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Rayleigh-Taylor Instability of a Two-fluid Layer Subjected to Rotation and a Periodic Tangential Magnetic Field

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Abstract: The Rayleigh-Taylor instability (RTI) of two superposed ferrofluids subjected to rotation and a periodic tangential magnetic field is considered. Relevant solutions and related dispersion relations are obtained by using the method of multiple scales.

Keywords: Ferrofluids, Rayleigh-Taylor instability, Rotation, Periodic tangential magnetic field.

1 Introduction

The Rayleigh-Taylor instability (RTI) of magnetic fluids has been the subject of much research [Chakraborty (1982); Shivamoggi (1988); Davalos-Orozco et al. (1989)] because of its implications on the stability of stellar and planetary interiors and many other problems related to materials science (production of semiconductor materials, metal alloys, etc., see also Gedik et al. (2012) and Lappa (2012) for other relevant references). The fundamental RTI problem consists of a heavier fluid supported by a lighter fluid. As the gravity destabilizes the interface, this configuration is unstable. However, if surface tension exists between the two fluids, it has a stabilizing effect on the configuration.

From a historical standpoint, RTI for two superposed fluids has been investigated by many authors. Cowley and Rosensweig (1967) considered the linear stability of two superposed magnetic fluids in the presence of an externally applied magnetic field. Zelazo and Melcher (1969) and Rosensweig (1985) investigated theoretically as well as experimentally the propagation of plane waves in the presence of a tangential magnetic field. Kant and Malik (1985) studied the propagation of weakly nonlinear waves in a Rayleigh-Taylor magnetic fluid system. They found that the tangential magnetic field plays a dual role in the stability criterion.

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Malik and Singh (1984) discussed the motion of an inviscid, incompressible ferrofluid under the influence of magnetic field, surface tension and gravity. They showed how the magnetic field and surface tension stabilize the interface, conserving contours and found that the nonlinear modulation instability cannot be suppressed by the application of a strong magnetic field. Hennenberg et al. (2007) considered the coupling between stationary Marangoni and Cowley-Rosensweig instabilities in a deformable Ferrofluid layer.

Lizuka and Wadati (1990) considered the nonlinear RTI under the effect of surface tension between two superposed fluids in the absence of a magnetic field. Elhefnawy (1993) investigated the nonlinear behaviour of two-dimensional RTI for two magnetic fluids of finite thickness, taking into account of the effect of surface tension and a tangential magnetic field.

Nonlinear wave propagation on the surface between two superposed magnetic fluids stressed by a tangential periodic magnetic field was also examined by El-Dib (1993) using the method of multiple scales.

Nonlinear RTI of two superposed magnetic fluids under parallel rotation and a tangential magnetic field was analysed by Anjalidevi and Jothimani (2001). Anjalidevi and Hemamalini (2007) considered the effect of parallel rotation and a normal magnetic field. RTI in three dimensions was studied by Stone and Gardiner (2007). Yu and Livescu (2008) discussed RTI in a cylindrical geometry with compressible fluids. RTI in dielectric fluids was examined by Joshi et al (2010). On the long time simulation of the RTI was analysed by Lee et al (2011). Wang et al (2012) considered density gradient effects in weakly nonlinear ablative RTI . Tao et al (2013) analysed the nonlinear RTI of rotating inviscid fluids. Rayleigh-Taylor stability boundary at solid-liquid interfaces was discussed by Priz et al. (2013). An experimental study of two-phase flow in porous media with measurement of relative permeability was conducted by Labed and Bennamoun (2012).

2 Basic equation and formulation

We consider finite-amplitude two-dimensional wave propagation on the interface z=0 separating two magnetic fluids. Fluid with density ρ_2 and permeability μ_2 occupies the region z >0.Whereas the region z <0 is occupied by a fluid of density ρ_1 and magnetic permeability μ_1. The fluids are assumed to be inviscid and incompressible. The motion is assumed to be rotational with gravity $\boldsymbol{g}$ acting in the negative y direction.

The system is stressed by a periodic tangential magnetic field $\boldsymbol{H}= \varepsilon^{\frac{1}{2}} H_0 \cos \omega t \vec{i}$ in the x direction where ε is a small dimensionless parameter, H_0 is the amplitude of the magnetic field, ω is the field frequency and $\vec{i}$ is the unit vector in the $x$direction.

