussed the effect of magnetic field-dependent viscosity on the velocity components and pressure profile. Further, the effect of porosity and variable viscosity on the ferrofluid flow due to a rotating disk by using power series approximation have studied by Ram et al. (2010a, 2010b).

In the present paper, the effects of porosity and MFD viscosity on the revolving ferrofluid flow in the presence of the stationary disk considering that the angular velocity (ω) is uniform at large distance from the plate. However, the centrifugal force is balanced by the pressure and the magnetization force in the radial direction.

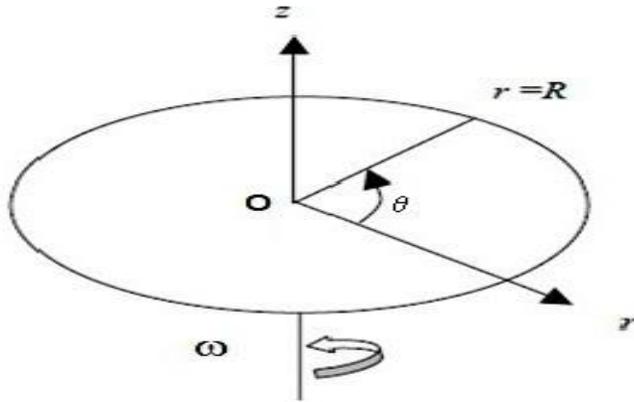


Figure 1: Revolving ferrofluid near a stationary disk.

We take cylindrical coordinates r, θ, z , where z -axis is normal to the plane and this axis is considered as the axis of rotation. The boundary layer equations together with boundary conditions are solved numerically. This problem of revolving ferrofluid flow in the presence of stationary disk, to the best of our knowledge, has not been investigated yet.

2 Mathematical formulation of the problem

The constitutive set of equations is as follows:

The equation of continuity

$$\nabla \cdot \mathbf{v} = 0 \tag{1}$$

The equation of motion

$$\frac{\rho}{\varepsilon} \left[\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\varepsilon} (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \frac{\mu}{\varepsilon} \nabla^2 \mathbf{v} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} + \frac{1}{\varepsilon} \frac{I}{2\tau_s} \nabla \times (\boldsymbol{\omega}_p - \boldsymbol{\Omega}) \tag{2}$$

The equation of rotational motion

$$I \frac{d\boldsymbol{\omega}_p}{dt} = \mu_0 (\mathbf{M} \times \mathbf{H}) - \frac{I}{\tau_s} (\boldsymbol{\omega}_p - \boldsymbol{\Omega}) \tag{3}$$

The Maxwell's Relations

$$\nabla \times \mathbf{H} = 0, \quad \nabla \cdot (\mathbf{H} + \mathbf{M}) = 0; \quad \text{with } \mathbf{M} = \chi \mathbf{H}, \quad \mathbf{M} \times \mathbf{H} = 0$$

Here ρ is the ferrofluid density, $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$, \mathbf{v} is the fluid velocity, p is the pressure, μ is the reference viscosity, μ_0 is magnetic permeability of free space, \mathbf{M} is the magnetization, \mathbf{H} is the magnetic field intensity, ε is the porosity parameter, $\boldsymbol{\omega}_p$ is the angular velocity of the particle, $\boldsymbol{\Omega}$ is the vorticity of the flow, \mathbf{B} is the magnetic induction, χ is the magnetic susceptibility, t is the time and τ_s is the Neel relaxation time and I is sum of the particle moment of inertia

Here, the inertial term is negligible in comparison with relaxation term i.e. $I \frac{d\boldsymbol{\omega}_p}{dt} \ll I \frac{\boldsymbol{\omega}_p}{\tau_s}$, therefore, the equation (3) can be written as:

$$\boldsymbol{\omega}_p = \boldsymbol{\Omega} + \mu_0 \frac{\tau_s}{I} (\mathbf{M} \times \mathbf{H}) \tag{4}$$

Now due to (4), equation (1) is modified as:

$$\frac{\rho}{\varepsilon} \left[\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\varepsilon} (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} + \frac{\mu}{\varepsilon} \nabla^2 \mathbf{v} + \frac{1}{\varepsilon} \frac{\mu_0}{2} \nabla \times (\mathbf{M} \times \mathbf{H}) \tag{5}$$

Two torques are acting on the particles which is called magnetic torque and viscous torque. Magnetic torque is denoted by $\mathbf{M} \times \mathbf{H}$ and the viscous torque is defined as the speed of the particles differing from the vorticity of the flow i. e. $(\boldsymbol{\omega}_p - \boldsymbol{\Omega})$ [Bacri et al. (1995)]. The equilibrium of both torques, which leads to the hindrance of the particle rotation, can thus be written from the equation (4) as:

$$\mu_0 (\mathbf{M} \times \mathbf{H}) = -6\mu\phi (\boldsymbol{\Omega} - \boldsymbol{\omega}_p) \tag{6}$$

