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Article

Adaptive Multi-Updating Strategy Based Particle Swarm Optimization

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Abstract: Particle swarm optimization (PSO) is a stochastic computation technique that has become an increasingly important branch of swarm intelligence optimization. However, like other evolutionary algorithms, PSO also suffers from premature convergence and entrapment into local optima in dealing with complex multimodal problems. Thus this paper puts forward an adaptive multi-updating strategy based particle swarm optimization (abbreviated as AMS-PSO). To start with, the chaotic sequence is employed to generate high-quality initial particles to accelerate the convergence rate of the AMS-PSO. Subsequently, according to the current iteration, different update schemes are used to regulate the particle search process at different evolution stages. To be specific, two different sets of velocity update strategies are utilized to enhance the exploration ability in the early evolution stage while the other two sets of velocity update schemes are applied to improve the exploitation capability in the later evolution stage. Followed by the unequal weightage of acceleration coefficients is used to guide the search for the global worst particle to enhance the swarm diversity. In addition, an auxiliary update strategy is exclusively leveraged to the global best particle for the purpose of ensuring the convergence of the PSO method. Finally, extensive experiments on two sets of well-known benchmark functions bear out that AMS-PSO outperforms several state-of-the-art PSOs in terms of solution accuracy and convergence rate.

Keywords: Particle swarm optimization; local optima; acceleration coefficients; swarm diversity; premature convergence

1 Introduction

Particle swarm optimization (PSO) is a swarm intelligent optimization algorithm that is based on the metaphors of social interaction and communication (e.g., bird flocking and fish schooling) [1]. Due to the merits of swarm intelligence, intrinsic parallelism, easy implementation, inexpensive computation and few parameters to be adjusted, PSO has drawn a great attention of numerous researchers in the field of evolutionary computation [2–8] and has been successfully applied in a wide

range of optimization problems in many different areas in the past two decades [9–21]. However, like other nature-inspired evolutionary algorithms (EA), PSO still suffers from loss of diversity and slow convergence caused by the contradiction between exploration and exploitation, especially when dealing with complex multimodal problems. This will inevitably impair the performance of particle swarm optimization to a large extent. To this end, a large number of PSOs have been developed from different perspectives to circumvent these issues in literature [2,4–12]. It is known that the performance of PSO is heavily dependent on the parameters associated with the update scheme. Concerning the inertia weight, it is believed that the overall performance of PSO is strongly affected by the inertia weight through balancing the exploration and exploitation during the search process. As a consequence, much research effort has been devoted to the development of various types of inertia weight, including fixed-value [22], linear [23], nonlinear [2], random [3], fuzzy rules [9], chaos [4], and others [5]. Regarding the acceleration coefficients, it is reported that the appropriate acceleration coefficients can enhance the global search in the early stage of the optimization and to encourage the particle to converge towards the global optima at the end of the search process. From the literature, it can be seen that a variety of acceleration coefficients, ranging from fixed-value [1], linear [6], nonlinear [10] and random [11] to adaptive paradigm [7], have been introduced into the update scheme of PSO to get a better trade-off between the global and local search abilities. Hence, this work will further investigate the effects of the acceleration coefficients and employ different values of them to enhance the performance of PSO according to the different statuses of particle. Besides, the two random numbers in PSO system also have a significant influence on the convergence behavior of the swarm [8,12], especially the chaotic sequence, instead of the random number, is used in the velocity update equation [12]. To strike a better balance between exploration and exploitation, this paper proposes an adaptive multi-updating strategy based particle swarm optimization (AMS-PSO). On the one hand, the chaos-based sequence is used to generate high-quality initial particles to accelerate the convergence rate of AMS-PSO. On the other hand, according to current iteration, different update schemes are leveraged to regulate the particle search process at different evolution stages. Specifically, two different sets of velocity update strategies are utilized to enhance the exploration ability in the early stage while the other set of update strategy as well as the Gaussian mutation is applied to improve the exploitation capability in the later stage. Followed by the unequal weightage of acceleration coefficients is harnessed to guide the search for the global worst position (*gworst*) of particle so as to strengthen the swarm diversity. Besides, an auxiliary update strategy is exclusively applied to the global best position (*gbest*) of particle in order to ensure the convergence of the proposal. In actual fact, the key idea behind AMS-PSO is somewhat similar to multi-swarm based PSOs, viz., different update scheme is used for different subgroup of particles so as to enhance the communications within and between subgroups. At length, conducted experiments validate the superiority of AMS-PSO over several existing competitive PSO variants in the literature.