The system is subjected to rotation with a constant angular velocity $\mathbf{\Omega}$(1,0,0) parallel to the direction of the flow. The interface between the two fluids is described by $z = \eta\ (x, t)$ or $z - \eta\ (x, t) = 0$. When it is completely flat then it is represented by $\eta = 0$.

The governing equations of the problem are:

$$\nabla \cdot \boldsymbol{q} = 0 \tag{1}$$

$$\rho \frac{d\boldsymbol{q}}{dt} + 2\rho(\mathbf{\Omega} \times \boldsymbol{q}) = -\nabla p + (\rho + \delta\rho)\boldsymbol{g} + \left(\frac{\mu - \mu_0}{2}\right) \nabla (\boldsymbol{H}_0 + \boldsymbol{h})^2 \tag{2}$$

$$\frac{d}{dt}(\rho + \delta\rho) = 0 \tag{3}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{4}$$

$$\nabla \times \boldsymbol{H} = 0 \tag{5}$$

where $\boldsymbol{q} = (u, v, w)$, $\mathbf{\Omega} = (\Omega, 0, 0)$, p is the total pressure and $\mu_0 = 1$.

The magnetic potential ψ ($\boldsymbol{h} = -\varepsilon^{\frac{1}{2}} \nabla \psi$) satisfies

$$\begin{aligned} \nabla^2 \psi^{(1)} &= 0, \quad -\infty < z < \eta(x,t) \\ \nabla^2 \psi^{(2)} &= 0, \quad \eta(x,t) < z < \infty \end{aligned} \tag{6}$$

where $\eta(x,t)$ is the elevation of the free surface from the unperturbed level.

The boundary conditions at the interface $z = \eta$ (x, t) are

$$\frac{\partial \eta}{\partial t} + u^i \frac{\partial \eta}{\partial x} - w^i = 0 \quad i = 1, 2 \tag{7}$$

$$\hat{n} \cdot \boldsymbol{q}^{(1)} = \hat{n} \cdot \boldsymbol{q}^{(2)} \tag{8}$$

$$\mu_1 H_{1n} = \mu_2 H_{2n} \tag{9}$$

$$H_{1t} = H_{2t} \tag{10}$$

$$[[-p + \frac{\mu}{2}(H_n^2 - H_t^2)]] = \frac{-T\eta_{xx}}{(1 + \eta_x^2)^{\frac{3}{2}}} \tag{11}$$

where $\hat{n} = \frac{(-\eta_x, 0, 1)}{\sqrt{1 + \eta_x^2}}$ and [[ ]] denotes the jump in the quantity across the interface.

To obtain an approximate solution to the above system of equations the method of multiple scales as formulated by Weissman (1979) and Murakami (1988) is employed to investigate the interaction of finite amplitude waves by introducing the scales $t_n = \varepsilon^n t$, $x_n = \varepsilon^n x\ (n = 0, 1, 2)$, assuming a smallness parameter ε expressing the steepness ratio of the wave.

The various physical quantities are

$$\eta(x,t) = \sum_{n=1}^{3} \varepsilon^n \eta_n(x_0, x_1, x_2\,;\, t_0, t_1, t_2) + O(\varepsilon^4) \tag{12}$$

$$\boldsymbol{q}(x,z,t) = \sum_{n=1}^{3} \varepsilon^n \boldsymbol{q}_n(x_0, x_1, x_2, z\,;\, t_0, t_1, t_2) + O(\varepsilon^4) \tag{13}$$

$$\psi(x,z,t) = \sum_{n=1}^{3} \varepsilon^n \psi_n(x_0, x_1, x_2, z\,;\, t_0, t_1, t_2) + O(\varepsilon^4) \tag{14}$$

and similarly for other variables also.

To evaluate boundary conditions (7) - (11), we use the Maclaurin series expansions at z = 0 for the quantities involved. Then on substitution of the above expansions (12) - (14) into the set of equations (1) - (11) and equating terms of equal powers of ε we obtain three sets of equations.

