

Mathematical Modeling of Two Dimensional Ferrofluid Flow in Thermal and Buoyant Conditions in a Trough

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Abstract The present problem addresses a thermally driven two-dimensional, buoyant flow in a vessel with the application of magnetic field directed in the radial and tangential direction. In the present study, a trough filled with ferrofluid is heated along the center strip by an applied heat flux. Thereby the convection pattern along with the heat distribution is observed. The half of the trough dynamics is calculated with the symmetric plane in the center. On the surface of the bowl, penalty function is applied to enforce the no-slip boundary condition. This problem is then modelled and the results are obtained for flow velocity, vorticity, density and temperature distribution. The results are then compared in the same scenario but in the absence of magnetic field.

Keywords: Ferrofluid, vorticity, magnetic field.

1 Introduction

Ferro fluids consists of stable colloidal mixture of nano-sized ferrous particles in a non-magnetic fluid carrier. For a stable suspension, the nano sized ferrous particles are coated with a suitable surfactant to avoid agglomeration[Xuan et al (2005), Sunil et al. (2011)]. Ferro fluids have a wide application in the field of mechanical, thermal and aerospace. It is specifically used in sealing of hard drives, rotating X-ray tubes and controlling heat generation in speakers. It also increases the acoustics and longevity of any speaker system. Recently ferrofluids have also found application in bio medical field, the nano

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sized ferrous particles when coated with hydro glucose polymers can be used in the treatment of cancer [Odenbach (2002), Alexiou et al. (2000)]

In the last five decades, efforts have been made to produce ferro fluids whose physical properties can be altered by introducing externally controlled electronic signals [Scherer (2005)]. Thereby introducing an external magnetic field can be used to enhance the viscose properties of ferro fluids. Hence the study of magneto viscous effects carries importance for the application of ferro fluids in engineering and other related departments [Rosensweig (1985), Engel et al. (2003)].

When a temperature gradient is introduced, there is a thermal motion between the nano sized ferrous particles and the carrier fluid and vice versa, thereby increases the transfer of temperature between the particles and the carrier fluid [Berkovsky et al. (1993, 1996)]. This results in an energy transfer, that is $U_T \sim k_B T$ where k_B represents the Boltzmann's constant and T corresponds to absolute temperature.

The effect of boundary layer in a general flow is extensively given in Schlichting [Schlichting (1960)]. The linear instability analysis of horizontal layer of ferrofluids rotating about its vertical axis, when heated from bottom, in the presence of magnetic field is discussed by Venkatasubramanian and Kaloni [1994]. The effect of alternating uniform magnetic field on convection in a horizontal layer of a ferrofluid within the framework of a quasi-stationary approach is studied by Belyaev and Simordin [2009]. Sekar et al [1993] used Darcy model to predict the effect of magnetic field along vertical axis on thermo-convective instability of ferrofluid in a rotating porous medium.

Nanjundappa et al. [2010] studied Benard-Marangoni ferroconvection in a ferrofluid layer in the presence of a uniform vertical magnetic field with magnetic field dependent (MFD) viscosity. Ram et al. [2010] solved the non-linear differential equations under Neuringer-Rosensweig model for ferrofluid flow by using power series approximations and discussed the effect of magnetic field-dependent viscosity on the velocity components and pressure profile. The effect of negative viscosity on ferrofluid flow due to a rotating disk under alternating magnetic field is studied by Bhandari and Kumar [2014]. Effect of magnetization force on ferrofluid flow due to a rotating disk in the presence of an external magnetic field is studied by Bhandari and Kumar [2015].

In the present problem, a trough filled with ferro-fluid is heated along the center strip by application of heat flux. The convection pattern and the heat distribution is predicted and

analyzed. The computation of the flow parameters for half of the trough is done with a symmetric plane in the center along with an external magnetic field.

2 Mathematical formulation of the problem

This problem addresses the problem of thermally driven buoyant flow of a ferrofluid in a vessel in two dimensions. In the Boussinesq approximation, fluid is assumed incompressible, except for thermal expansion effects, which generate a buoyant force. In the present study, the Shliomis model [1994] is used in the problem formulation and this model leads to the governing equations which are considered from Navier-Stokes equations of magnetization. The constitutive set of equations is as follows:

The equation of continuity is

$$\nabla \cdot \mathbf{v} = 0 \quad (1)$$

The equation of motion is

$$\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \nu \nabla^2 \mathbf{v} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} + \frac{I}{2\tau_s} \nabla \times (\boldsymbol{\omega}_p - \boldsymbol{\Omega}) + \mathbf{F} \quad (2)$$

The equation of rotational motion is

$$I \frac{d\boldsymbol{\omega}_p}{dt} = \mathbf{M} \times \mathbf{H} - \frac{I}{\tau_s} (\boldsymbol{\omega}_p - \boldsymbol{\Omega}) \quad (3)$$

\mathbf{v} is the fluid velocity, p is the pressure, ν is the kinematic viscosity, μ_0 magnetic permeability of free space, \mathbf{M} is the magnetization, \mathbf{H} is the magnetic field intensity, $\boldsymbol{\omega}_p$ is the angular velocity of the particle, $\boldsymbol{\Omega}$ is the vorticity of the flow, \mathbf{F} is the vector of body forces, χ is the magnetic susceptibility, t is the time, τ_s is the Neel relaxation time and I is the sum of the particle moment of inertia.

