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Combined MHD and Pulsatile Flow on Porous Medium

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Abstract: This work investigates the dynamic behavior of a pulsatile flow electrically conducting through porous medium in a cylindrical conduit under the influence of a magnetic field. The imposed magnetic field is assumed to be uniform and constant. An exact solution of the equations governing magneto hydro-dynamics (MHD) flow in a conduit has been obtained in the form of Bessel functions. The analytical study has been used to establish an expression between the Hartmann number, Darcy number and the stress coefficient. The numerical method is based on an implicit finite difference time marching scheme using the Thomas algorithm and Gauss Seidel iterative method for solving the resulting algebraic system of equations. The results show that the flow behavior is strongly affected by the permeability parameter of medium porosity and the Hartmann number. It has also shown that the stress coefficient has a sinusoidal aspect and it increases with decreasing Darcy number.

Keywords: Pulsatile flow, stress coefficient, permeability, magnetic field, analytical solution, finite difference.

1 Introduction

The study of magneto hydro-dynamics through a uniform porous medium has attracted attention for its applications in diverse domains. Common examples in medical science include peristaltic food motion in the intestine, motion of urine in urethra, and blood flow through an artery. In engineering we find its applications in MHD pump. It is also very common in astrophysical theoretical stellar structure and in geophysics, as well as in cores terrestrial and solar plasma. In biological systems, the control of blood pressure through human arterial system is being possible by using porous effect with an application of external magnetic field on blood flow, in cases of chole sterol and related diseases, which contributes to an increase in the friction of flowing blood.

Vardanyan [Vardanyan (1973)] have developed several theoretical models on MHD effects on a pulsatile flow. They found that the application of a constant and uniform magnetic field decreases the flow rate. Their work had a significant impact on biological research.
Amos et al. [Amos and Ogulu (2002)] conducted a numerical study on the pulsatile flow in a conduit with a constriction in the presence of an external uniform magnetic field. Since

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the speed decreases when the magnetic field increases, the only way to circumvent this problem of speed reduction is to increase the flow pressure. This corresponds to the increase in the work load of the heart which can lead to heart attacks. Mansutti et al. [Mansutti, Pontrelli and Rajagopal (1993)] discussed the case of fluid with a steady flow past a porous plate with suction or injection. Ruunge et al. [Ruunge and Rusetski (1993)] investigated the effects of magnetic fields on bio magnetic fluid flow with ample applications in bioengineering and medical sciences. Ramamurthy et al. [Ramamurthy and Shanker (1994)] have studied the magneto hydrodynamics effects on blood flow through a porous channel. The pulsatile flow of blood through a porous medium under periodic body acceleration was investigated by Elshehawey et al. [Elshehawey, Elbarbary and Elshahed (2000)]. Saynal et al. [Saynal and Biswas (2010)] analyzed the effect of uniform transverse magnetic field on its pulsatile motion through an axisymmetric tube, assuming blood to be an incompressible fluid.

The Porous medium is defined by its permeability, which is a measure of the flow conductivity in the material volume consisting of solid matrix with an interconnected void. It is characterized by the porosity, the ratio of void space to void volume of the medium and the Darcy law, which relates linearly the flow velocity to the pressure gradient across the porous medium. The combination of the fluid and the porous medium is the tortuosity which represents the hindrance to flow diffusion imposed by local boundaries or local viscosity. The tortuosity is especially important in medical applications as works investigated by Khaled et al. [Khaled and Vafai (2003)]. Sinha et al. [Sinha and Misra (2011)] investigated a mathematical modeling of blood flow in porous vessel, having double stenosis in the presence of an external magnetic employing Darcy's law. The peristaltic flow of a compressible non-Newtonian Maxwellian fluid through porous medium in a tube was investigated by Eldesoky et al. [Eldesoky and Mousa (2010)]. Abeer et al. [Abeer and Ahmed (2011)] studied the pulsatile flow of a non-Newtonian fluid, through a non-Darcy porous medium between two permeable parallel plates. Majdalani [Majdalani (2008)] obtained the analytical solution for the evolution of the velocity profiles and the stress coefficient in the case of a pulsatile flow. Hatami et al. [Hatami, Hatami and Ganji (2014)] studied the third-grade non-Newtonian blood conveying gold nanoparticles in a porous and hollow vessel. The authors used two analytical methods called Least Square Method and Galerkin Method (GM). Asim and Taha (2013) investigated the solution and modeling of the unsteady flow of an incompressible third grade fluid over a porous plate within a porous medium by considering that the fluid is electrically conducting in the presence of a uniform magnetic field applied transversely to the flow. Das et al. [Das and Saha (2009)] studied the arterial MHD pulsatile flow of blood under periodic body acceleration. Under the influence of an imposed pressure gradient and periodic body acceleration, the flow of combined two phase motion of viscous ideal medium, through a parallel plate channel was analyzed by Rao et al. [Rao, Ravikumar, Vasudev et al. (2011)]. Adesanya [Adesanya (2012)] studied the effect of couple stresses under a uniform external magnetic field on an unsteady magneto hydrodynamic (MHD) flow between two parallel fixed porous plates. For that, Eyring-Power model was used and they found that the flow is damped with the increase of the couple stresses effect. The mathematical frame work for the laminar flow in the presence of external magnetic fields, with the effect as a means to various medical application of a non-Newtonian fluid, was analyzed by Tzirakis et al. [Tzirakis, Botti, Vavourakis et al. (2016)]. Bouvier [Bouvier (2005)] studied experimentally the heat transfer in oscillating flow from an analytical development. They have reported a fundamental system in oscillating flow.