The expression for mean magnetic torque becomes as:

$$\mu_0 (\overline{\mathbf{M} \times \mathbf{H}}) = -6\mu\phi g \boldsymbol{\Omega} \tag{7}$$

Now, we calculate

$$\begin{aligned} \frac{1}{\varepsilon} \frac{\mu_0}{2} \nabla \times \overline{\mathbf{M} \times \mathbf{H}} &= \frac{1}{2\varepsilon} \nabla \times -6\mu\phi g \boldsymbol{\Omega} \\ &= -\frac{3}{2\varepsilon} \mu\phi g \nabla (\nabla \cdot \mathbf{v}) + \frac{3}{2} \frac{\mu}{\varepsilon} \phi g \nabla^2 \mathbf{v} = \frac{3}{2} \frac{\mu}{\varepsilon} \phi g \nabla^2 \mathbf{v} \end{aligned} \tag{8}$$

Here g is the effective magnetization parameter given by Bacari(1995) and ϕ is the volume fraction. Now with the help of (8), the equation of motion can be written as:

$$\frac{\rho}{\varepsilon} \left[\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{\varepsilon} (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} + \frac{\mu}{\varepsilon} \left(1 + \frac{3}{2} \phi g \right) \nabla^2 \mathbf{v} \tag{9}$$

In equation (9), $\frac{3}{2} \phi g = \boldsymbol{\delta} \cdot \mathbf{B}$ where $\boldsymbol{\delta}$ is the linear measurement of the viscosity in the direction of the applied magnetic field.

Equations (1) and (9) can be written in the cylindrical form:

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0 \tag{10}$$

$$-\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\mu}{\rho} |\mathbf{M}| \frac{\partial}{\partial r} |\mathbf{H}| + \frac{v_1}{\varepsilon} \left[\frac{\partial^2 v_r}{\partial r^2} + \frac{\partial}{\partial r} \left(\frac{v_r}{r} \right) + \frac{\partial^2 v_r}{\partial z^2} \right] = \frac{1}{\varepsilon^2} \left[v_r \frac{\partial v_r}{\partial r} + v_z \frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} \right] \tag{11}$$

$$\frac{v_1}{\varepsilon} \left[\frac{\partial^2 v_\theta}{\partial r^2} + \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{\partial^2 v_\theta}{\partial z^2} \right] = \frac{1}{\varepsilon^2} \left[v_r \frac{\partial v_\theta}{\partial r} + v_z \frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r} \right] \tag{12}$$

$$-\frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\mu}{\rho} |\mathbf{M}| \frac{\partial}{\partial z} |\mathbf{H}| + \frac{v_1}{\varepsilon} \left[\frac{\partial^2 v_z}{\partial r^2} + \frac{1}{r} \frac{\partial v_z}{\partial r} + \frac{\partial^2 v_z}{\partial z^2} \right] = \frac{1}{\varepsilon^2} \left[v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} \right] \tag{13}$$

where v_r , v_θ and v_z are velocity components in the radial, tangential and axial direction, respectively. Here, $v_1 = \frac{\mu(1+\delta.B)}{\rho}$ is kinematic variable viscosity, $\delta.B$ is a dimensionless parameter.

Here the fluid at large distance from the disk is revolving with uniform angular velocity ω . The boundary conditions for the revolving flow of ferrofluid in the presence of the stationary disk used by Schlichting (1960) are given as follows

$$\left. \begin{aligned} &\text{at } z = 0; \quad v_r = 0, \quad v_\theta = 0, \quad v_z = 0 \\ &\text{at } z \rightarrow \infty; \quad v_r \rightarrow 0, \quad v_\theta \rightarrow r\omega \end{aligned} \right\} \tag{14}$$

Here, v_z does not vanish at $z \rightarrow \infty$, but tends to a finite value.

Using the boundary layer approximation $\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{\mu_0}{\rho} |\mathbf{M}| \frac{\partial}{\partial r} |\mathbf{H}| = r\omega^2$ and similarity transformations $v_r = r\omega E(\beta)$, $v_\theta = r\omega F(\beta)$, $v_z = \sqrt{v\omega} G(\beta)$, where $\beta = z\sqrt{\frac{\omega}{v}}$, the system reduces into a set of nonlinear coupled differential equations in the form of E, F and G as follows:

$$G' + 2E = 0 \tag{15}$$

$$\varepsilon k E'' - GE' - E^2 + F^2 - \varepsilon^2 = 0 \tag{16}$$

$$\varepsilon k F'' - GF' - 2EF = 0 \tag{17}$$

Here,

$$\left. \begin{aligned} &E(0) = 0, \quad F(0) = 0, \quad G(0) = 0 \\ &E(\infty) = 0, \quad F(\infty) = 1 \end{aligned} \right\} \tag{18}$$

In equations (15)-(17), $E'(\beta) = \frac{dE}{d\beta}$, $E''(\beta) = \frac{d^2E}{d\beta^2}$, $F'(\beta) = \frac{dF}{d\beta}$, $F''(\beta) = \frac{d^2F}{d\beta^2}$, $G'(\beta) = \frac{dG}{d\beta}$, and $\frac{v_1}{v} = 1 + \delta.B = k$, where k represents a MFD viscosity parameter.