Here, the inertial term is negligible in comparison with relaxation term as: $I \frac{d\boldsymbol{\omega}_p}{dt} \ll I \frac{\boldsymbol{\omega}_p}{\tau_s}$,

therefore, equation (3) can be written as:

$$\boldsymbol{\omega}_p = \boldsymbol{\Omega} + \frac{\tau_s}{I} (\mathbf{M} \times \mathbf{H}) \quad (4)$$

Now due to (4), equation (2) is modified as

$$\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \nu \nabla^2 \mathbf{v} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} + \frac{\mu_0}{2} \nabla \times (\mathbf{M} \times \mathbf{H}) + \mathbf{F} \quad (5)$$

Two torques are acting on the particles which are called magnetic torque and viscous torque. The magnetic torque is denoted by $\mathbf{M} \times \mathbf{H}$ and the viscous torque is defined as the speed of the particles differing from the vorticity of the flow i.e. $(\boldsymbol{\omega}_p - \boldsymbol{\Omega})$. The equilibrium of both torques, which leads to 2athe hindrance of particle rotation, can thus be written as:

$$\mu_0(\mathbf{M} \times \mathbf{H}) = -6\nu\varphi(\boldsymbol{\Omega} - \boldsymbol{\omega}_p) \tag{6}$$

The expression for mean magnetic torque becomes

$$\mu_0(\overline{\mathbf{M} \times \mathbf{H}}) = -6\nu\varphi\boldsymbol{\Omega} \tag{7}$$

Now, we calculate

$$\begin{aligned} \frac{\mu_0}{2} \nabla \times (\overline{\mathbf{M} \times \mathbf{H}}) &= \frac{1}{2} \nabla \times -6\nu\varphi e_m \boldsymbol{\Omega} \\ &= -\frac{3}{2} \nu\varphi e_m \nabla(\nabla \cdot \mathbf{v}) + \frac{3}{2} \nu\varphi e_m \nabla^2 \mathbf{v} = \frac{3}{2} \nu\varphi e_m \nabla^2 \mathbf{v} \end{aligned} \tag{8}$$

Here e_m is the effective magnetization parameter, φ is the volume fraction. Now with the help of (8), the equation of motion can be written as:

$$\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\nabla p + \nu \left(1 + \frac{3}{2} \varphi e_m \right) \nabla^2 \mathbf{v} + \mu_0 (\mathbf{M} \cdot \nabla) \mathbf{H} \tag{9}$$

If the flow is steady, we may drop the time derivative and we take the curl of the (steady-state) momentum equation, we get

$$\nabla \times (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \times \nabla p + \nu \left(1 + \frac{3}{2} \varphi e_m \right) \nabla \times \nabla^2 \mathbf{v} + \mu_0 \nabla \times (\mathbf{M} \cdot \nabla) \mathbf{H} \tag{10}$$

Using the continuity equation $\nabla \cdot \mathbf{v} = 0$ and $\nabla \cdot (\nabla \times \mathbf{v}) = 0$ and defining the vorticity $\boldsymbol{\Omega} = \nabla \times \mathbf{v}$, we get

$$(\mathbf{v} \cdot \nabla) \boldsymbol{\Omega} = (\boldsymbol{\Omega} \cdot \nabla) \mathbf{v} + \nu \left(1 + \frac{3}{2} \varphi e_m \right) \nabla \times \nabla^2 \mathbf{v} + \mu_0 \nabla \times (\mathbf{M} \cdot \nabla) \mathbf{H} \tag{11}$$

$(\boldsymbol{\Omega} \cdot \nabla) \mathbf{v}$ represents the effective vortex stretching, and it is zero in two-dimensional systems. Furthermore, in two dimensions the velocity has only two components, say v_1 and v_2 , and the vorticity has only one, which we shall write as ω .

If we define a scalar function ψ such that

$$v_1 = \frac{\partial \psi}{\partial y} \text{ and } v_2 = -\frac{\partial \psi}{\partial x} \quad (12)$$

Hence the equation of continuity can be written as:

$$\nabla \cdot \mathbf{v} = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

Now, we may write:

$$\nabla \cdot (\nabla \psi) = -\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} = -\omega \quad (13)$$

Using ψ and ω , we may write the equation (2.10) as:

$$\frac{\partial \psi}{\partial y} \times \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \times \frac{\partial \omega}{\partial y} = \nu \left(1 + \frac{3}{2} \varphi e_m \right) \nabla^2 \boldsymbol{\Omega} + \mu_0 \nabla \times (\mathbf{M} \cdot \nabla) \mathbf{H} + \nabla \times \mathbf{F} \quad (14)$$

If \mathbf{F} is a gravitational force, then

$$\mathbf{F} = (0, -g * \rho) \text{ and } \nabla \times \mathbf{F} = -\mathbf{g} \frac{\partial \rho}{\partial x} \quad (15)$$

where ρ is the fluid density and g is the acceleration of gravity.

The temperature of the system may be found from the heat equation

$$\rho c_p \left[\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right] = \nabla \cdot (k \nabla T) - \mu_0 T \frac{\partial \mathbf{M}}{\partial T} \cdot \left(\frac{\partial \mathbf{H}}{\partial T} + \mathbf{v} \cdot \nabla \right) \mathbf{H} \quad (16)$$

Here, c_p is the specific heat at constant pressure, T is the temperature, k is the thermal conductivity..

If we assume linear expansion of the fluid with temperature, then

$$\rho = \rho_0 [1 + \alpha(T - T_0)] \text{ and } \nabla \times \mathbf{F} = -\mathbf{g} \rho_0 \alpha \frac{\partial T}{\partial x} \quad (17)$$

Here α is the thermal expansion constant and T_0 is the initial temperature.