A survey of oscillating flow in stirling engine heat exchangers was studied by Simon et al. [Simon and Seume (1988)]. Ikbal. Md et al.[Ikbal, Chakravarty, Kelvin et al. (2009)] investigated atherosclerotic arteries deals with mathematical models that represent non-Newtonian flow of blood through a stenosed artery in the presence of a transverse magnetic field. They reported that the magnetic field causes substantial reduction of the flow rate. The form of magnetic field gradient plays an important role and substantially determines the flow field. Khechiba et al. [Khechiba and Ghezal (2013)] studied the influence of magnetic field on the pulsatile flow through cylindrical conduit. Norzieha et al. (2009) investigated unsteady magneto hydrodynamic blood flow through irregular multi-stenosed arteries. The results obtained show that the flow separates mostly towards the downstream of the multi-stenoses. However, the flow separation region keeps on shrinking with the increasing intensity of the magnetic-field which completely disappears with sufficiently large value of the Hartmann number. Malathy et al. [Malathy and Srinivas (2008)] studied Pulsating flow of a hydro magnetic fluid between permeable beds. The effects of the magnetic field on the velocity fields are calculated numerically for different values of the parameter. It is interesting to note that the velocity can attain its maximum even at the lower permeable bed in the case of some specific choice of parameters. When the Hartmann number tends to zero, Mkheimer et al. [Mkheimer, Haroun, Elkot et al. (2011)] studied the effects of magnetic field and porosity for anisotropically elastic multi stenosis arteries on blood flow characteristics. They reported that the trapping bolus increases in size toward the line center of the tube as the Darcy number increases and decreases when the Hartmann number increased.

Chand et al. [Chand, Rana and Hussein (2015)] studied the Instability in a low Prandtl number nanofluid layer in a porous medium. They reported that Prandtl number and Darcy number have a destabilizing effect on stationary convection. A 3D finite element analysis of incompressible fluid flow and contaminant transport through a porous landfill was studied by Adegun et al. [Adegun, Komolafe, Hussein et al. (2014)]. Wakif [Wakif and Boulahia (2017)] studied numerical analysis of the onset of longitudinal convective rolls in a porous medium saturated by an electrically conducting fluid in the presence of an external magnetic field. Chitra et al. [Chitra and Karthikeyan (2018)] studied oscillatory flow of blood in porous vessel of a stenosed artery with variable viscosity effects of magnetic field. Hatami et al. [Hatami, Hatami and Ganji (2014)] studied the effect of a variable magnetic field (VMF) on the natural convection heat transfer. As a main outcome, results confirm that in low Eckert numbers, increasing the Hartmann number make a decrease on the Nusselt number due to Lorentz force resulting from the presence of stronger magnetic field. Zhou et al. [Zhou, Hatami, Song et al. (2016)] investigated the heat transfer in heat sink by a numerical method. They reported that a result should have practical value for designing the compact heat exchanger. The proposed optimization method is supposed to have a wide application for the time-efficient optimization of heat transfer through irregular configurations.

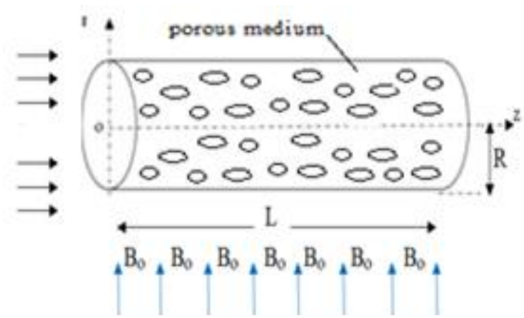


Figure 1: Flow field geometry

2 Mathematical formulation

2.1 Physical problem

The pulsatile flow of a viscous incompressible fluid, through porous medium in a cylindrical conduit of length L and radius R, in the presence of a transverse uniform magnetic field B, is considered. $B = B_0 + B_1$, represents the total magnetic field with the induced magnetic field B_1 assumed to be negligible to the external magnetic field B_0 in MHD flow at small magnetic Reynolds number. The fluid flow is subjected to a pulsatile dimensionless pressure gradient parallel to the axis given by:

$$\frac{\partial P}{\partial z} = -(A_0 + A_1 e^{i\omega t}) \quad (1)$$

A_0: Represents the amplitude of the steady part, and A_1: represents the amplitude of the not stationary part of the pressure gradient.

The fluid which is laminar, Newtonian and with constant physical properties, flows through porous medium with permeability parameter k.The Lorentz electromagnetic force is expressed after neglecting gravity for reasons of axial symmetry of the problem, in the following way:

$$F = J \times B \quad (2)$$

J: Represents the current density vector due to the movement of the conductive fluid, and according to Ohm's law we have:

$$J = \sigma(E + V \times B) \quad (3)$$

Therefore $F = -\sigma \times B^2 \times V = \sigma {B_0}^2 w$ (4)

Where: σ represents the electrical conductivity, w the velocity and E the electric field which is considered as negligible. In order to solve the problem analytically, we assume that the flow occurs only in the axial direction and is fully developed.

Boundary Conditions:

The associated initial and boundary conditions to the model are given here by:

At $t = 0$, $W(r, z, 0) = U(r, z, 0) = 0$ (5)

Where $(r,\ z,\ U,\ W,\ t)$ are the cylindrical coordinates system, the normal and the axial components of the velocities and the time. The velocity component in r and z direction represented by U and W correspondingly.

Along the axis of symmetry, the axial velocity gradient and the normal component of the velocity are given as:

For $\quad r = 0: \frac{\partial W(r,z,t)}{\partial r} = 0, U(r,z,t) = 0$ (6)

At the outlet $\quad z = L: \frac{\partial W(r,z,t)}{\partial z} = \ \frac{\partial W(r,z,t)}{\partial z} = 0$ (7)

Given all the assumptions described above and after projection of the equations on cylindrical coordinates $(r,\ z)$, the equation governing the flow is reduced to [Hatami, Hatami and Ganji (2014)]:

Momentum equation

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \frac{\mu}{\rho}\left[\frac{\partial^2 w}{\partial r^2} + \frac{\partial w}{r\partial r}\right] - \frac{\sigma {B_0}^2 w}{\rho} - \frac{\mu w}{k\rho} \tag{8}$$

The terms μ and ρ represents the dynamic viscosity and the density of fluid.