The rest of this paper is organized as follows. Section 2 discusses the related work, involving the update strategies based on acceleration coefficients, *gworst* particle, and *gbest* particle, respectively. In Section 3, the standard particle swarm optimization is briefly introduced. Section 4 elaborates on the proposed AMS-PSO from three aspects of swarm initialization, update scheme for the *gbest* particle and update scheme for the *gworst* and other particles, respectively. In Section 5, extensive experiments on two sets of well-known benchmark functions are reported and analyzed. Finally, the concluding remarks and future work are discussed in Section 6.

2 Related Work

Ever since the introduction of particle swarm optimization in 1995, researchers have been working hard to explore more and more efficient PSOs from different aspects to enhance their performance. Loosely speaking, most of the current existing PSO variants can be roughly classified into the following five categories:

(i) Swarm initialization. As a crucial step in any evolutionary computation, swarm initialization has a great effect on the convergence rate and the quality of the final solution. The most common method is random initialization which is used to generate initial particles without any prior information about the solution to the problems [24]. To produce high-quality initial particles, several advanced swarm initialization methods, including chaos-based sequence [25], opposition-based learning method [12], chaotic opposition-based learning method [26], and other initial strategies [27], have been developed and widely applied in the field of swarm intelligence optimization with substantial performance improvement.

(ii) Parameter selection. Proper parameters can significantly affect the performance of particle swarm optimization. In general, an appropriate inertia weight can achieve an effective balance between exploration and exploitation. A larger inertia weight is beneficial for global exploration whereas a smaller one is conducive to local exploitation for fine-tuning the current search region [23], such as fitness-based dynamic inertia weight [12], sigmoid-based inertia weight [2] and chaotic-based inertia weight [28], to name a few. By changing inertia weight dynamically, the search capability of PSO can be also dynamically adjusted. Besides, both acceleration coefficients [7] and random numbers [12] have a significant effect on the convergence behavior of swarm. All of them have been deeply studied in the literature and widely applied in many real-world optimization problems.

(iii) Topology structure. The neighborhood topology aims to enrich the swarm diversity and undoubtedly influence the flow rate of the best information among particles. The early notable topologies include circle, wheel, star, and random topology [29]. Instead of using the fixed neighborhood topology, the dynamic multi-swarm PSO variants are developed in literature [13], whose basic idea is that a larger swarm is divided into many small-sized sub-swarms that can be regrouped after a predefined regrouping period. Besides, a ring topology in exemplar generation is adopted to enhance the swarm diversity and exploration ability of the genetic learning PSO [30]. Compared with basic PSO, it is not difficult to see that the topology structure-based PSOs have a more powerful ability in keeping the swarm diversity and preventing premature convergence.

(iv) Learning strategy. PSO adopts different learning strategies to regulate global exploration and local exploitation that has attracted considerable attention. It's known that a particle usually learns from its history personal best and the global best of the entire swarm, but such a learning strategy tends to cause particle to oscillate between the two exemplars when they are located on the opposite sides of the candidate particle [14]. Hence, a variety of learning strategy-based PSO variants have been proposed in this regard, such as heterogeneous CLPSO [31], hybrid learning strategies-based PSO [32], PSO with elite and dimensional learning strategies [33], PSO with enhanced learning strategy [34], to keep swarm diversity and to avoid premature convergence.

(v) Hybrid mechanism. Since each algorithm has its own merits and demerits, two methods are usually complementary to each other and the combination makes them benefit from each other. Based on this recognition, the goal of hybrid PSO with other meta-heuristics or auxiliary search techniques is to well balance the global search and local search without being trapped

into local optima. Classical work includes hybrid PSO with genetic algorithm [15], hybrid PSO with artificial bee colony [35], integrating PSO with firefly [36], hybrid PSO with gravitational search algorithm [16], hybrid PSO with chaos searching technique [17] and so forth. Besides, the most often used genetic operators (crossover [34] and mutation [26]) are integrated into PSO to enhance the diversity of swarm.