3 Numerical solution

The numerical solution of nonlinear-coupled ode in COMSOL Multiphysics is obtained by finite elements. The following steps are taken in the solution:

- (i). Create geometry (Draw)

- (ii). Define the PDE/ ODE (Physics)
- (iii). Set boundary conditions (Physics)
- (iv). Make the mesh (Mesh)
- (v). Solve (Solve)
- vi). Plot (Post-processing)

The geometry is built in a drawing program, from elementary bodies (2D arc). Note that the objects are snapped to the grid, change the grid under Options>Grid Settings if needed. In our problem x axis is dimensionless distance, therefore, in the geometry we draw a line of length 6 unit. The general form of pde equation is defined in COMSOL is:

$$e_a \frac{\partial^2 \mathbf{u}}{\partial t^2} + d_a \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (-c \nabla \mathbf{u} - \alpha \mathbf{u} + \gamma) + \beta \cdot \nabla \mathbf{u} + a \mathbf{u} = f \quad (18)$$

where $\nabla = \frac{\partial}{\partial x}$

In the present problem, we define the coefficient involved in the equation (18) and it is modeled as given in (11),(14),(16). Under the global definition in the COMSOL, we add the parameters there and then parametric sweep is applied to obtain the results for different values of magnetic field intensity. To getting the more accurate solution, we have to select different kind of mesh category in the toolbar. In COMSOL, we can use any one of the meshes from Extremely Fine Mesh, Finer Mesh, Fine Mesh, Normal Mesh, Coarse Mesh, Coarser Mesh, Extra Coarse and Extremely Coarse Mesh. Without sacrificing the relevant phenomena, we have used extremely fine mesh. The solution is obtained for two different size of mesh, in first case we take 0.06 element size and in second one the size of the element is taken is 0.0001. Finally, under the result category, we obtain the 2-D plot for different values of the parameter.

In the present problem, thermally driven buoyant flow of ferrofluid in a vessel in two dimensions is studied. Here, we have computed one fourth part of the bowl. The length of the region in x direction is 1 and in y direction is 0.5. This problem is solved using finite element method and the process of mesh generation is represented in figure 1.

Along the symmetry plane, velocity in the z direction is zero, since on this plane

$$\frac{\partial v_2}{\partial x} = 0 \quad \text{and} \quad v_1 = 0, \quad \text{so that} \quad \frac{\partial v_1}{\partial y} = 0.$$

Applying the boundary condition $\psi = 0$ forces the streamlines to be parallel to the boundary, enforcing no flow through the boundary. On the surface of the bowl, a penalty function is applied to enforce a no-slip boundary condition. This is used by using a natural boundary conditions to introduce a surface source of vorticity to counteract the tangential velocity. The penalty weight was arrived at by trial and error. Larger weights can force the surface velocity closer to zero, but this has no perceptible effect on the temperature distribution in the flow of magnetic fluid.

On the free surface, the proper boundary condition for the vorticity is problematic, because this implies no vorticity transport across the free surface. This problem is modeled with the following parameters:

Length along x axis:	1
Length along y axis:	0.5
Surface heat coefficient at the top:	0.001
Bowl heat loss coefficient:	1
Acceleration due to gravity in y direction:	980
Thermal conductivity:	0.004
Viscosity:	1.1
Density:	1
Thermal expansion coefficient	0.001
Heat In:	10
Initial temperature:	50
Specific heat:	1
Magnetic field intensity: 0, 10, 100	

The region of the flow at boundaries is taken as follows:

Region 1

{on the arc of the bowl, set $\psi = 0$, and apply a conductive loss to T.

Apply a penalty function to w to force the tangential velocity to zero }

start "outer" (0,0)

natural (Temperature) = -bowl heat loss coefficient * temperature

value (ψ) = 0

natural(w)= penalty*tangential (curl(ψ))

arc (center=0, Rad) to (Lx,Ly)

{on the top, continue the $\psi = 0$ BC, but add the heat in put term to T,
and apply a natural=0 BC for w}

natural(w)=0

load(temp) = heatin*exp(-(10*x/Lx) ^2) – heat loss coefficient*temp

line to (0, Ly)

{in the symmetry plane assert w=0, with a reflective BC for T }

value(w)=0

load(temp) = 0

line to close

Equations (11), (14) and (16) are modeled in PDE interface of COMSOL and the solution is obtained using finite element method.

4 Results and discussions

In present problem, density, vorticity, flow velocity and temperature distribution are studied for different values of magnetic field intensity. Here, $H = 0$ is the ordinary case where magnetic field is absent.

The figure 1 shows the mesh generation for the fluid flow. Triangular mesh is taken into consideration for smooth profile. Figure 2 represents the density profile of the ferrofluid flow in the circular trough in the absence of magnetic field intensity. Here the flow behavior is of Newtonian model. In the subsequent figures Figure 3 and 4 the density profile is observed with different magnetic field intensities. It is perceived that the density in the flow decreases with increase in the external magnetic field intensity. The minimum value of the density profile in the flow was 0.57 in the absence of the external magnetic field but the minimum value of density decreased to 0.56 at $H = 10$ and further 0.52 at H

= 100. The maximum density in the flow did not get affected with the application of the external magnetic field. It is observed that the density decreases in the radial direction when an external magnetic field is applied to the ferrofluid flow.

The figures (Fig. 5-7) show the velocity distribution of the flow field. In Figure 5, the velocity distribution of the flow is in the absence of any external magnetic field. Figure 6 and 7 represents the velocity of the flow field at different magnetic field intensities viz. $H = 10$ and $H = 100$. It is observed that in the presence of the external magnetic field the velocity of the flow decreases with the increase in magnetic field intensity.

Figures 8, 9 and 10 represent the temperature distribution over quarter of the trough. Figure 8 shows the temperature distribution in the absence of any external magnetic field. The temperature gradient is more towards the centre of the strip and decreases at the edges. Figure 9 and 10 represents the temperature distribution when an external magnetic field is applied with different intensities. It is observed that the temperature gradient increases with the increase in the external magnetic field intensity.

Lastly, figure 11, 12 and 13 shows the vorticity of the magnetic fluid flow over the quarter of the trough. The figure 11 shows the vorticity profile in the absence of magnetic field. As the magnetic field is applied to the flow, the angular velocity decreases and at higher intensities the flow is disturbed as the vorticity is splits in different regions of the flow field. At low magnetic field the angular velocity of the flow can be controlled.

