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Numerical Solutions of Unsteady MHD Flow Heat Transfer over a Stretching Surface with Suction or Injection

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Abstract: The objective of the present problem is to investigate a two-dimensional unsteady flow of a viscous incompressible electrically conducting fluid over a stretching surface taking into account a transverse magnetic field of constant strength. Applying the similarity transformation, the governing boundary layer equations of the problem converted into nonlinear ordinary differential equations and then solved numerically using fourth order Runge-Kutta method with shooting technique. The effects of various parameters on the velocity and temperature fields as well as the skin-friction coefficient and Nusselt number are presented graphically and discussed qualitatively.

Keywords: MHD, heat transfer, incompressible fluid, R-K method, nusselt number, skin friction.

1 Introduction

Boundary layer flow on a moving continuous surface is an important type of flow occurring in a number of engineering processes. Aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, which would be in the form of an electrolyte, crystal growing, the boundary layer along a liquid film in condensation process and polymer sheet extruded continuously from a die are the practical applications of moving surfaces and also the materials manufactured by extrusion processes and heat treated materials traveling between a feed roll and wind up roll or on a conveyer belt possesses the characteristics of a moving continuous surfaces. During the past few decades, the phenomenon of hydromagnetic flow of an electrically conducting fluid towards a stretching surface has attracted a lot of attention of many researchers due to their widespread applications in industrial manufacturing, modern metallurgical and metal-working processes such as hot rolling, glass blowing, paper production, drawing of wire and plastic films, metal spinning, liquid composite molding metal and polymer extrusion etc. Crane [Crane (1970)] became the first to obtained boundary layer flow past a continuous solid surface which moves with a constant speed. Elbashbeshy [Elbashbeshy (1998)] has analyzed the heat transfer over a stretching surface with variable surface heat flux. Exact solutions for self-similar boundary layer flows induce by permeable stretching walls have been analyzed by Magyari et al. [Magyari and Keller (2000)]. Further, the effects of heat transfer for steady and unsteady flow past a stretching sheet have been

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presented by several researchers, like Mahapatra et al. [Mahapatra and Gupta (2002)], and Chaudhary et al. [Chaudhary and Kumar (2014)] in the presence of different physical parameters.

For a long time, a major subject in fluid dynamics is the problem of steady and unsteady laminar flow over a permeable surface because of its importance from both theoretical and practical point of view and has been comprehensively studied. It also has many applications in engineering and technological processes, such as petroleum industries, groundwater flows, extrusion of a polymer sheet from a dye and boundary layer control. The flow over a surface has been in an analyzed by Ishak et al. [Ishak, Nazar and Pop (2009)] under the limiting cases. Khader [Khader (2014)] studied about flow and heat transfer over a permeable surface with second-order slip and viscous dissipation.

The rate of stretching and the cooling liquid are the main fluid properties of various manufacturing processes desired for a better outcome. The stretching rate is of utmost importance as rapid stretching results in sudden solidification, thereby destroying the characteristics expected for the final product. The use of electrically conducting fluid and applications of a magnetic field can control the rate of cooling and the desired properties of the end product. The magnetic field has been used in the process of purification of molten metal from non-metallic inclusions. The study of hydromagnetic flow for an electrically conducting fluid over a heated sheet has attracted considerable interest in view of its diverse applications in many technological processes, such as plasma studies, foodstuff processing, solidification of liquid crystals, cooling of nuclear reactors, exotic lubricants and suspension solutions, the boundary layer control in aerodynamics, MHD power generators and in the field of planetary magnetosphere. Andersson [Andersson (1992)] presented an exact analytical solution of the MHD flow of Walters liquid B past a stretching sheet. Later, several researchers such as Chaudhary et al. [Chaudhary and Kumar (2015)], Singh et al. [Singh and Singh (2012)] have focused their attention to the various aspects of the problem of heat transfer and hydromagnetic flow of stretching surface. Ganesh kumar et al. [Ganesh Kumar, Gireesha, Krishanamurthy et al. (2017)] has studied an unsteady squeezed flow of a tangent hyperbolic fluid over a sensor surface in the presence of variable thermal conductivity. Radiative heat transfers of Carreau fluid flow over a stretching sheet with fluid particle suspension and temperature jump have been analyzed by Ganesh Kumar et al. [Ganesh Kumar, Gireesha and Manjunatha et al. (2017)]. Thammannam et al. [Thammanna, Ganesh Kumar and Gireesha et al. (2017)] were presented in the three-dimensional MHD flow of couple stress Casson fluid past an unsteady stretching surface with chemical reaction. Effects of nonlinear thermal radiation on double-diffusive mixed convection boundary layer flow of viscoelastic nanofluid over a stretching sheet have been analyzed by Makinde et al. [Makinde, Kumar and Manjunatha et al. (2017)]. Effect of nonlinear thermal radiation on MHD boundary layer flow and melting heat transfer of micro-polar fluid over a stretching surface with fluid particles suspension have been studied by Makinde et al. [Makinde, Kumar and Manjunatha et al. (2017)].