3 Linear theory

The solution of the first- order problem comes out to be

$$\eta_1 = A(x_1, x_2; t_1, t_2)\exp i(kx_0 - \omega_0 t_0) + c.c \tag{15}$$

$$u_1^{(1)} = \frac{A\delta}{k}\omega_0 \exp(i\theta + \delta z), \quad z < 0 \tag{16}$$

$$u_1^{(2)} = -\frac{A\delta}{k}\omega_0 \exp(i\theta - \delta z), \quad z > 0 \tag{17}$$

$$v_1^{(1)} = 2\Omega A \exp(i\theta + \delta z), \quad z < 0 \tag{18}$$

$$v_1^{(2)} = 2\Omega A \exp(i\theta - \delta z), \quad z > 0 \tag{19}$$

$$w_1^{(1)} = -iA\omega_0 \exp(i\theta + \delta z), \quad z < 0 \tag{20}$$

$$w_1^{(2)} = -iA\omega_0 \exp(i\theta - \delta z), \quad z > 0 \tag{21}$$

$$\psi_1^{(1)} = \frac{H_0 \cos\omega t\, iA(1-\mu)}{(1+\mu)} \exp(i\theta + kz), \quad z < 0 \tag{22}$$

$$\psi_1^{(2)} = \frac{H_0 \cos\omega t\, iA(1-\mu)}{(1+\mu)} \exp(i\theta - kz), \quad z > 0 \tag{23}$$

where $\theta = kx_0 - \omega_0 t_0$, $\delta = k\left(1 - \frac{4\Omega^2}{\omega_0^2}\right)^{\frac{1}{2}}$.

The first- order problem leads to the **dispersion relation**

$$\delta_1 \omega_0^2(\rho_1+\rho_2) = (\rho_1 - \rho_2)gk - \frac{H_0^2\cos^2\omega t(1-\mu)^2\mu_2}{(1+\mu)}k^2 + Tk^3 \tag{24}$$

where $\mu = \frac{\mu_1}{\mu_2}$, $\delta_1 = \left(1 - \frac{4\Omega^2}{\omega_0^2}\right)^{\frac{1}{2}}$.

The linear dispersion relation (24) was initially discussed as the result of a linear perturbation by Chandrasekhar (1961) in the absence of rotation and magnetic field. Further in the absence of rotation, the neutral stability occurs at $\omega_0 = 0$ and the neutral stability curve H_0^2 Vs k is then furnished by (24).

This curve has a minimum H_c^2 for a finite

$$K_c = \left(\frac{(\rho_1 - \rho_2)\,g}{T}\right)^{\frac{1}{2}}$$

where

$$H_c^2 = \frac{2(1+\mu)}{(1-\mu)^2}\,[(\rho_1 - \rho_2)\,gT]^{\frac{1}{2}}$$

The instability sets in $H^2 \geq H_c^2$. It is interesting to note that the system is hydro dynamically stable for $\rho_1 > \rho_2$, introduction of the magnetic field H greater than H_c renders the system is unstable. These aspects are not restricted with respect to rotation.

4 The second- order problem

If we carry the problem to the second-order set of equations, we can substitute the solution of the first-order problem into the second-order and solve the resulting equations.

Second order solutions are

$$\eta_2 = \Lambda A^2 \exp(2i\theta) + c\cdot c \tag{25}$$

$$\begin{aligned} u_2^{(1)} =& \frac{i}{k}\left\{\left(\frac{-\omega_0\delta^2}{k}z\right)\frac{\partial A}{\partial x_1} + \left(\delta + \frac{c}{2\delta}(1+z\delta)\right)\frac{\partial A}{\partial t_1}\right\}e^{i\theta+\delta z} \\ &+ \frac{2\omega_0 A^2}{k}(\Lambda-\delta)\,\alpha\, e^{2i\theta+2\alpha z}, \quad z<0 \end{aligned} \tag{26}$$

$$\begin{aligned} u_2^{(2)} =& -\frac{i}{k}\left\{\left(\frac{\omega_0\delta^2}{k}z\right)\frac{\partial A}{\partial x_1} + \left(\delta + \frac{c}{2\delta}(1-z\delta)\right)\frac{\partial A}{\partial t_1}\right\}e^{i\theta-\delta z} \\ &- \frac{2\omega_0 A^2}{k}(\Lambda+\delta)\alpha e^{2i\theta-2\alpha z}, \quad z>0 \end{aligned} \tag{27}$$

$$v_2^{(1)} = \frac{i}{k}\left\{-2\Omega\delta z\frac{\partial A}{\partial x_1} + \frac{\Omega kcz}{\omega_0\delta}\frac{\partial A}{\partial t_1}\right\} e^{i\theta+\delta z} + 2\Omega A^2(\Lambda-\delta)\, e^{2i\theta+2\alpha z}, \quad z<0 \tag{28}$$

$$v_2^{(2)} = \frac{i}{k}\left\{2\Omega\delta z\frac{\partial A}{\partial x_1} - \frac{\Omega kcz}{\omega_0\delta}\frac{\partial A}{\partial t_1}\right\} e^{i\theta-\delta z} + 2\Omega A^2(\Lambda+\delta)\, e^{2i\theta-2\alpha z}, \quad z>0 \tag{29}$$