5 Conclusions

The present study revealed that introduction of the magnetic field in the ferrofluid flow changes the field parameters significantly. The temperature and velocity gradient can be controlled in the flow when the magnetic field intensity is controlled. This is utilized in various engineering applications like aerospace, magnetic speakers, rotating machinery and in ocean engineering. If the magnetic field intensity is carefully applied to the flow, the characteristics of the flow can be obtained as per the requirement. There lies a vast scope of research to precisely control the flow field as per need of the application.

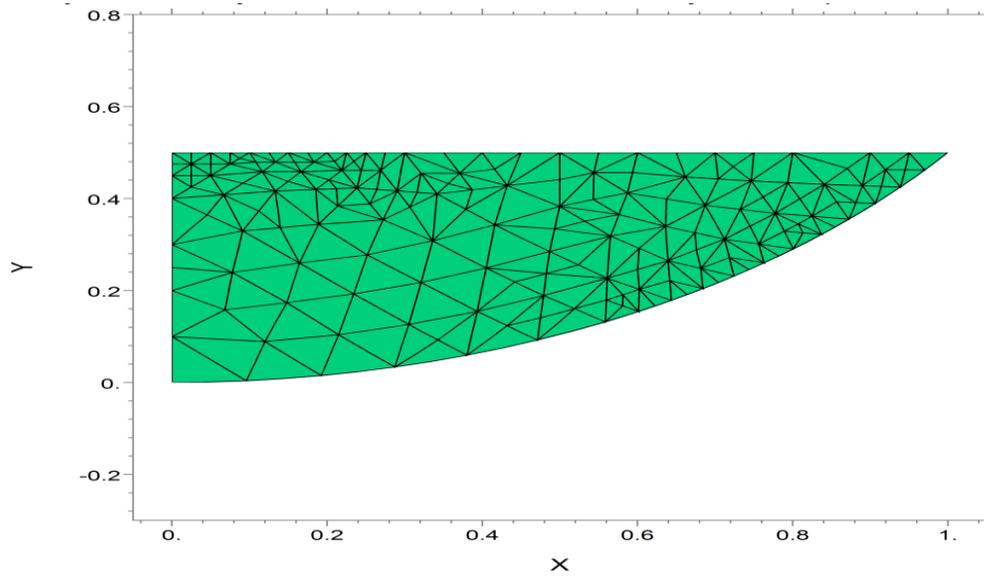


Figure 1: Mesh generation process