Using the following dimensionless variables:

$$W^* = \frac{w}{W}, r^* = \frac{r}{R}, z^* = \frac{z}{R}, t^* = \frac{tW}{R}, P^* = \frac{P}{\rho W^2}$$

Therefore, the equation of motion, governing the considered phenomenon can be written as follows:

$$\frac{\partial W}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{R_e}\left[\frac{\partial^2 W}{\partial r^2} + \frac{\partial W}{r\partial r}\right] - \frac{Ha^2 W}{R_e} - \frac{W}{R_e D_a} \tag{9}$$

The parameters Ha, D_a and R_e represents the Hartmann number, Darcy number and Reynolds number respectively, and are defined by:

$$Ha^2 = \frac{\sigma {B_0}^2 R^2}{\mu}, D_a = \frac{k}{R^2}, R_e = \frac{\rho W R}{\mu}$$

2.2 Analytical study

In addition to the previous simplifying assumptions, we assume that the flow occurs only in the axial direction. Under these conditions, the conservation equation governing the pulsated flow is modeled as follows [Khechiba and Ghezal (2013)]:

$$\frac{\partial W}{\partial t} = -\frac{\partial P}{\partial z} + \frac{1}{R_e}\left[\frac{\partial^2 W}{\partial r^2} + \frac{\partial W}{r\partial r}\right] - \frac{1}{R_e}\left(Ha^2 + \frac{1}{D_a}\right) W \tag{10}$$

Steady state

The dimensionless equation to be solved in the steady state is a modified Bessel equation [Vardanyan (1973)]:

$$\frac{d^2 W}{dr^2} + \frac{dW}{rdr} - \left(Ha^2 + \chi^2\right) W = -A_0 R_e \tag{11}$$

Where $\chi^2 = \frac{1}{D_a}$ (12)

The solution is a combination of Bessel functions I_0, K_0 of first and second kind respectively; with homogeneous solution:

$$W_0(r) = C_1\, I_0\left(\sqrt{Ha^2 + \chi^2}\,.r\right) + C_2\, K_0\left(\sqrt{Ha^2 + \chi^2}\,.r\right) \quad (13)$$

and $\frac{A_0\, R_e}{Ha^2 + \chi^2}$ as a particular solution, where A_0 is a known constant. (14)

Boundary Conditions

At the wall of the conduit:

$$r = \mp 1,\ W = 0 \quad (15)$$

C_1, C_2 are constants calculated from the previous boundary conditions:

For: $r = 0:\ W_0(0) = C_1\, I_0(0) + C_2\, K_0(0) + \frac{A_0\, R_e}{Ha^2 + \chi^2}$ (16)

With: $I_0(0) = 1$, $K_0(0) \to \infty$, the constant C_2 must be equal to zero, since the velocity cannot be infinite at $r = 0$.

For: $r = 1:\ W_0(1) = C_1\, I_0\left(\sqrt{Ha^2 + \chi^2}\right) + \frac{A_0\, R_e}{Ha^2 + \chi^2} = 0$ (17)

$$C_1 = -\frac{A_0\, R_e}{Ha^2 + \chi^2}\left(\frac{1}{I_0\left(\sqrt{Ha^2 + \chi^2}\right)}\right) \quad (18)$$

Then, we find:

$$W_0(r) = \frac{A_0\, R_e}{(Ha^2 + \chi^2)}\left(1 - \frac{I_0\left(\sqrt{Ha^2 + \chi^2}\,.r\right)}{I_0\left(\sqrt{(Ha^2 + \chi^2}\right)}\right) \quad (19)$$

This expression is the same founded by Khechiba et al. [Khechiba and Ghezal (2013)], with non-porous medium $\chi^2 = 0$

Unsteady state

To characterize the oscillatory flow, we include the dimensionless frequency $R_{e\omega}$ and we use dimensionless variables of time $t = \omega t$

Where:

$R_{e\omega} = \frac{\rho R^2 \omega}{\mu}$ is equal to the square of the Womersley number $R_{e\omega} = \alpha^2$ (20)

Under these conditions, the dimensionless velocity profile is: $W_1(r,\ t) = R_e(f(r)e^{it})$

The dimensionless equation can be written as follow:

$$\frac{d^2}{dr^2}f + \frac{d}{rdr}f - ((Ha^2 + \chi^2) + i\alpha^2)f = -A_1\alpha^2 \quad (21)$$

The solution of the homogeneous equation $f_h(r)$ is written in the form:

$$f_h(r) = C_1\, I_0\left(\sqrt{Ha^2 + \chi^2 + i\alpha^2}.r\right) + C_2\, K_0\left(\sqrt{Ha^2 + \chi^2 + i\alpha^2}.r\right) \tag{22}$$

and $f_p = \frac{A_1\alpha^2}{(Ha^2 + \chi^2) + i\alpha^2}$ as a particular solution (23)

The constant C_2 must be set equal to zero, since the velocity cannot be infinite at $r = 0$

C_1: Is determined by the condition of velocity $W_1 = 0$ at the wall expressed by:

$f_g(1) = 0$

Such as: $f_g(r) = f_h(r) + f_p$ (24)

The general solution is as follows:

$$f_g(r) = -\frac{A_1\alpha^2}{(Ha^2 + \chi^2) + i\alpha^2}\left(\frac{I_0\left(\sqrt{(Ha^2 + \chi^2) + i\alpha^2}.r\right)}{I_0\left(\sqrt{(Ha^2 + \chi^2) + i\alpha^2}\right)} - 1\right)$$

Letting: $\beta_k = \sqrt{(Ha^2 + \chi^2) + i\alpha^2}$ (25)