$$w_2^{(1)} = \left\{-\frac{\omega_0\delta z}{k}\frac{\partial A}{\partial x_1} + \left(1+\frac{cz}{2\delta}\right)\frac{\partial A}{\partial t_1}\right\} e^{i\theta+\delta z} - 2i\omega_0 A^2(\Lambda-\delta)e^{2i\theta+2\alpha z}, \quad z<0 \tag{30}$$

$$w_2^{(2)} = \left\{\frac{\omega_0\delta z}{k}\frac{\partial A}{\partial x_1} + \left(1-\frac{cz}{2\delta}\right)\frac{\partial A}{\partial t_1}\right\} e^{i\theta-\delta z} - 2i\omega_0 A^2(\Lambda+\delta)e^{2i\theta-2\alpha z}, \quad z>0 \tag{31}$$

where

$$\delta = k\left(1-\frac{4\Omega^2}{\omega_0^2}\right)^{\frac{1}{2}}, \quad \alpha = k\left(1-\frac{\Omega^2}{\omega_0^2}\right)^{\frac{1}{2}}, \quad c = k^2-\delta^2+\frac{4\Omega^2 k^2}{\omega_0^2} = \frac{8\Omega^2 k^2}{\omega_0^2}$$

$$\psi_2^{(1)} = -\frac{(\mu-1)H_0\cos\omega t}{(\mu+1)} z\frac{\partial A}{\partial x_1}e^{i\theta+kz} + iH_0\cos\omega t A^2\frac{(1-\mu)}{(1+\mu)}(\Lambda-k)e^{2i\theta+2kz} \tag{32}$$

$$\psi_2^{(2)} = \frac{(\mu-1)H_0\cos\omega t}{(\mu+1)} z\frac{\partial A}{\partial x_1}e^{i\theta-kz} + iH_0\cos\omega t A^2\frac{(1-\mu)}{(1+\mu)}(\Lambda+k)e^{2i\theta-2kz} \tag{33}$$

where

$$\Lambda = \frac{\left\{\frac{2\omega_0^2\alpha\delta}{k^2}(\rho_2-\rho_1) - \frac{\omega_0^2\delta^2}{k^2}(\rho_2-\rho_1) + H_0^2\cos^2\omega t k^2\left(\frac{1-\mu}{1+\mu}\right)\left[4-\frac{\mu_2(1-\mu)^2}{(1+\mu)}\right]\right\}}{-\frac{2\omega_0^2\alpha}{k^2}(\rho_1+\rho_2) + 2H_0^2\cos^2\omega t k\mu_2\left(\frac{(1-\mu)^2}{1+\mu}\right) - (\rho_2-\rho_1)g + 4Tk^2}$$

5 Third order problem and nonlinear evolution equation

Substituting from the first and second order solution already derived into the equations governing the third-order problem, the third-order **dispersion relation** is obtained.