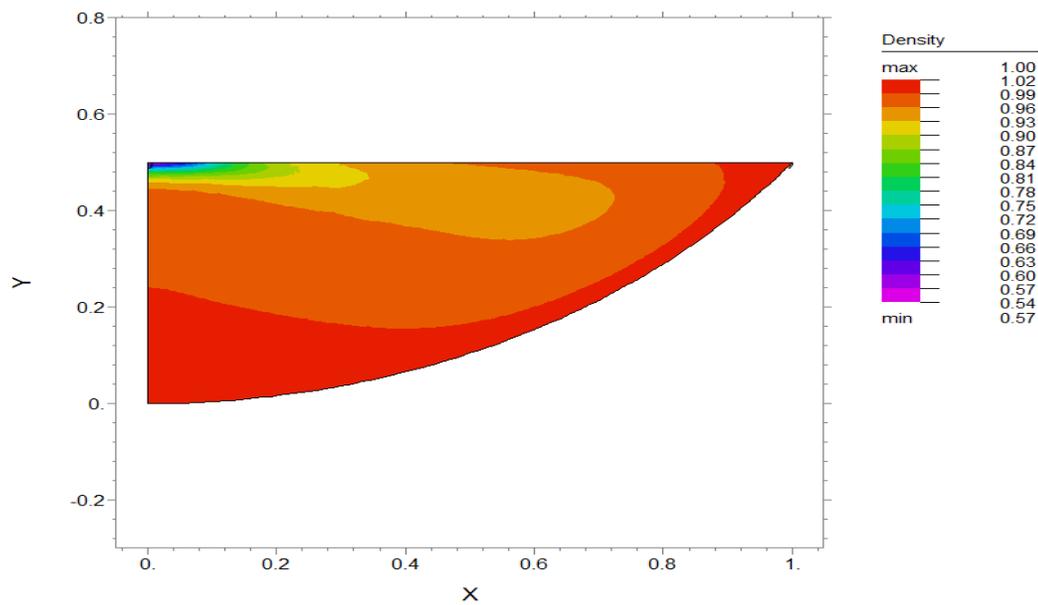


Figure 2: Density profile at $H = 0$

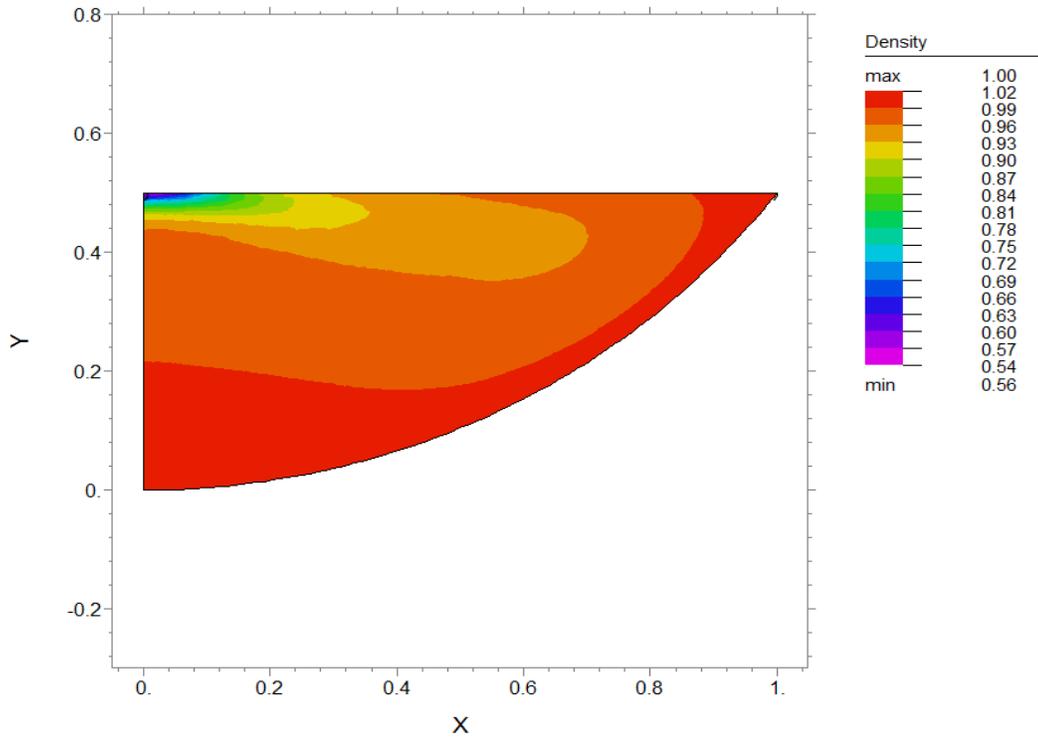


Figure 3: Density profile at $H = 10$

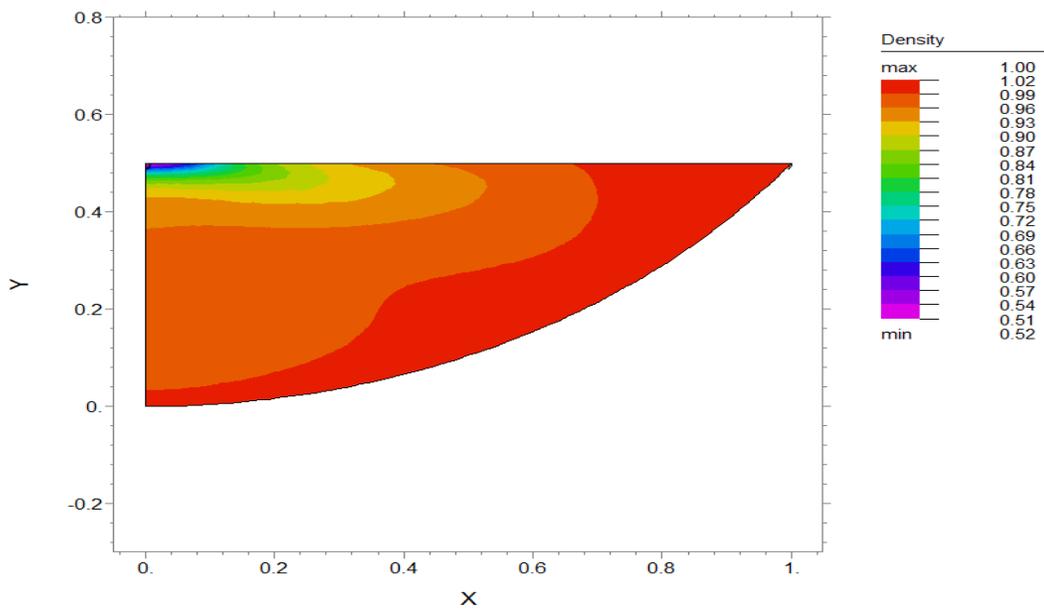


Figure 4: Density profile at $H = 100$

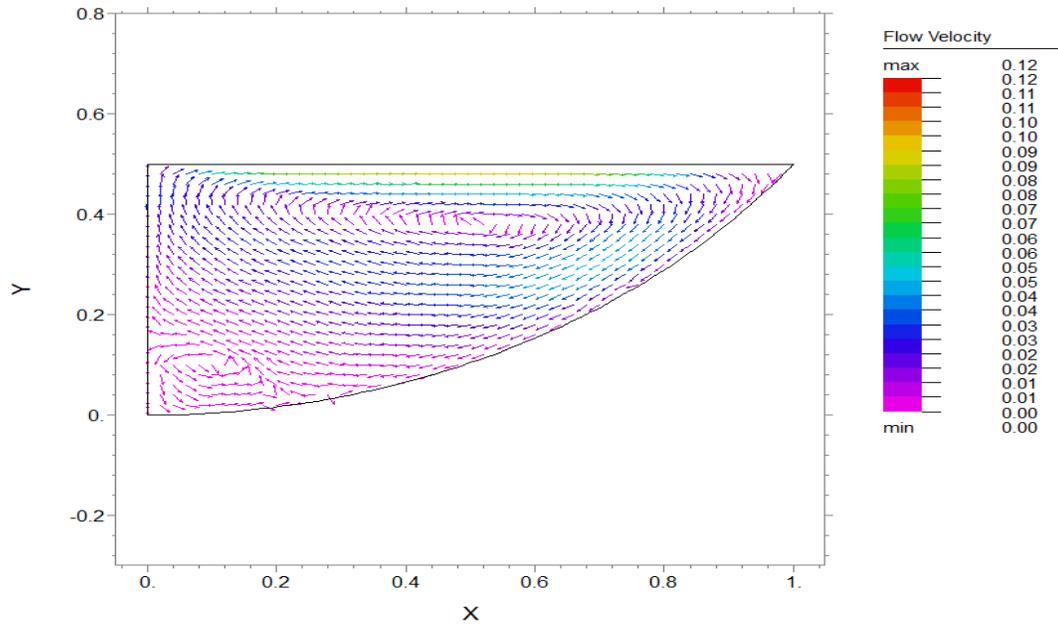


Figure 5: Flow velocity at $H = 0$

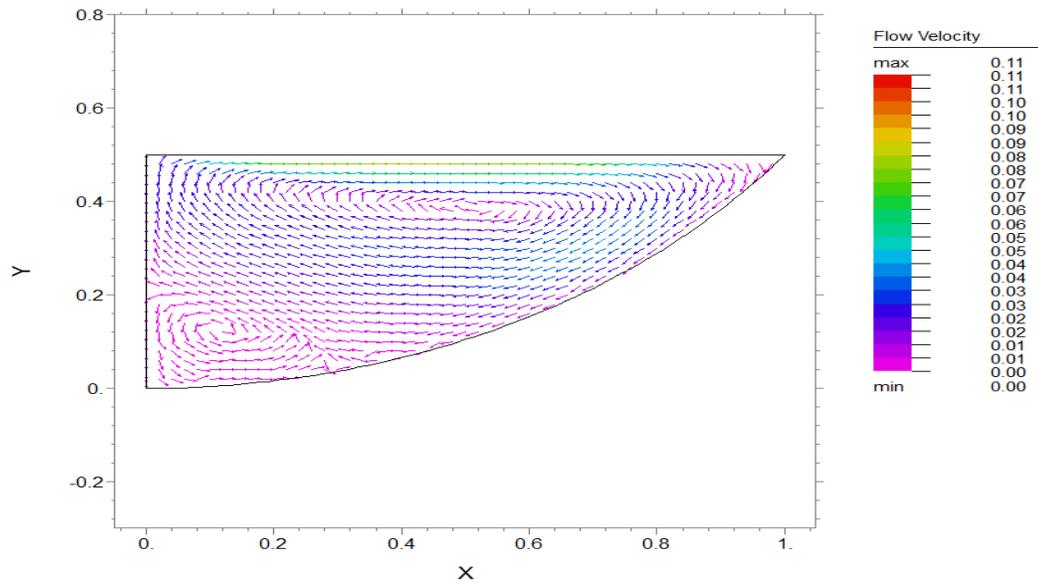