We obtain the expression of the dimensionless velocity:

$$W_1(r,t) = -\frac{A_1\alpha^2}{{\beta_k}^2}\left(\frac{I_0(\beta_k.r)}{I_0(\beta_k)} - 1\right)e^{it} \tag{26}$$

The value of A_1 is not known so, we use the flow rate velocity $W = R_e(W_{max}e^{it})$ with the expression given by Bouvier [Bouvier (2005)].

$$W_{max} = -\frac{A_1\alpha^2\omega R}{{\beta_k}^2}\left(\frac{2I_1(\beta_k)}{\beta I_0(\beta_k)} - 1\right) \tag{27}$$

We deduce: $A_1 = -\frac{W_{max}{\beta_k}^2}{\alpha^2\omega R}\left(\frac{\beta_k I_0(\beta_k)}{2I_1(\beta_k) - \beta_k I_0(\beta_k)}\right) = -\frac{R_{emax}{\beta_k}^2}{2\alpha^4}\left(\frac{\beta_k I_0(\beta_k)}{2I_1(\beta_k) - \beta_k I_0(\beta_k)}\right)$ (28)

Where: $R_{emax} = \frac{2\alpha^2 W_{max}}{\omega R}$ (29)

Finally: $W_1(r,t) = \frac{R_{emax}}{2\alpha^2}\left(\frac{\beta_k I_0(\beta_k.r) - \beta_k I_0(\beta_k)}{2I_1(\beta_k) - \beta_k I_0(\beta_k)}\right)e^{it}$ (30)

This equation is similar to the expression that was given by Tzirakis et al. [Tzirakis, Botti, Vavourakis et al. (2016)] in the absence of magnetic field with: $\beta_k = \sqrt{\chi^2 + i\alpha^2}$

It was also that found by Bouvier [Bouvier (2005)], for non-porous medium, and in the absence of magnetic field with: $\beta_k = \sqrt{i\alpha^2}$

Pulsatile state:

The expression of the velocity field in the pulsatile state $W(r,\ t)$ is the superposition of the steady $W_0(r)$ and unsteady component $W_1(r,\ t)$.

$$W(r,\ t) = W_0(r) + W_1(r,\ t) \tag{31}$$

$$W(r,\ t) = \frac{A_0\, R_e}{(Ha^2 + \chi^2)} \left(1 - \frac{I_0\left(\sqrt{Ha^2 + \chi^2}\, .r\right)}{I_0\left(\sqrt{(Ha^2 + \chi^2}\right)} \right) + \frac{R_{emax}}{2\alpha^2} \left(\frac{\beta_k I_0(\beta_k .r) - \beta_k I_0(\beta_k)}{2I_1(\beta_k) - \beta_k I_0(\beta_k)} \right) e^{it} \quad (32)$$

2.2.1 Stress coefficient study

The dimensionless stress coefficient at the wall τ is calculated from the previous expression of the velocity:

$$\tau = -\frac{\partial W}{\partial r}\bigg|_{r=1} = \frac{A_0 R_e}{(Ha^2 + \chi^2)} \frac{I_1\left(\sqrt{Ha^2 + \chi^2}\right)}{I_0\left(\sqrt{(Ha^2 + \chi^2}\right)} - \frac{\beta_k R_{emax}}{2\alpha^2} \left(\frac{\beta_k I_1(\beta_k)}{2I_1(\beta_k) - \beta_k I_0(\beta_k)} \right) e^{it} \quad (33)$$

It can be written by the following formulation:

$$\tau = \tau_S + \tau_{OSC} \quad (34)$$

Where: τ_S and τ_{OSC} are the stationary and oscillatory stress coefficients respectively.

$$\tau_S = \frac{A_0\, R_e}{(Ha^2 + \chi^2)} \frac{I_1\left(\sqrt{Ha^2 + \chi^2}\right)}{I_0\left(\sqrt{(Ha^2 + \chi^2}\right)} \quad (35)$$

$$\tau_{OSC} = -\frac{\beta_k R_{emax}}{2\alpha^2} \left(\frac{\beta_k I_1(\beta_k)}{2I_1(\beta_k) - \beta_k I_0(\beta_k)} \right) e^{it} \quad (36)$$

The normalized stress coefficient

The normalized stress coefficient computation is not possible directly from the equations of governing flow, which led to use an indirect method involving the balance of forces in a volume element of fluid commonly used in such studies.

If we consider an incompressible fluid with a flow rate velocity $W = R_e(W_{max} e^{it})$ in a cylindrical conduit through porous medium in the presence of a magnetic field, we can write for a given volume element, the equation of balance of forces acting on it as follows:

$$\pi R^2 \rho \frac{\partial W}{\partial t} + \pi R^2 \frac{\partial P}{\partial z} = 2\pi R \tau_p - \pi R^2 \sigma B^2 W - \pi R^2 \frac{\mu}{k} W \quad (37)$$

Adding the dimensionless variable C_{fn} called normalized stress coefficient as follows:

$C_{fn} = \frac{\tau_p}{\tau_s}$, with the expression given by Bouvier [Bouvier (2005)]:

$$\tau_s = 4\mu \frac{W}{R} \quad (38)$$

Eq. (7) can be written using the previous dimensionless variables:

$$iR\omega^2\, W_{max} e^{it} + R\omega^2 \frac{\partial P}{\partial z} = -\frac{8\omega\mu}{\rho R} C_{fn}\, W_{max} e^{it} - \frac{\sigma B^2}{\rho}\, R\omega W_{max} e^{it} - \frac{\mu}{k\rho} R\omega W_{max} e^{it} \quad (39)$$

The previous expression becomes:

$$i\alpha^2 + \frac{\alpha^2}{W_{max}e^{it}}\frac{\partial P}{\partial z} + Ha^2 + \chi^2 = -8C_{fn} \quad (40)$$