3 Solution of the problem

Equations (16)-(17) can be written with the help of (15) as:

$$2\epsilon k G''' - 2GG'' + (G')^2 - 4F^2 + 4\epsilon^2 = 0 \tag{19}$$

$$\epsilon k F'' - GF' + G'F = 0 \tag{20}$$

Finite difference method is used to discretize (19)-(20) as:

$$2\epsilon k \left[\frac{1}{2h^3} (G_{i+2} - 2G_{i+1} + 2G_{i-1} - G_{i-2}) \right] - 2G_i \left[\frac{1}{h^2} (G_{i+1} - 2G_i + G_{i-1}) \right] + \frac{1}{4h^2} (G_{i+1} - G_{i-1})^2 + 4F_i^2 + 4\epsilon^2 = 0 \tag{21}$$

$$\epsilon k \left[\frac{1}{h^2} (F_{i+1} - 2F_i + F_{i-1}) \right] - G_i \left[\frac{1}{2h} (F_{i+1} - F_{i-1}) \right] + F_i \left[\frac{1}{2h} (G_{i+1} - G_{i-1}) \right] = 0 \tag{22}$$

The solution of this boundary value problem is obtained in the range $\beta = 0$ to $\beta = 5$, where the interval is differing by h . We divide the whole length into 100 equal parts as:

We take $h = \frac{1}{20}$; $\beta_0 = 0$, $\beta_{100} = 5$

Now, (21) and (22) can be written as:

$$8 \times 10^3 \epsilon k (G_{i+2} - 2G_{i+1} + 2G_{i-1} - G_{i-2}) - 8 \times 10^2 G_i (G_{i+1} - 2G_i + G_{i-1}) + 10^2 (G_{i+1} - G_{i-1})^2 - 4F_i^2 + 4\epsilon^2 = 0 \tag{23}$$

$$4 \times 10^2 \epsilon k (F_{i+1} - 2F_i + F_{i-1}) - 10G_i (F_{i+1} - F_{i-1}) + 10(G_{i+1} - G_{i-1}) F_i = 0 \tag{24}$$

The equations (23) and (24) for $1 \leq i \leq 99$, we get a set of 99 equations as:

From Equation (23):

$$\left. \begin{aligned} &8 \times 10^3 \epsilon k (G_3 - 2G_2 + 2G_0 - G_{-1}) - 8 \times 10^2 G_1 (G_2 - 2G_1 + G_0) + 10^2 (G_2 - G_0)^2 - 4F_1^2 + 4\epsilon^2 = 0 \\ &8 \times 10^3 \epsilon k (G_4 - 2G_3 + 2G_1 - G_0) - 8 \times 10^2 G_2 (G_3 - 2G_2 + G_1) + 10^2 (G_3 - G_1)^2 - 4F_2^2 + 4\epsilon^2 = 0 \\ &\dots\dots\dots \\ &8 \times 10^3 \epsilon k (G_{101} - 2G_{100} + 2G_{98} - G_{97}) - 8 \times 10^2 G_{99} (G_{100} - 2G_{99} + G_{98}) + 10^2 (G_{100} - G_{98})^2 - 4F_{99}^2 + 4\epsilon^2 = 0 \end{aligned} \right\} \tag{25}$$

From Equation (24):

$$\left. \begin{aligned}
 4 \times 10^2 \varepsilon k (F_2 - 2F_1 + F_0) - 10G_1 (F_2 - F_0) + 10(G_2 - G_0)F_1 &= 0 \\
 4 \times 10^2 \varepsilon k (F_3 - 2F_2 + F_1) - 10G_2 (F_3 - F_1) + 10(G_3 - G_1)F_2 &= 0 \\
 \dots\dots\dots & \\
 \dots\dots\dots & \\
 4 \times 10^2 \varepsilon k (F_{100} - 2F_{99} + F_{98}) - 10G_{99} (F_{100} - F_{98}) + 10(G_{100} - G_{98})F_{99} &= 0
 \end{aligned} \right\} \tag{26}$$

Using the boundary conditions, the following relations are obtained as:

$$E(\beta) = -\frac{G'(\beta)}{2} \text{ so that } G'(0) = \frac{1}{2h}(G_1 - G_{-1}) \text{ and } G'(100) = \frac{1}{2h}(G_{101} - G_{99})$$