$$
\begin{aligned}
&\frac{\partial^2 A}{\partial {x_1}^2}\left\{\frac{\omega_0\rho_1 c_1\, i}{k^2}-\frac{\omega_0\rho_2 c_4\, i}{k^2}+\frac{\omega_0^2\,\delta(\rho_1+\rho_2)}{k^4}\right.\\
&-T-H_0\cos\omega t\left(\frac{1-\mu}{1+\mu}\right)[iH_0\cos\omega t\, k(\mu_2-1)(4k\mu-i(3+\mu))\\
&-\frac{1}{4k}(2k-1)(\mu_2-1)+iH_0\cos\omega t\, k(\mu_1-1)\,(4k+i(3-\mu))\\
&\left.+\frac{H_0\cos\omega t}{4k}(1-2k)(\mu_1-1)]\right\}\\
&+\frac{\partial^2 A}{\partial x_1\partial t_1}\left\{\frac{\omega_0\rho_2}{k}\left(\frac{\delta}{k^2}+\frac{4\Omega^2}{\delta\omega_0^2}-\frac{c_5\, i}{k}\right)+\frac{\omega_0\rho_1}{k}\left(\frac{\delta}{k^2}+\frac{4\Omega^2}{\delta\omega_0^2}-\frac{c_2\, i}{k}\right)\right.\\
&\left.+\frac{2\omega_0\delta}{k^3}(\rho_1+\rho_2)+\frac{c}{k^3}(\rho_1+\rho_2)\right\}\\
&+\frac{\partial^2 A}{\partial {t_1}^2}\left\{-\frac{\omega_0\rho_2 c_6\, i}{k^2}+\frac{\omega_0\rho_1 c_3\, i}{k^2}-\frac{i}{k}(\delta+c)(\rho_1+\rho_2)\right\}\\
&-\frac{\partial A}{\partial x_2}\left\{\frac{2i\omega_0^2\,\delta}{k^3}(\rho_1+\rho_2)+H_0\cos\omega t\left(\frac{1-\mu}{1+\mu}\right)\left[\frac{2iH_0\cos\omega t\, k}{1+\mu}(\mu_2-1)\right.\right.\\
&\left.\left.+\left(1+ik\mu\frac{H_0\cos\omega t\, k\mu_1(1-\mu)}{\mu}\right)-2iH_0\cos\omega t\frac{(\mu_1-1)}{1+\mu}\right]+2T\, ik\right\}\\
&+\frac{\partial A}{\partial t_2}\left\{\frac{2i\omega_0\delta}{k^2}(\rho_1+\rho_2)\right\}\\
&+A^2\bar{A}\left\{\frac{4\omega_0^2\alpha^2}{k^2}(\rho_2(\Lambda+\delta)-\rho_1(\Lambda-\delta))\right.\\
&+\frac{2\omega_0^2}{k}(\rho_2(\Lambda+\delta)-\rho_1(\Lambda+\delta))\left(\alpha\delta-2\alpha^2-i\delta^2\right)\\
&-\frac{4H_0^2\cos^2\omega t}{k^2}\left(\frac{(1-\mu)}{1+\mu}\right)[(\mu_2-1)(\Lambda+k)+(\mu_1-1)(\Lambda-k)]\\
&-\frac{1}{2k^2}\omega_0^2\delta^3(\rho_1+\rho_2)+\frac{H_0^2\cos^2\omega t k^2}{2}\frac{(1-\mu)^2}{(1+\mu)}\mu_2\\
&-4\mu_2(1-\mu)k^2\Lambda+2k^3H_0^2\cos^2\omega t\frac{(1-\mu)^2}{(1+\mu)}\mu_2\\
&-H_0^2\cos^2\omega t k^3\mu_2\frac{(1-\mu)}{\mu}\left[2i\left(\frac{1-\mu}{1+\mu}\right)+4\Lambda\left(\frac{1-\mu}{1+\mu}\right)-2\right]+\frac{9}{2}k^4\\
&-2k^2H_0^2\cos^2\omega t\left(\frac{(1-\mu)}{1+\mu}\right)^2[(\Lambda+k)(2\mu+1)-(\Lambda-k)(2\mu-1)+2k]
\end{aligned}
\qquad (34)
$$

$$-4k^2H_0^2\cos^2\omega t\left(\frac{(1-\mu)}{1+\mu}\right)[(\Lambda+k)(\mu_2-1)+(\Lambda-k)(\mu\mu_2-1)]$$
$$+2k^2H_0^2\cos^2\omega t\,\Lambda\left(\frac{(1-\mu)}{1+\mu}\right)-\frac{kH_0^2\cos^2\omega t(1-\mu)}{2(1+\mu)^2}$$
$$[-k^2(\mu_2-1)(2k(\mu+3)+(1-\mu)(1+6\Lambda)-4\Lambda)$$
$$\left.+(\mu_1-1)\left(2k^2(\mu+3)+k(1-\mu)(1+6\Lambda)-4(1+\mu)\right)]\right\}$$

where

$$c_1=\frac{i}{2k^2\delta},\quad c_2=\frac{-ik^2}{4\delta^2}\left[\frac{8\Omega^2\delta}{k\omega_0^2}-\frac{\delta^3}{k^3}+\frac{\delta}{k}+\frac{4\Omega^2\delta}{k\omega_0^2}\right],$$

$$c_3=\frac{-ik^2}{4\delta^2\omega_0}\left[\frac{4\Omega^2k\delta}{\omega_0^2}-\frac{16\Omega^4k^2}{\delta\omega_0^4}-\frac{4\Omega^2k^2}{\delta\omega_0^2}\right],\quad c_4=\frac{-i\omega_0\delta}{2k^2},$$

$$c_5=\frac{-ik}{4\delta^2}\left[\frac{\delta^3}{k^2}-\frac{12\Omega^2}{\omega_0^2}-\delta\right]$$

$$c_6=\frac{-ik}{4\omega_0\delta^2}\left[\frac{4\Omega^2k^2}{\delta^2\omega_0^2}-4\Omega^2\delta-1\right]$$