Where: $Ha^2 = \frac{\sigma {B_0}^2 R^2}{\mu}$, $D_a = \frac{k}{R^2}$, $\alpha^2 = \frac{\rho R^2 \omega}{\mu}$ (41)

We deduce:

$$C_{fn} = -\frac{1}{8}(Ha^2 + \chi^2) - \frac{i\alpha^2}{8}\left(\frac{\alpha^2 2I_1(\beta_k) - \beta_k I_0(\beta_k)(\alpha^2 + i{\beta_k}^2)}{\alpha^2(2I_1(\beta_k) - \beta_k I_0(\beta_k))}\right)e^{it} \quad (42)$$

In the absence of magnetic field through non-porous medium: $Ha^2 = 0$, $\chi^2 = 0, \beta_k = \sqrt{i\alpha^2}$

We find the same expression given by Seume et al. [Seume and Simon (1988)].

$$C_{fn} = -\frac{i\alpha^2}{8}\left(\frac{2I_1(\beta_k)}{2I_1(\beta_k) - \beta_k I_0(\beta_k)}\right)e^{it} \quad (43)$$

2.3 Numerical study

The computations were carried out by using FORTRAN code. The discretization of the equations using a finite difference implicit time marching used by Ghezal [Ghezal (2007)] leads to an algebraic equations, which were solved using an iterative Gauss-Seidel technique, with a relaxation coefficient. At each new time, the system of the algebraic equations resulting from FDM discretization is solved by TDMA Algorithm.

Due to the axisymmetry, the computational domain is reduced to the mesh grid domain. In the vicinity of the ducts, the mesh is refined by replacing the mesh situated near the wall by sub decreasing mesh size following geometric sequences.

A numerical investigation has been performed to study the effects of Hartmann and Darcy numbers on streamlines, pressure isolines and stress coefficient by using Tecplot code.

The computer code and mathematical model have been validated by comparison with the available results in the literature [Majdalani (2008); Malathy and Srinivas (2008); Seume and Simon (1988); Ikbal, Chakravarty, Kelvin et al. (2009)].

On the other hand, these results are in good agreement between our results and those of previous works. The agreement between these results allows us to ensure the validity of our Code concerning the study of this type of flow.

3 Results and discussion

Fig. 2a shows the effects of the porosity on streamlines, in the absence of the magnetic field $Ha = 0$. It should be noticed, from the streamlines contours, it can be observed that all the streamlines get gradually perturbed more towards the wall, and follow the straight line path near the axis which for $Da = 0.1$, indicating the occurrence of the annular phenomenon. When the porosity decrease $Da = 0.001$, we noted that the dimensionless stream function values increase and the streamlines are parallel to the cylindrical axis, the results are in agreement with the corresponding ones of Ikbal et al. [Ikbal, Chakravarty, Kelvin et al. (2009)].

Fig. 2b shows the Hartmann number effects for various Darcy number on streamlines. It should be noted, that in the presence of the magnetic field with $Da = 0.1$, the streamlines have less curve than in the preceding case, without magnetic field. It indicates that the magnetic field tends to decrease the effect of porosity on the parietal constraint. It has been observed that the increase in the porosity with presence of magnetic field, results in a progressive flattening of the axial velocity W. The main difference in this figure is on the streamline values, as seen by increasing the Ha, the values on the stream lines reduce significantly, this reduction on the stream line may be due to Lorentz force resulting from the presence of the magnetic field which reducing the stream component along the axis of symmetry and slowdown the fluid motion. Our results are in agreement with the corresponding ones of Khechiba et al. [Khechiba and Ghezal (2013)]. It is also noted that the flow lines presents certain similarity between the case of a porous medium, $Da = 0.1$ subjected to an intense magnetic field, and the case of a medium of low porosity, $Da = 0.001$ not subjected to the magnetic field. These results indicate the combined of external magnetic forces and those due to the porosity, and are in agreement with the corresponding ones of Ikbal et al. [Ikbal, Chakravarty, Kelvin et al. (2009)].

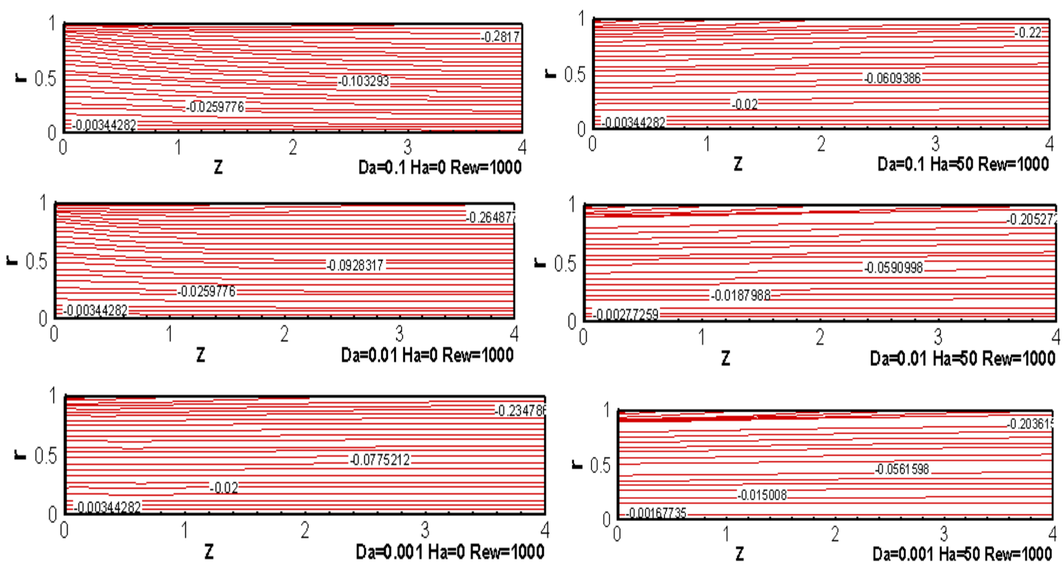
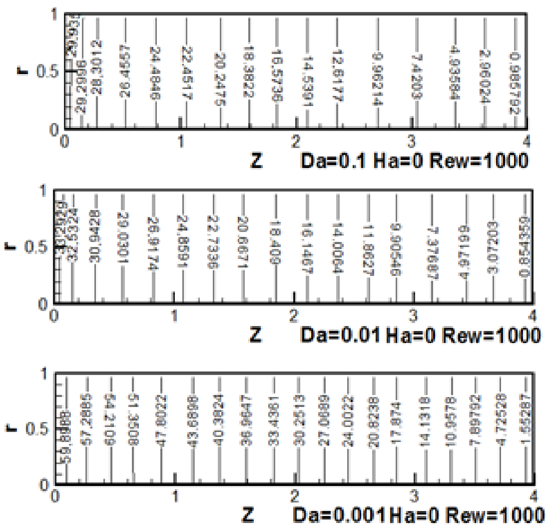