Implying $G_1 = G_{-1}$ and $G_{101} = G_{99}$

Also, we have $F_{100} = 1, F_0 = 0; G_0 = 0$

Using the above relations in equations (25) and (26), a system of nonlinear equations is formed of n variables and n equations. Their solution can be found with the help of Newton method as:

Let f_i and g_i , respectively, are functions defined for equations (25) and (26) as:

$$\left. \begin{aligned}
 f_i(G_1, G_2, \dots, G_{99}; F_1, F_2, \dots, F_{99}) &= 0 \\
 g_i(G_1, G_2, \dots, G_{99}; F_1, F_2, \dots, F_{99}) &= 0
 \end{aligned} \right\} \text{ for } 1 \leq i \leq 99 \tag{27}$$

By using equation (25):

$$\frac{\partial f_1}{\partial G_1} = -8000\varepsilon k - 800G_2 + 3200G_1 \tag{28}$$

Similarly, other derivative of functions f_1 are calculated as:

$$\frac{\partial f_1}{\partial G_i}; \text{ for } 2 \leq i \leq 99 \text{ and } \frac{\partial f_1}{\partial F_i}; \text{ for } 1 \leq i \leq 99 \tag{29}$$

As above, the partial derivative of f_j can be computed as:

$$\frac{\partial f_j}{\partial G_i}, \frac{\partial f_j}{\partial F_i} \text{ for } 2 \leq j \leq 99, 1 \leq i \leq 99 \tag{30}$$

By using equation (26):

$$\frac{\partial g_1}{\partial G_1} = -10F_2 \tag{31}$$

Similarly, other derivative of functions g_1 are calculated as :

$$\frac{\partial g_1}{\partial G_i}; \text{ for } 2 \leq i \leq 99 \text{ and } \frac{\partial g_1}{\partial F_i}; \text{ for } 1 \leq i \leq 99 \tag{32}$$

As above, the partial derivative of g_j can be computed as:

$$\frac{\partial g_j}{\partial G_i}, \quad \frac{\partial g_j}{\partial F_i} \quad \text{for } 2 \leq j \leq 99, \quad 1 \leq i \leq 99 \tag{33}$$

Now, we initialize the values of G_1, G_2, \dots, G_{99} and F_1, F_2, \dots, F_{99} as follows:

$$G_1^1 = G_1^0 + a_1, \quad G_2^1 = G_2^0 + a_2, \quad \dots, \quad G_{99}^1 = G_{99}^0 + a_{99} \tag{34}$$

$$F_1^1 = F_1^0 + b_1, \quad F_2^1 = F_2^0 + b_2, \quad \dots, \quad F_{99}^1 = F_{99}^0 + b_{99} \tag{35}$$

Similarly, we write set of 2^{nd} iterated values as:

$$G_1^2, \quad G_2^2, \quad \dots, \quad G_{99}^2; \quad F_1^2, \quad F_2^2, \quad \dots, \quad F_{99}^2. \tag{36}$$

Here, superscript 0 denote the initial value and superscript 1 denote the improved value after iteration. As in present case, the initial guess is very close to the actual value, therefore it converges after 2^{nd} iteration only. However, if the initial guess is too far from the actual value, the solution may not converge.

In equations (34) and (35), $a_1, a_2, \dots, a_{99}; b_1, b_2, \dots, b_{99}$ are the perturbations from the actual values. It may be either positive or negative. If the initial guess is close to the actual value, the perturbation will be close to zero; as perturbation tends to zero, actual solution of the problem is obtained.

Equations (25) and (26) are transformed in linear equations as:

$$\left. \begin{aligned} f_1 (G_1 \dots G_{99}; F_1 \dots F_{99}) + \sum_{i,j=1}^{99} \left(a_i \frac{\partial f_1}{\partial G_i} + b_j \frac{\partial f_1}{\partial F_j} \right) \\ f_2 (G_1 \dots G_{99}; F_1 \dots F_{99}) + \sum_{i,j=1}^{99} \left(a_i \frac{\partial f_2}{\partial G_i} + b_j \frac{\partial f_2}{\partial F_j} \right) \\ \dots \dots \dots \\ f_{99} (G_1 \dots G_{99}; F_1 \dots F_{99}) + \sum_{i,j=1}^{99} \left(a_i \frac{\partial f_{99}}{\partial G_i} + b_j \frac{\partial f_{99}}{\partial F_j} \right) \end{aligned} \right\} \tag{37}$$

$$\left. \begin{aligned} g_1 (G_1 \dots G_{99}; F_1 \dots F_{99}) + \sum_{i,j=1}^{99} \left(a_i \frac{\partial g_1}{\partial G_i} + b_j \frac{\partial g_1}{\partial F_j} \right) \\ g_2 (G_1 \dots G_{99}; F_1 \dots F_{99}) + \sum_{i,j=1}^{99} \left(a_i \frac{\partial g_2}{\partial G_i} + b_j \frac{\partial g_2}{\partial F_j} \right) \\ \dots \dots \dots \\ g_{99} (G_1 \dots G_{99}; F_1 \dots F_{99}) + \sum_{i,j=1}^{99} \left(a_i \frac{\partial g_{99}}{\partial G_i} + b_j \frac{\partial g_{99}}{\partial F_j} \right) \end{aligned} \right\} \tag{38}$$