The above third order dispersion relation is reduced to the well known Schrödinger equation

$$i\frac{\partial A}{\partial\tau}+\alpha_{1k}^*\frac{\partial^2A}{\partial\xi^2}+\alpha_k^*\frac{\partial A}{\partial\xi}+Q_k|A|^2A=0 \tag{35}$$

where

$$\alpha_{1k}^*=\frac{i}{J_{5k}}[J_{1k}-C_{gk}J_{2k}+C_{gk}^2J_{3k}]$$

$$\alpha_k^*=i\left(\frac{J_{4k}}{J_{5k}}-C_{gk}\right)$$

$$Q_k=\frac{i}{J_{5k}}Q_{1k}\quad\text{where}\quad Q_{1k}=\frac{\partial D}{\partial|A|^2}\quad\text{and}\quad C_{gk}=-\frac{Dk}{D\omega}$$

with

$$Dk=(\rho_1-\rho_2)g-2kH_0^2\cos^2\omega t\frac{(1-\mu)^2}{(1+\mu)}\mu_2+3Tk^2$$

$$D\omega = 2(\rho_1+\rho_2)(\omega_0^2 - 2\Omega^2)(\omega_0^2 - 4\Omega^2)^{\frac{-1}{2}}.$$

J_{1k}^*, J_{2k}^*, J_{3k}^*, J_{4k}^* and J_{5k}^* respectively represent the coefficients of the terms $\frac{\partial^2 A}{\partial x_1^2}$, $\frac{\partial^2 A}{\partial x_1^0 t_1}$, $\frac{\partial^2 A}{\partial t_1^2}$, $\frac{\partial A}{\partial x_2}$ and $\frac{\partial A}{\partial t_2}$ of equation (34).

Following the lines of El-Dib (1993), we can examine the stability criteria by assuming that the solution varies with time only.

$$i.e., \; A = m\exp(iQ_k m^2\tau), \quad \text{where } m \text{ is a constant.} \tag{36}$$

The above time-dependent solution is now perturbed as

$$A = [m + A_{1k}(\xi,\tau) + iB_{1k}(\xi,\tau)]\exp(iQ_k m^2\tau) \tag{37}$$

where A_{1k} and B_{1k} are real functions.

Substitution of (37) in Nonlinear Schrödinger equation (35) yields the **dispersion relation**

$$\omega_{1k}^2 + \left[\alpha_{1k}^{*2}k_1^4 - k_1^2\alpha_k^{*2} - 2ik_1^3\alpha_k^*\alpha_{1k}^*\right] = 0 \tag{38}$$

where k_1 is the wave number and ω_{1k} is the disturbance frequency.

6 Discussions and Conclusions

The linear dispersion relation for the case of a periodic tangential magnetic field coincides with the linear dispersion relation in the case of tangential magnetic field when the field frequency $\omega = 0$. It is found that in general, the system is unstable when the effects of rotation and tangential magnetic field are coupled (see also Anjalidevi and Jothimani, 2001).

The instability sets in when $H \geq H_c$. The system bifurcates into a new steady state for the post critical values of the magnetic field. Cowley and Rosensweig (1967) have experimentally verified the value of the critical magnetic field is given by $H_c^2 = \frac{2(1+\mu)}{(1-\mu)^2}[(\rho_1 - \rho_2)gT]^{\frac{1}{2}}$.

It was pointed out by Malik and Singh(1984) and Kant and Malik (1985) that when $k^2 = \frac{1}{2}[(\rho_1 - \rho_2)g/T]$ the second order solutions have a singularity. Physically this represents a second harmonic internal resonance. The wave number $'k'$ here is independent of the magnetic field. For ordinary surface capillary gravity waves, the resonant wave number is given by $k = (\rho_1 g/2T)^{\frac{1}{2}}$ and the perturbation expansions are no longer uniformly valid for the second order problem due to nonlinear focusing [MCGoldrick (1970)].

The homogeneous part of the third-order problem has a nontrivial solution that is the same as the first order problem. The inhomogeneous problem has a solution if and only if the inhomogeneous part is orthogonal to every solution of the adjoint homogeneous problem.

A nonlinear Schrödinger equation with complex coefficients has been derived from the solvability conditions and used to analyse the stability of the system. A quadratic dispersion relation with complex coefficients has been obtained.

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