The aim of current analysis to investigate a two-dimensional unsteady flow of a viscous incompressible electrically conducting fluid flow over a stretching surface taking into account a transverse magnetic field of constant strength was considered. Applying the similarity transformation, the governing boundary layer equations of the problem converted into nonlinear ordinary differential equations and then solved numerically using fourth order Runge-Kutta method with shooting technique. The effects of various parameters on the velocity and temperature fields as well as the skin-friction coefficient and Nusselt number are presented graphically and discussed qualitatively.

2 Formulation of problem

Consider a two-dimensional unsteady boundary layer flow of an incompressible electrically conducting fluid over a stretching surface coinciding with the plane $y = 0$, the flow being confined to $y > 0$. The x-axis is chosen along the sheet, and a uniform magnetic field B_0 is imposed along y-axis. The continuous stretching surface is assumed to have the velocity $u_w = \frac{ax}{1-ct}$, the transpiration velocity through the permeable wall is v_w with suction and injection for $\mp v_w > 0$ and the temperature $T_w = T_\infty + \frac{b}{a}u_w$, where a, b and c constants with are $a > 0, b \geq 0$ and $c \geq 0$ and also $c < \frac{1}{t}, t$ is the time, x is the coordinate measured along stretching surface and T_∞ is the temperature of the fluid far away from the stretching sheet. Under the boundary layer approximation, the unsteady two-dimensional boundary layer equations can be written as:

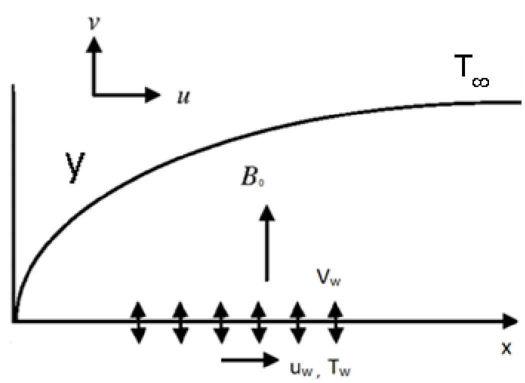

Figure 1: Geometry of the problem

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e \mu_e B_0^2}{\rho}u \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 u}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The corresponding end point conditions are given by:

$$\begin{aligned} &u = u_w(x,t), v = v_w(x,t), T = T_w(x,t) \text{ at } \quad y = 0 \\ &u \to 0, T \to T_\infty \text{ as } \quad y \to \infty \end{aligned} \tag{4}$$

where u and v are the velocity components in the x and y directions respectively, ν is the kinematic viscosity, σ_e is the electrical conductivity, μ_e is the magnetic permeability, ρ is the fluid density, T is the temperature of the fluid, α is the thermal diffusivity, μ is the coefficient of viscosity and Cp is the specific heat at constant pressure.

The mathematical analysis of the problem is simplified by introducing the similarity variable; non-dimensional function and also temperature are given as

$$\xi(x,y,t)=\sqrt{\nu x u_w}\, f(\eta), \eta=\sqrt{\frac{u_w}{\nu x}}y \text{ and } T=T_\infty+\frac{b}{a}u_w\theta(\eta) \tag{5}$$

Using the relations (5), the boundary layer governing Eqs. (2) and (3) can be written in a non-dimensional form as

$$f'''+ff''-A\left(\frac{1}{2}\eta f''+f'\right)-f'^2-Mf'=0 \tag{6}$$

$$\theta''+\Pr f\theta'-A\left(\frac{1}{2}\eta\theta'+\theta\right)-f'\theta+Ecf''^2=0 \tag{7}$$