Figure 2a **Figure 2b**

Effect of Darcy number on streamlines for various Darcy number when $R_{e\omega} = 1000$

Figure 2a. $Ha = 0$ Figure 2b. $Ha = 50$

Fig. 3a shows the effects of the porosity on pressure isoline, in the absence of the magnetic field $Ha = 0$.The pressure gradient is constant, and does not depend on radial coordinate, but only on azimuthal coordinate. It must be noticed, that the pressure gradient increase by decreasing the number of Darcy. One may be notice that the increase in the pressure gradient may be caused by the increasing in the apparent viscosity of the flow, resulting from the decreasing porosity. Fig. 3b shows the Hartmann number effects for various Darcy number on pressure isolines. It should be noted, that the pressure gradient increase in the presence of a magnetic field, and this increasing remains practically constant, by increasing the number of Darcy, the results are in good agreement with the corresponding ones of Moustapha et al. [Moustapha, Amin, Chakravarty et al. (2009)]. One may be notice that at great values of Hartmann the effect of porosity can be considered as neglected.



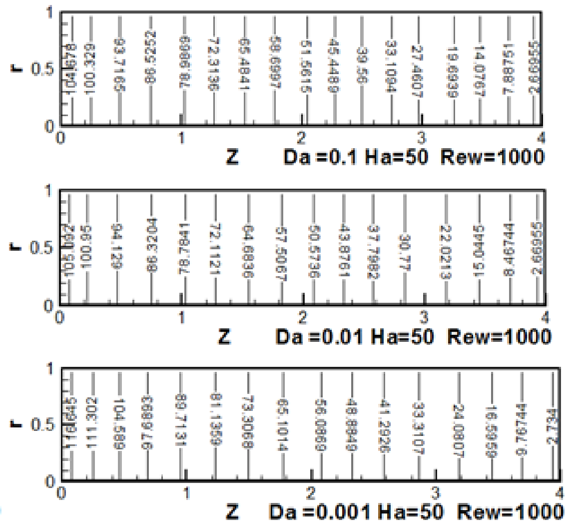
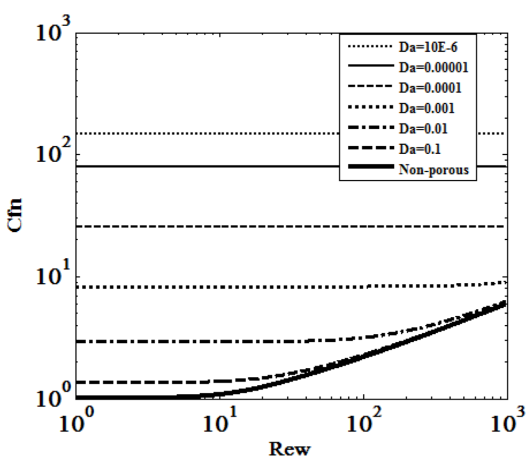


Figure 3a **Figure 3b**

Effect of Darcy number on pressure isolines for various Darcy number when $R_{e\omega} = 1000$

Figure 3a. $Ha = 0$ Figure 3b. $Ha = 50$



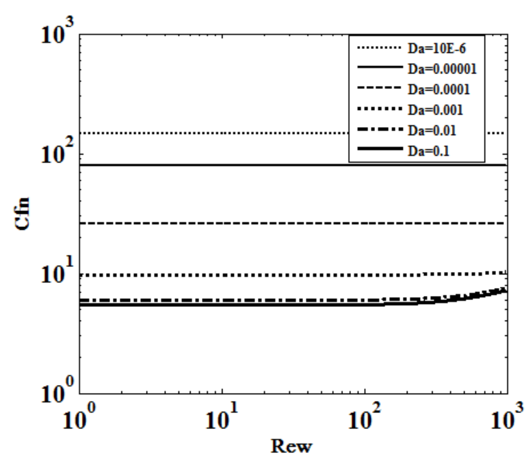


Figure 4a **Figure 4b**

The effect of Darcy number on stress coefficient C_{fn}: Figure 4a. $Ha = 0$, Figure 4b. $Ha = 50$