We write the equations (37) and (38) in 98×98 matrix form as:

$$\begin{bmatrix} \frac{\partial f_1}{\partial G_1} & \dots & \frac{\partial f_1}{\partial F_{99}} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_{99}}{\partial G_1} & \dots & \frac{\partial g_{99}}{\partial F_{99}} \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ b_{99} \end{bmatrix} = \begin{bmatrix} f_1(G_1, G_2, \dots, G_{99}; F_1, F_2, \dots, F_{99}) \\ \vdots \\ g_{99}(G_1, G_2, \dots, G_{99}; F_1, F_2, \dots, F_{99}) \end{bmatrix} \tag{39}$$

Here $a_1, a_2, \dots, a_{99}; b_1, b_2, \dots, b_{99}$ can be calculated numerically with the help of MATLAB. After 2^{nd} iteration, the convergent result is obtained; however, the initial guess is taken with the help of Flex PDE. The Numerical data, which was taken from Flex PDE, are verified by MATLAB. The actual values of E, F and G come very close when $a_1, a_2, \dots, a_{99}; b_1, b_2, \dots, b_{99}$ tend to zero.

The boundary-layer displacement thickness is calculated as

$$d = \frac{1}{r\omega_0} \int_0^\infty v_\theta dz = \int_0^\infty F(\beta) d\beta \tag{40}$$

The displacement thickness of the boundary layer is presented in the following tables as:

Table 1: Displacement thickness (d) for various values of porosity and MFD viscosity parameters.

$\epsilon \backslash k$	1.1	1.2	1.3
0.7	3.3896	3.2456	3.1287
0.8	3.8658	3.7425	3.6328
0.9	4.3268	4.1628	4.0698

However, the displacement thickness (d) is 4.872905 for ordinary case where there is no effect of porosity and MFD viscosity parameters on the ferrofluid flow.

The expressions for shear stress on the wall of the disk (τ_w) and its surface (τ_s) are as follows:

$$\left. \begin{aligned} \tau_w &= \mu \left[\frac{\partial v_\theta}{\partial z} + \frac{1}{r} \frac{\partial v_z}{\partial \theta} \right]_{z=0} \\ \tau_s &= \mu \left[\frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z} \right]_{z=0} \end{aligned} \right\} \tag{41}$$

By using similarity transformation, the skin friction coefficient in the tangential direction (c_w) and in the radial direction (c_s) can be calculated as:

Table 2: Skin friction coefficient c_w for different values of ε and k .

$k \backslash \varepsilon$	$\varepsilon = 1$	$\varepsilon = 0.7$	$\varepsilon = 0.8$	$\varepsilon = 0.9$
1	0.7736	0.5414	0.6188	0.6962
1.1	0.7375	0.5161	0.5899	0.6637
1.2	0.7061	0.4939	0.5647	0.6354
1.3	0.6784	0.4743	0.5423	0.6104

Table 3: Skin friction coefficient c_s for different values of ε and k .

$k \backslash \varepsilon$	$\varepsilon = 1$	$\varepsilon = 0.7$	$\varepsilon = 0.8$	$\varepsilon = 0.9$
1	0.9429	0.6602	0.7545	0.8487
1.1	0.8988	0.6295	0.7192	0.8090
1.2	0.8603	0.6027	0.6886	0.7745
1.3	0.8263	0.5790	0.6614	0.7438

4 Results and Discussion

The problem considered here involves a number of parameters, on the basis of which a wide range of numerical results has been derived. Of these results, a small section is presented here for brevity. A case of motion near a stationary disk, which is being at $z = 0$, the fluid at large distance from it rotates at constant angular velocity (ω). The fluid particles which rotate at a large distance from the wall are in equilibrium due to centrifugal force.

Figures 1-3 show the radial velocity profile for different values of MFD viscosity and porosity parameters. It is evident from the figures 1-3, for $\varepsilon = 0.7$, the radial velocity gets less negative value in comparison to ordinary case. However, increasing values of porosity parameter (ε), flow is directed radially inward. And, at $\varepsilon = 1$, we reach the state, where there is no effect of porosity parameter. At a large distance from the plate (large values of β), the flow is directed radially outward. Negative values of the radial velocity indicate that the flow is directed radially inward and positive values shows that it is directed radially outward.. Also it is clear from the figures that MFD viscosity parameter has less effect on the velocity profiles in comparison to porosity parameter. Further, we observe that at porosity $\varepsilon = 0.9$, radial velocity becomes positive at less axial distance from the disk in comparison to the porosity $\varepsilon = 0.7, 0.8$. However, for $k = 1$, $\varepsilon = 1$, the

problem reduces to the ordinary case (Schlichting 1960), where there is no effect of porosity and MFD viscosity parameters. In the ordinary case, the radial velocity is negative for starting values of β because the particle near the wall to flow radially inwards so the tangential velocity is reduced thus decreasing the centrifugal force.