Subject to the corresponding boundary conditions are:

$$\begin{aligned} &f=f_0,\ f'=1,\ \theta=1 \text{ at } \eta=0\\ &f'\to 0,\ \theta\to 0 \quad \text{as } \eta\to\infty \end{aligned} \tag{8}$$

where prime denotes the differentiation with respect to η, $M=\frac{\sigma_e\mu_e B_0^2\nu \operatorname{Re} x}{\rho u_0^2}$ is the magnetic parameter, $\operatorname{Re}_x=\frac{u_w x}{\nu}$ is the local Reynolds number, $\Pr=\frac{\nu}{\alpha}$ is the Prandtl number, $Ec=\frac{u_w^2}{C_w(T_w-T_\infty)}$ is the Eckert number, $S=-\left(\frac{v_w}{u_w}\right)\sqrt{\operatorname{Re}_x}$ is the suction or injection parameter and $A=\frac{c}{a}$ is the unsteadiness parameter.

The Eqs. (6) and (7) with boundary conditions (8) are solved by Runge-Kutta fourth order method along with shooting technique for a better approximation. The step size is taken as $\Delta\eta=0.001$ and accuracy of the six decimal places as the criterion of convergence.

The physical quantities of primary interest are the local skin-friction coefficient (C_f) and the Nusselt number (Nu), which are defined as

$$C_f=\frac{2\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho u_w^2}=2f''(0)\sqrt{\operatorname{Re}_x} \tag{9}$$

$$Nu_x=-x\frac{\left(\frac{\partial T}{\partial y}\right)_{y=0}}{(T_w-T_\infty)}=-\theta'(0)\sqrt{\operatorname{Re}_x} \tag{10}$$

3 Results and discussion

A numerical solution for the system of non-linear differential Eqs. (6)-(7), subject to the boundary condition Eq. (8) is obtained by the method of the Runge-Kutta with shooting technique. The effects of various parameters such as magnetic parameter (M), unsteadiness parameter (A), Prandtl number (Pr), Eckert number (Ec), Skin-friction and Nusselt number on non-dimensional velocity components and temperature distributions are discussed through graphs and table respectively.

Figs. 2 and 3 illustrated that the effect of velocity and temperature profiles for different values of magnetic parameter (M). It is observed that the velocity decreases with an increasing the the magnetic parameter and reverse trend is observed in temperature profiles. From the physical point of view, known as the Lorentz force arises and an increase of the magnetic parameter makes stronger, which ultimately slow down the fluid flow and increases the temperature.

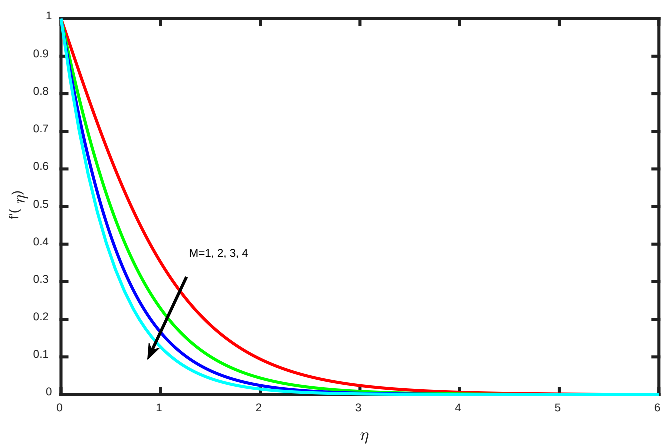

Figure 2: Effect of velocity profiles for different values of the magnetic parameter (M)

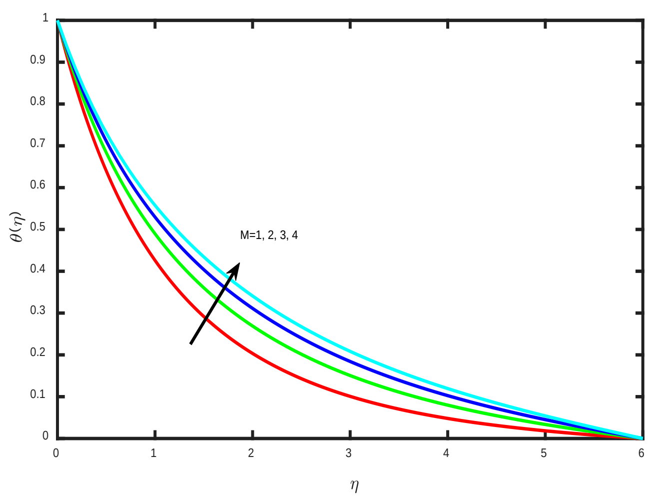

Figure 3: Effect of temperature profiles for different values of the magnetic parameter (M)