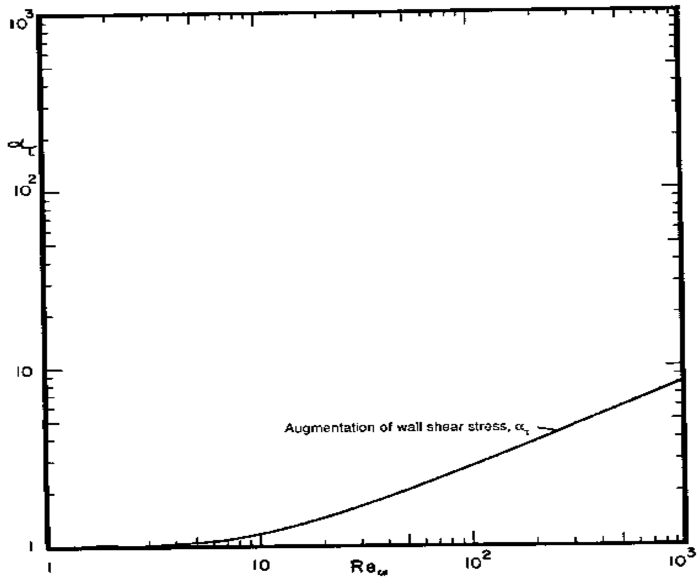


Figure 4c: Seume and Simon results (1988)

Fig. 4a shows that the maximum amplitude of the normalized stress coefficient C_{fn}, in the absence of magnetic field, through non-porous medium increases with frequency about 10 times its value in the case of steady flow. These results are in agreement with those obtained by Seume et al. [Seume and Simon (1988)], see Fig. 4c. However there was a slight increase for low frequencies. This can be justified by the contribution of the second component of the velocity. The frequency effect is appreciable for frequencies $R_{e\omega}$ between 100 and 150, which corresponds to the initiation of the annular effect. Another important result is that the normalized stress coefficient C_{fn} increases since the Darcy number Da decreases, for a given frequency and this increasing is accentuated by the presence of magnetic field $Ha = 20$. Fig. 4b shows that the normalized stress coefficient C_{fn} increases with an increase of the porosity, and the results are in agreement with the corresponding ones of Malathy et al. [Malathy and Srinivas (2008)]. The normalized stress coefficient C_{fn} is approximately equal to 1 for low frequencies $R_{e\omega} < 100$, which corresponds to the case of steady flow. This result is found by the limited development of the Eq. (43) in the absence of the magnetic field through non-porous medium and for $R_{e\omega} \to 0$. In fact, the limited development near zero of the function $I_1(\beta)$ which is given by: $I_1(\beta) = \frac{\beta}{2} + \frac{\beta^3}{16}$ leads to an expression of the normalized stress coefficient such that:

$$C_{fn} = \frac{i\alpha^2}{8}\left(1 + \frac{8}{\beta^2}\right) = 1 + \frac{iR_{e\omega}}{8} \tag{44}$$

This gives for $R_{e\omega} = 0 \to C_{fn} = 1$ (45)

In the presence of the magnetic field, the normalized stress coefficient C_{fn} increases when the Darcy number decreases. For a given frequency, the maximum amplitude varies as a function of the frequency from $R_{e\omega} > 100$. Another important result is that, the variation of normalized stress coefficient C_{fn} in the presence of the magnetic field is less dependent on frequency as the porosity decreases. The variation becomes totally independent of frequency for values of $Da = 0.001$. The expression of the normalized stress coefficient C_{fn} as a function of time (42) is given in Fig. 4d:

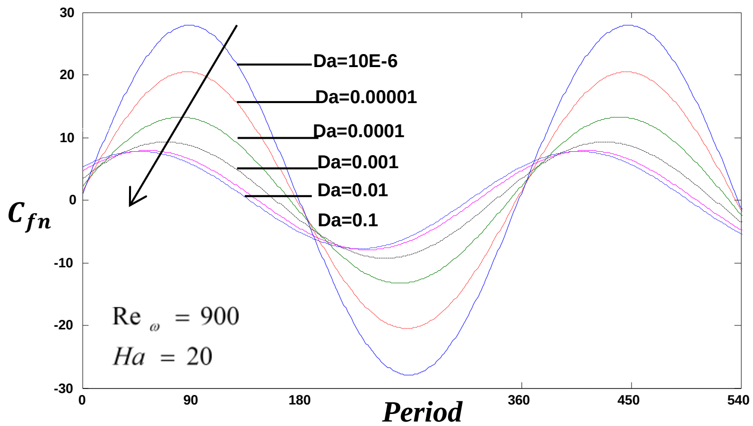


Figure 4d: The stress coefficient for various Darcy numbers, when $Ha = 20, R_{e\omega} = 900$

Fig. 4d shows that the normalized stress coefficient C_{fn} is sinusoidal and its amplitude decreases by increasing the Darcy numbers at high frequencies. The addition of the magnetic field $Ha = 20$ increase the normalized stress coefficient, this increase makes it possible to shows this decrease due to Darcy number better than in the case of the absence of magnetic field. A very important result is that the normalized stress coefficient decreases with increasing Da, but the opposite effect can be noticed with increasing of the magnetic field.

4 Vorticity study

Dimensionless vorticity is given by:

$$\Omega = -\frac{dW}{dr} = -\frac{d}{dr}\left(W_0(r) + W_1(r,\ t)\right) \tag{46}$$

Where: $W(r,\ t) = \frac{A_0\, R_e}{(Ha^2 + \chi^2)}\left(1 - \frac{I_0\left(\sqrt{Ha^2 + \chi^2}\,.r\right)}{I_0\left(\sqrt{(Ha^2 + \chi^2}\right)}\right) - \frac{A_1 \alpha^2}{{\beta_k}^2}\left(\frac{I_0(\beta_k.r)}{I_0\,(\beta_k)} - 1\right) e^{it}$

The final expression of the vorticity is given by:

$$\Omega = \frac{A_0\, R_e}{\sqrt{Ha^2 + \chi^2}} \frac{I_1\left(\sqrt{Ha^2 + \chi^2}\,.r\right)}{I_0\left(\sqrt{(Ha^2 + \chi^2}\right)} + \frac{A_1 \alpha^2}{\beta_k}\left(\frac{I_1(\beta_k.r)}{I_0\,(\beta_k)}\right) e^{it} \tag{47}$$

The vorticity is plotted using MATLAB as a function of radius *r*, for different values of Darcy number, frequency, phases and different ratios of amplitudes.