Figure 6: Flow velocity at $H = 10$

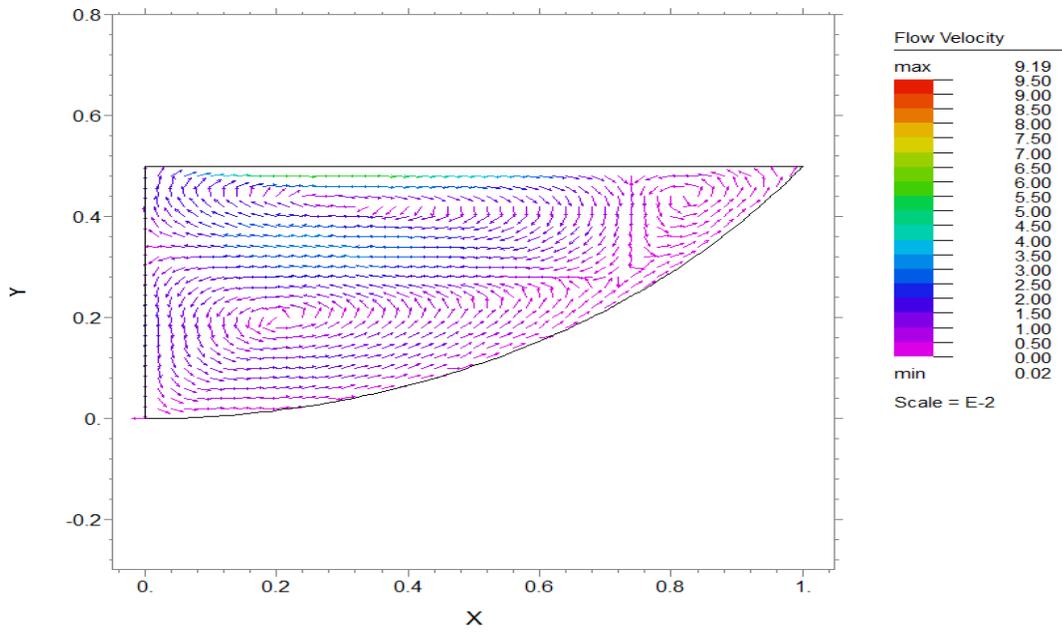


Figure 7: Flow velocity at $H = 100$

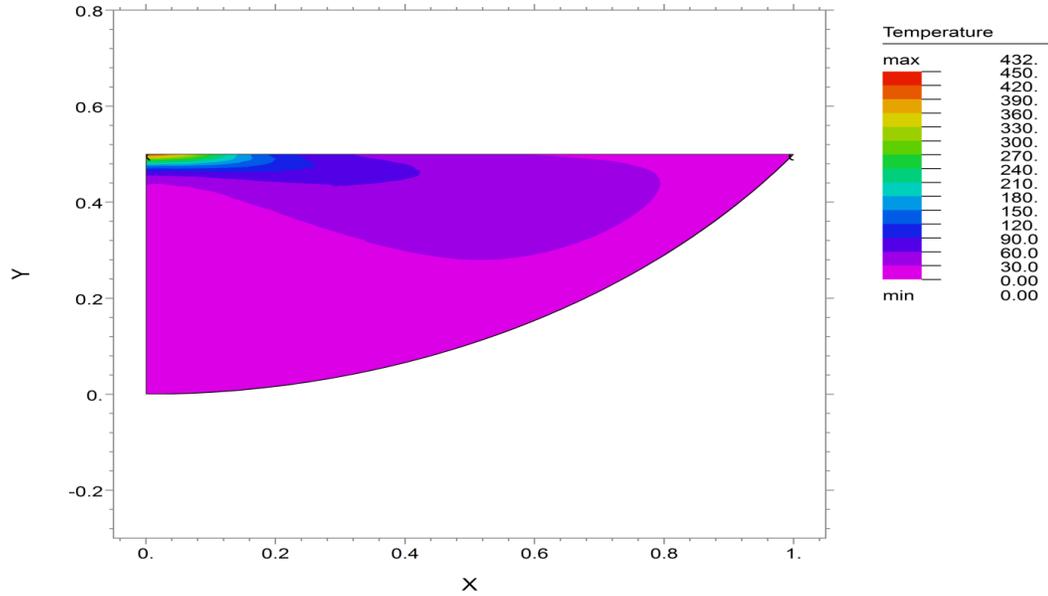


Figure 8: Temperature distribution at $H = 0$

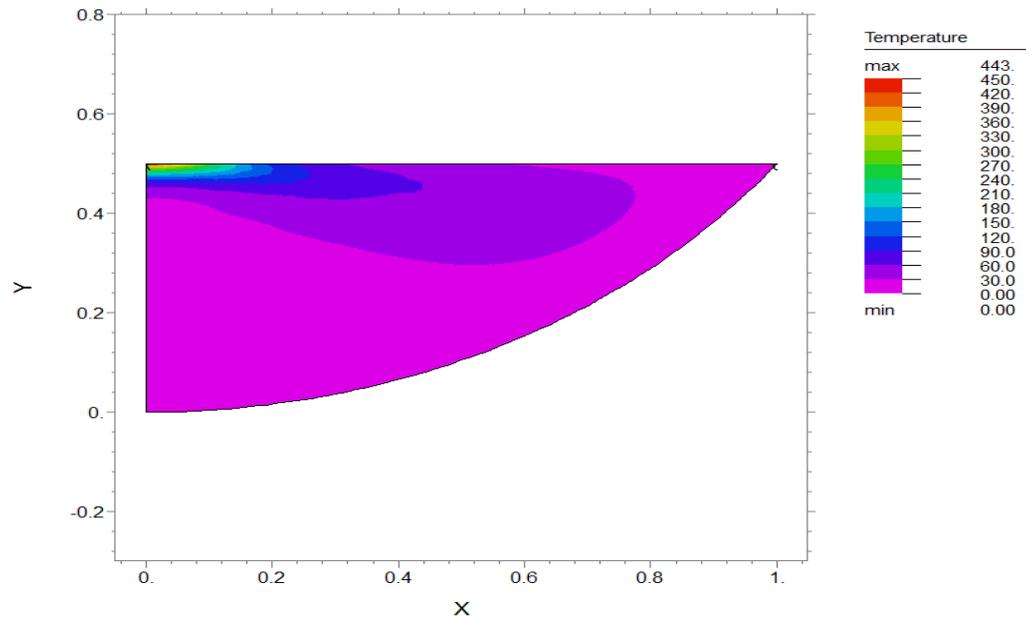


Figure 9: Temperature distribution at $H = 10$

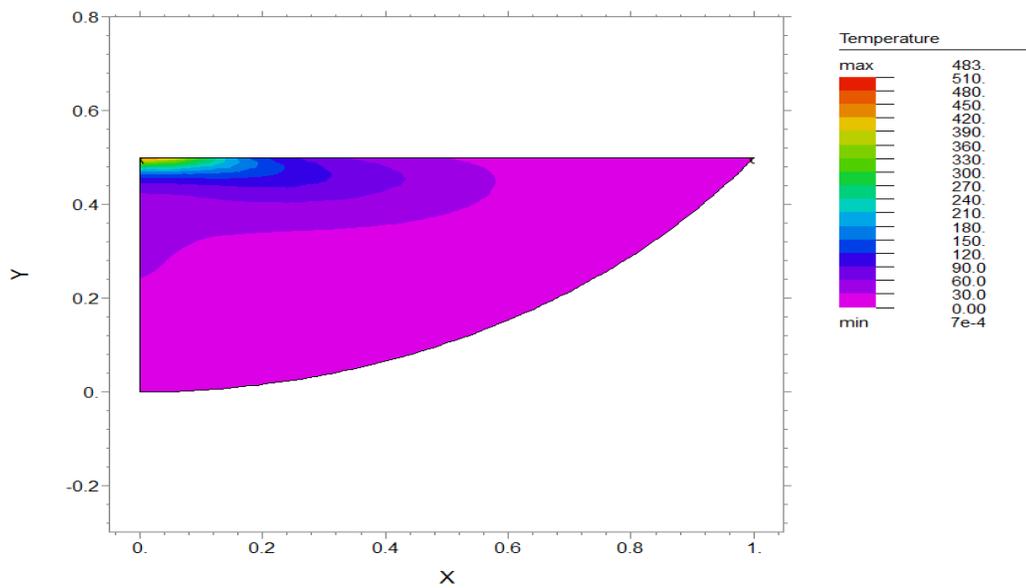