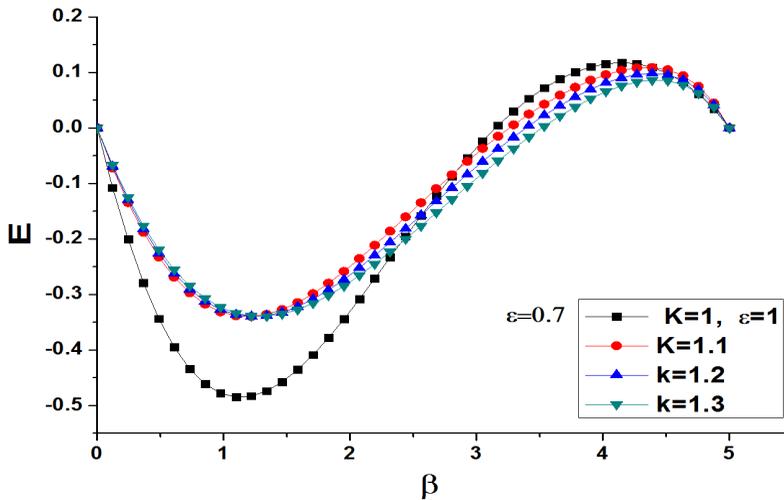


Figure 2: Radial velocity profile for various values of k at $\varepsilon = 0.7$.

Figures 4-6 show the tangential velocity profile. In the ordinary case, the tangential velocity increases continuously for increasing values of β . However, the fluid particle rotates at large distance from the plate about the z axis with constant angular velocity ω . Due to porosity parameter, the tangential velocity decreases in comparison to the ordinary case. For increasing values of porosity parameter, the tangential velocity increases and at $\varepsilon = 1$, the state is reached where there is no effect of porosity

Figures 7-9 indicate the axial velocity profile. Here it is observed that the axial velocity does not depend much on radius of the disk but only on the distance from the ground. Due to the effect of porosity, the axial velocity decreases, however, for increasing values of MFD viscosity parameter, the axial velocity increases.

The displacement thickness is also calculated for different values of porosity and MFD viscosity parameters by Simpson's three-eight's rule. From the table, it is clear that for increasing the porosity parameter the displacement thickness of the boundary layer increases. However, in the ordinary case, the displacement thickness becomes 4.8729. In tables 2 and 3, the skin friction coefficients are calculated in the radial and tangential directions respectively. In case of without porosity, the

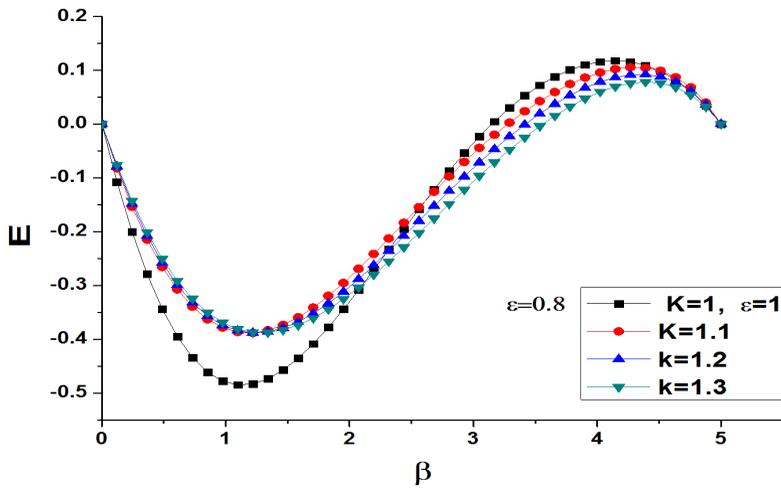


Figure 3: Radial velocity profile for various values of k at $\varepsilon = 0.8$.

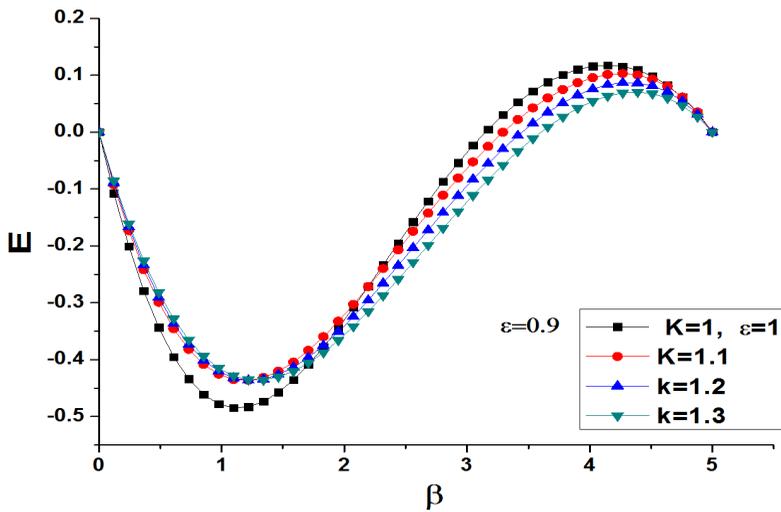


Figure 4: Radial velocity profile for various values of k at $\varepsilon = 0.9$.