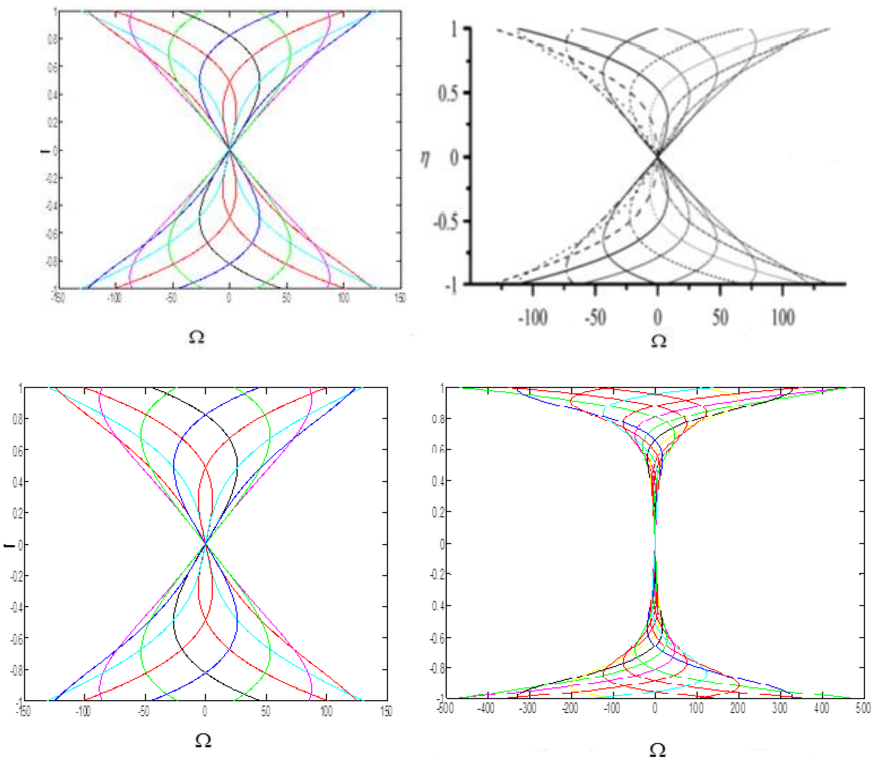


Figure 6: Vortex profiles when $Ha = 0, R_{e\omega} = 10, A_1 = 50$, (a) $\chi^2 \to 0$, (b) $\chi^2 = 100$

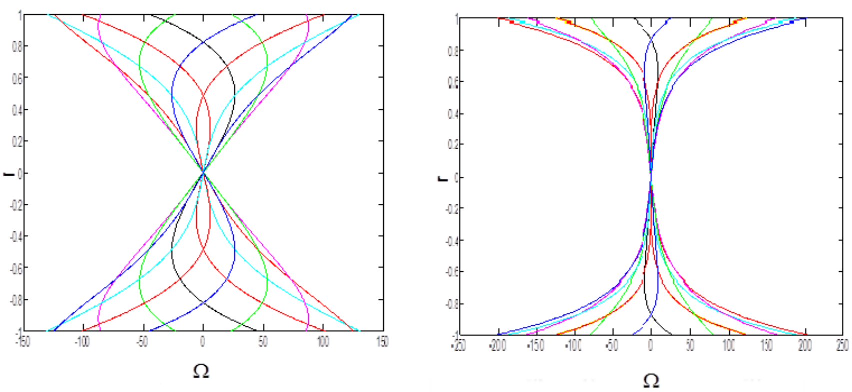


Figure 7: Vortex profiles when $R_{e\omega} = 10, A_1 = 50$, (a) $Ha = 0$, (b) $Ha = 5$

Then we can develop the following discussion:

For high amplitudes through non-porous medium

The influence of the amplitude is shown in Fig. 5. The results indicate that the values of the vorticity are higher at high amplitude values. By comparing our results with those found by Majdalani [Majdalani (2008)], without magnetic field through non-porous medium, for an amplitude $A_1 = 50$ and frequency $R_{e\omega} = 10$, we note that they are in good agreement.

For a low Darcy number and high amplitudes

Fig. 6 shows the effect of the porosity on the radial profile of vorticity. We note that the magnitude of the vortices is higher at low Darcy numbers. It should be noted that the effect of the porosity parameter, is significant only in the vicinity of the wall and when the porosity increases at the Darcy number $D_a = 0.01$.The vorticity can expect values up to five times the value in the absence of the porosity. This causes a swirling movement within the flow.

For high amplitudes through non-porous medium in the presence of magnetic field (Ha=5)

The influence of the amplitude is shown in Fig. 7. The results indicate that the values of the vorticity are higher in the presence of magnetic field. By comparing our results with those found by Khechiba et al. [Khechiba and Ghezal (2013)], in the presence of magnetic field through non-porous medium, for an amplitude $A_1 = 50$ and frequency $R_{e\omega} = 10$, we note that they are in good agreement. A very important result is that the low Hartmann number or low Darcy number have the same effect on the vorticity.

5 Conclusion

The dynamic behavior of a pulsatile flow, electrically conducting through porous medium in a cylindrical conduit under the influence of a magnetic field has been carried out. we have computed the stress coefficient based on the one-dimensional case used in some works, since the calculation in the case of bi-dimensional unsteady flow presents some difficulties due to the change of sign of the velocity value over a cycle. The analytical study has permitted us to establish expressions reflecting the velocity profile, the variation of the stress coefficient and vorticity as a function of Darcy and Hartmann numbers. It has also shown that the stress coefficient has a sinusoidal aspect and it increases with the decrease of the Darcy number and the amplitude of the vorticity is higher when the Darcy number is low. It should be noted that the effect of magnetic field on the vorticity is visible only from $D_a = 0.01$.

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