Figure 10: Temperature distribution at $H = 100$

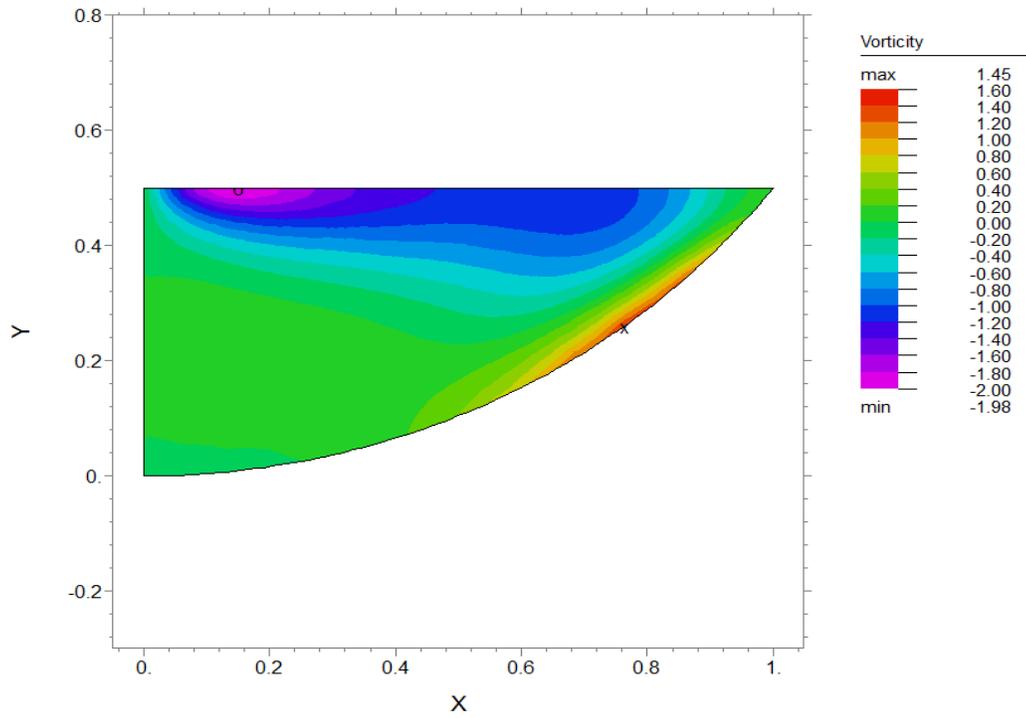


Figure 11: Vorticity in the flow at $H = 0$

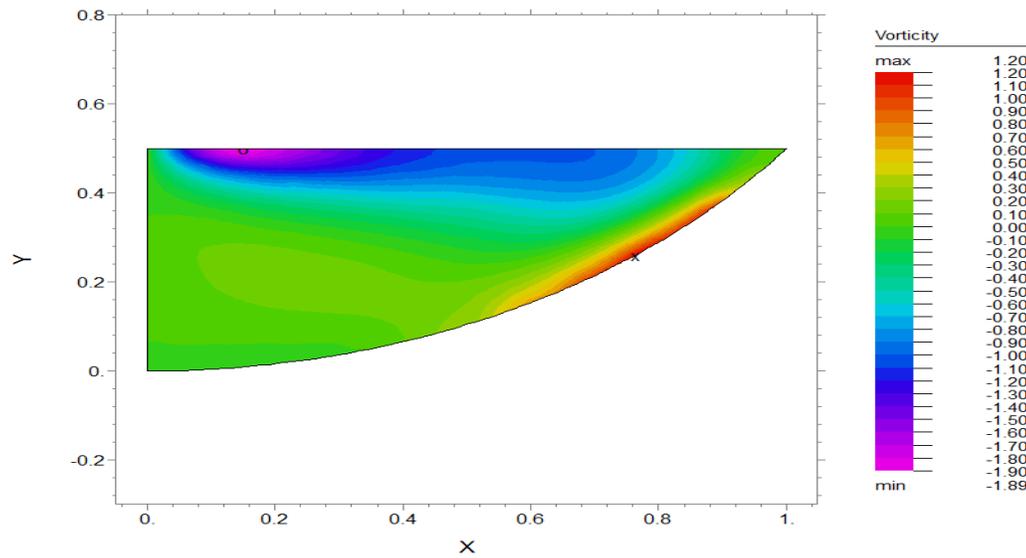


Figure 12: Vorticity in the flow at $H = 10$

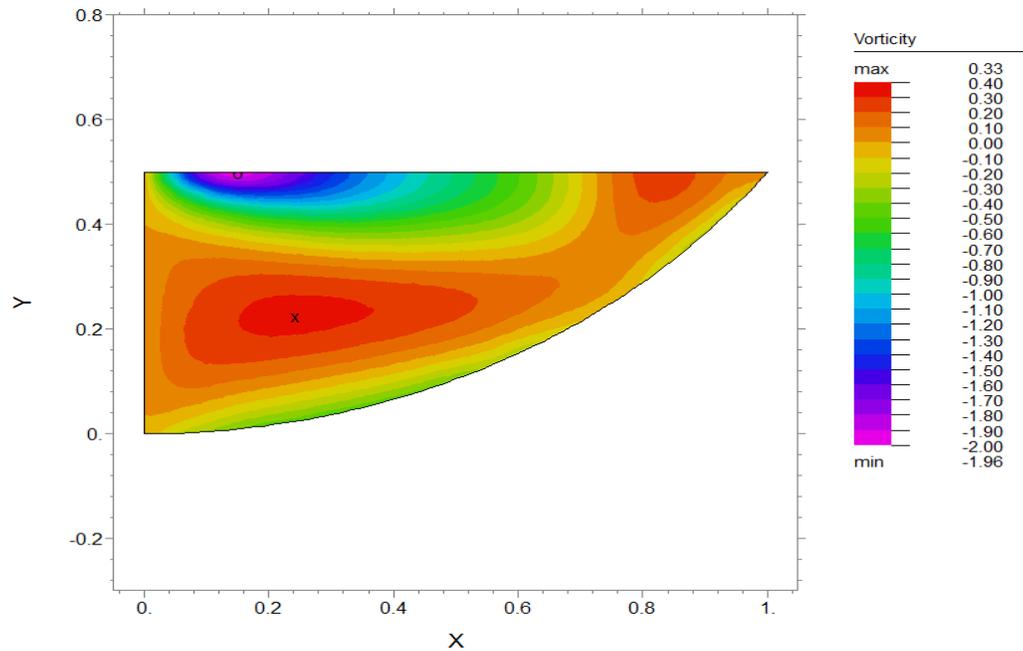


Figure 13: Vorticity in the flow at $H = 100$

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