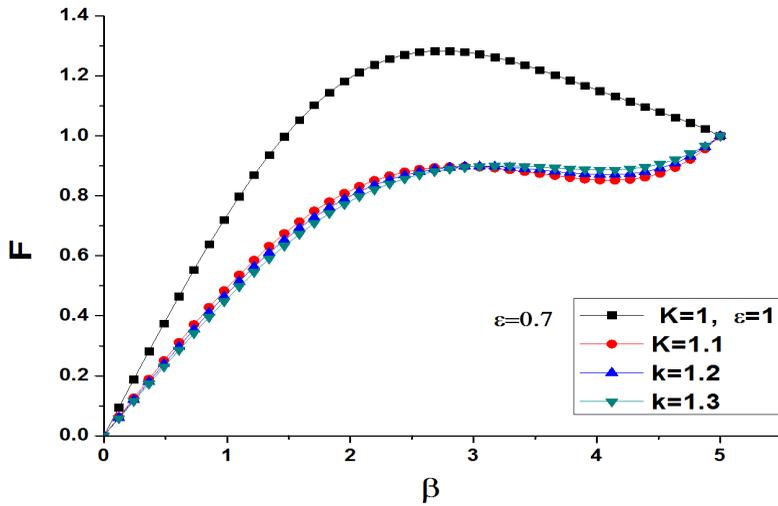


Figure 5: Tangential velocity profile for various values of k at $\varepsilon = 0.7$.

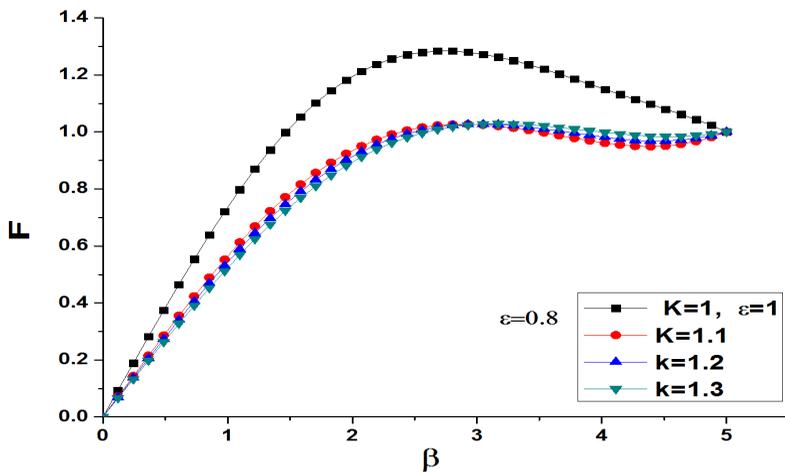


Figure 6: Tangential velocity profile for various values of k at $\varepsilon = 0.8$.

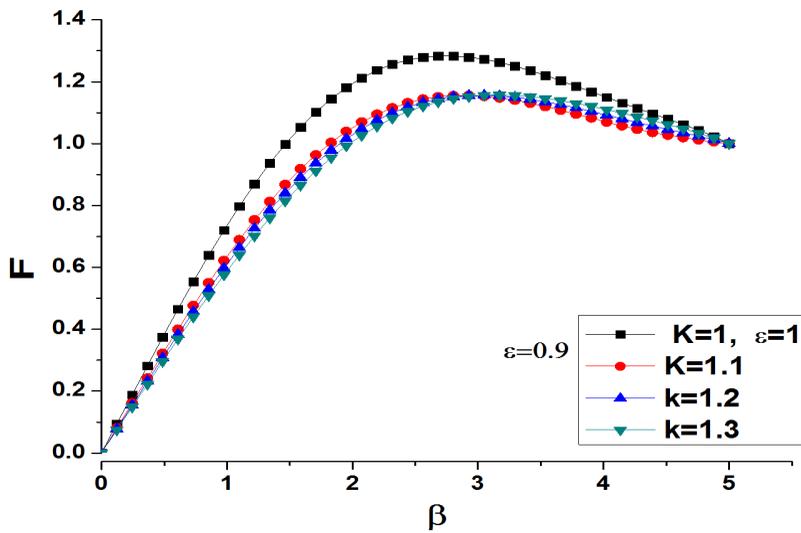


Figure 7: Tangential velocity profile for various values of k at $\varepsilon = 0.8$.