The velocity and temperature profiles for various values the unsteadiness parameter (A) as shown in Figs. 4 and 5 respectively. It is noticed that the velocity increases with an increasing unsteadiness parameter and a reverse trend is observed in the temperature profiles. Effect of Prandtl number Pr over the temperature profiles has been demonstrated in Fig. 6, and it is clear from the figure that the temperature falls with the increasing effect of Prandtl number. That is to say, Prandtl number plays an opposite effect on the temperature field. The influence Eckert number (Ec) on temperature presented in Fig. 7. As Eckert number increases, the temperature also increases due to heat addition by frictional heating.

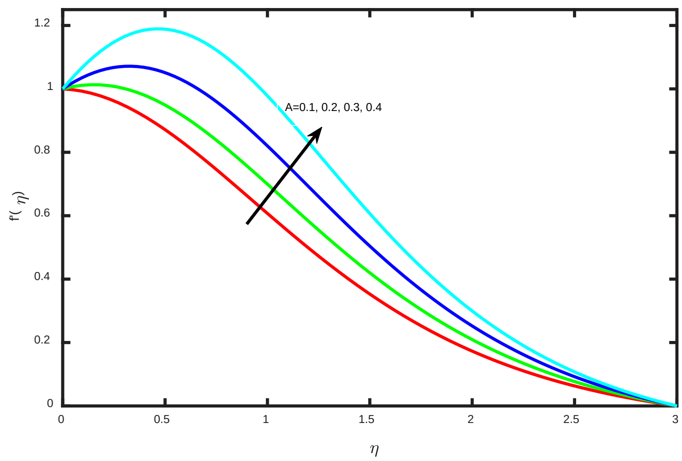

Figure 4: Effect of velocity profiles for different values of unsteadiness parameter (A)

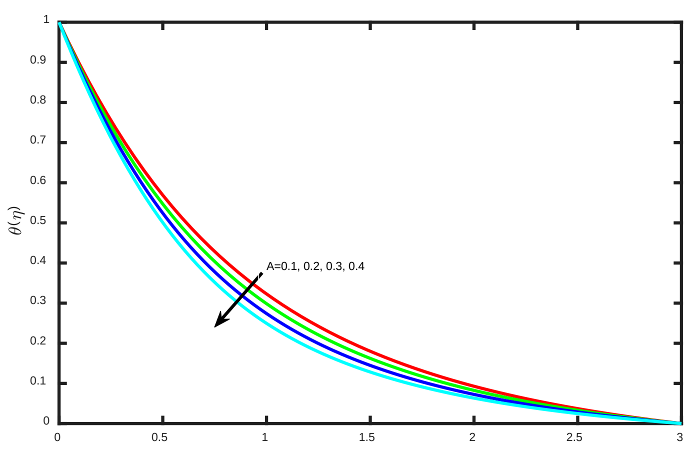

Figure 5: Effect of temperature profiles for different values of unsteadiness parameter (A)

Table 1: Values of $-f''(0)$ and $-\theta'(0)$ for various values of S, M, A, Pr and Ec

S	M	A	Pr	Ec	Present Results $-f''(0)$	Present Results $-\theta'(0)$
1.0	1	0.1	0.71	0.1	0.774413	0.934832
	2				1.252389	0.820191
	3				1.599671	0.743916
	4				1.885308	0.686877
1.0	1.0	0.1	0.71	0.1	0.029365	1.109522
		0.2			0.148490	1.088454
		0.3			0.254576	1.068841
		0.4			0.350019	1.050605
1.0	1.0	0.1	0.71	0.1	0.781910	0.964313
				1.0	0.781890	0.644192
				2.0	0.781890	0.288998
				3.0	0.781890	0.111402
-1.0	1.0	0.1	0.71	0.1	0.781890	0.963867
-0.5					0.781890	0.963867
0.5					0.781882	0.963893
1.0					0.781910	0.964313
1.0	1.0	0.1	0.71	0.1	0.781910	0.964313
			1.00		0.781910	1.028050
			3.00		0.781890	1.449294
			7.00		0.781890	2.099029