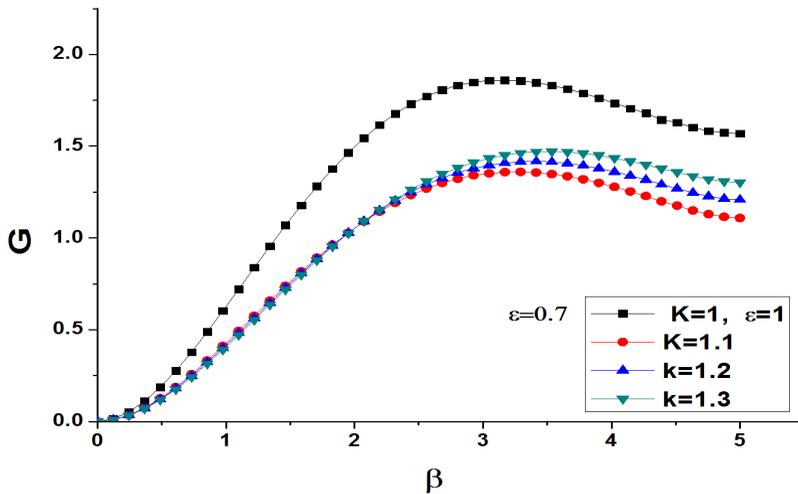


Figure 8: Axial velocity profile for various values of k at $\varepsilon = 0.7$.

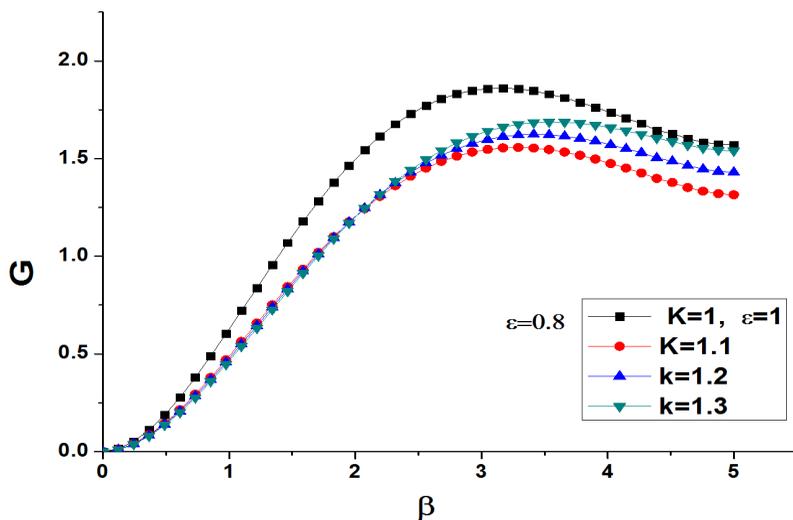


Figure 9: Axial velocity profile for various values of k at $\varepsilon = 0.8$.

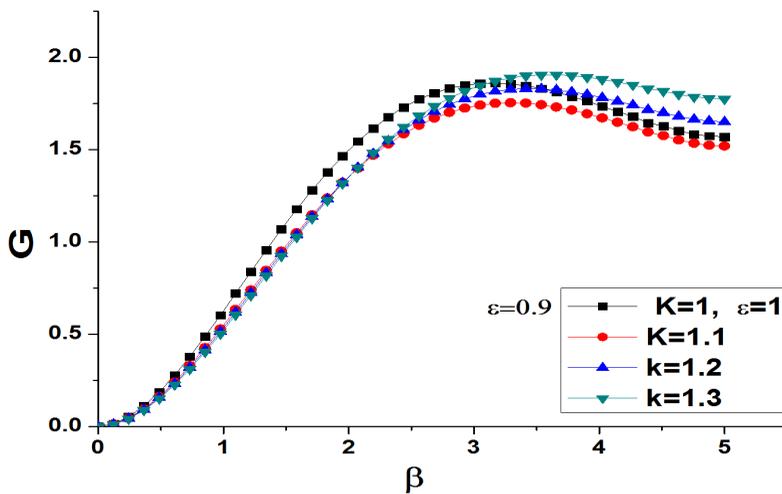


Figure 10: Axial velocity profile for various values of k at $\varepsilon = 0.9$.

radial skin friction coefficient, c_s , and tangential friction coefficient, c_w , decrease for increasing values of MFD viscosity parameter. However, these coefficients i.e. c_s and c_w , increases for increasing the porosity parameter.

5 Conclusions

The present results clarify the effect of porosity and MFD viscosity on the velocity components of a revolving ferrofluid flow in the presence of a stationary disk. These results indicate that there is a dominant effect of the porosity parameter on the flow characteristics while the influence of the MFD viscosity parameter is rather limited.

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