Finally Tab. 1 shows the effects of the suction or injection (S), the magnetic parameter (M), the unsteadiness parameter A, the Prandtl number (Pr), and the Eckert number (Ec), on the local skin friction coefficient $f''(0)$ and the local Nusselt number $\theta'(0)$. It is observed that the skin friction and Nusselt number coefficients decrease with an increase of suction or injection parameter when other parameters are constants. Moreover, it is also observed that the Nusselt number decrease with the increasing values of the magnetic parameter and a reverse trend is observed in the skin-friction coefficient. Further, it is also noticed that the skin friction increases when unsteadiness parameter increases but the Nusselt number decreases. Moreover, it is quite that the values of the Nusselt number increase with an increasing Prandtl number.

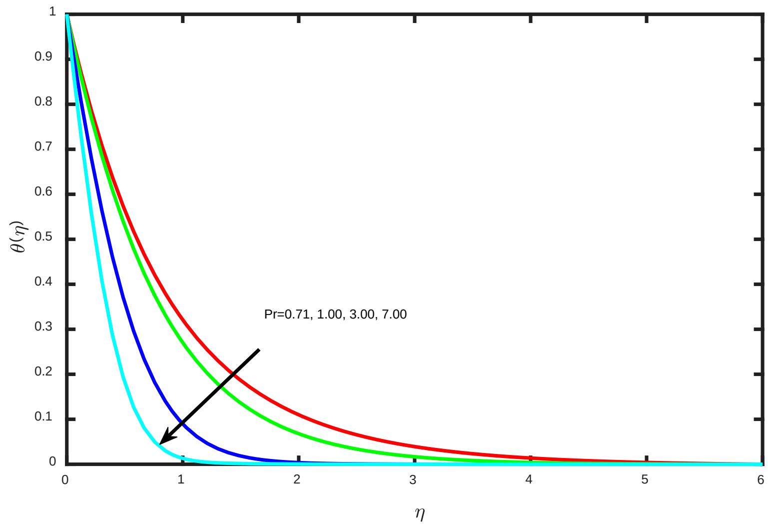

Figure 6: Effect of temperature profiles for different values of Prandtl number (Pr)

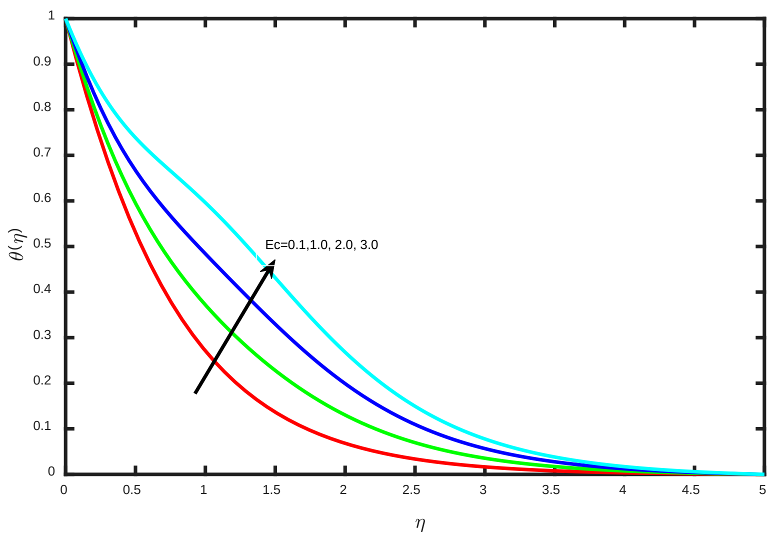

Figure 7: Effect of temperature profiles for different values of Eckert number (Ec)

5 Conclusions

The effects of the various parameters on the velocity, temperature, skin friction coefficient and Nusselt number are illustrated and discussed. It can be concluded that the velocity boundary layer thickness, the thermal boundary layer thickness, the surface gradient and the rate of heat transfer decrease as the suction or injection parameter and the unsteadiness parameter increase while the reverse behavior is noted after the crossing over point for velocity as well as thermal boundary layer thickness for the unsteadiness parameter. Moreover, in case of increase in the magnetic parameter the velocity and the surface gradient decrease while the opposite phenomenon occurs for the thermal boundary layer thickness and the rate of heat transfer. Finally, the thermal boundary layer thickness and the rate of heat transfer decrease with the increase in the Prandtl number but the effects of the Eckert number are quite opposite.

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