As reviewed above, most of the PSOs can achieve state-of-the-art performance and motivate us to explore more efficient particle swarm optimization algorithms with the help of their excellent experiences and knowledge. Different from previous studies of the related work, here, we exclusively focus on the existing PSO update schemes in the literature from the aspects of acceleration coefficients, *gworst* particle, and *gbest* particle, respectively. More details can be found in the following sections.

2.1 Review of Acceleration Coefficients-Based Updating Strategy

From the literature, it can be clearly observed that acceleration coefficients can be divided into two categories: fixed-value [1] and time-varying [7]. For the limited space, here we only provide a brief survey on the constant acceleration coefficients. Since the introduction of the acceleration factors into the update scheme of PSO, they come into effect on the movement of particle, particularly they are greatly importance to the success of PSO in both the social-only model and the cognitive-only model. A large value of the acceleration factor c_1 makes a particle fly towards its own ever-known best position (*pbest*) faster and a large value of the acceleration factor c_2 makes a particle move towards the best particle (*gbest*) of the entire swarm faster. In general, c_1 and c_2 are set equal ($c_1 = c_2 = 2$) [1] to give equal weightage to both the cognitive component and the social component simultaneously. In [37], the acceleration coefficients are also set equal ($c_1 = c_2 = 2.05$) to control magnitudes of particle's velocity and to ensure convergence of particle swarm optimization. As for the values of c_1 and c_2, it is reported that acceleration coefficient with big values tends to cause particle to separate and move away from other, while taking small values causes limitation of the movements of particle, and not being able to scan the search space adequately [38].

Alternatively, it is to be noted that particles are allowed to move around the search space rather than move toward the *gbest* with a large cognitive component and a small social component in the early stage. On the contrary, a small cognitive component and a large social component allow particle to converge to the global optima in the latter stage of the search process. Based on this argument, much research effort is devoted to the development of unequal acceleration coefficients for PSO [12,18,39,40]. In the early work [39], the cognitive and social values of $c_1 = 2.8$ and $c_2 = 1.3$ yield good results for their chosen set of problems. In literature [40], the acceleration coefficients c_1 and c_2 are first initialized to 2 and adaptively controlled according to four evolutionary states: increasing c_1 and decreasing c_2 in an exploration state, increasing c_1 slightly and decreasing c_2 slightly in an exploitation state, increasing c_1 slightly and increasing c_2 slightly in a convergence state, and decreasing c_1 and increasing c_2 in a jumping-out state. To accelerate the convergence, a larger value is assigned to the social component ($c_1 = 0.5$, $c_2 = 2.5$) [18], which enables the convergence rate of PSO to be markedly improved than that with an equal setting of $c_1 = 1.5$ and $c_2 = 1.5$. Besides, the unbalanced setting of c_1 and c_2 ($c_1 = 3$, $c_2 = 1$) is exclusively used for *gworst* particle whereas the balanced setting of c_1 and c_2 ($c_1 = c_2 = 2$) is used for all the other particles [12]. It draws direction of the newly updated particle towards its *pbest*. That is, the unequal weightage is only applied in the case of stagnation of the particle that helps the newly generated particle to avoid drifting towards the *gbest* and explore a new region in the search space. Thus it can avoid stagnation of swarm at a suboptimal solution. In sum, proper adaptation and proper value setting of c_1 and c_2 can be promising in enhancing the performance

of PSO. This is the main motivation to develop AMS-PSO in this work by varying the value of acceleration coefficients to control the influence of cognitive information and social information, respectively.

2.2 Review of the gworst Particle-Based Updating Strategy

In PSO system, the personal best position (*pbest*) and global best position (*gbest*) play an important role in regulating particle search around a region. In contrast, there is almost no research and analysis about the global worst position (termed as *gworst*) in literature. However, just as the *cask effect* says that how much water a wooden bucket can hold does not depend on the longest board but the shortest board, which can increase the water storage of the wooden bucket by replenishing the length of the shortest board and eliminating the restricting factors formed by this short board. Similarly, *gworst* particle does not mean that the particle does not contain any valuable information for the search process. All particles may carry useful information, and the performance of the entire swarm may be enhanced to some extent by strengthening the worst particle. Inspired by this argument, the personal worst position (*pworst*) and the *gworst* of particles are introduced into the velocity update equation as a means to dictate the motion of particle [41]. Afterward, only the *pworst* is integrated into the basic update scheme to help particles to remember their previously visited worst position [19]. This modification is beneficial for exploring the search space very effectively to identify the promising solution region. Likewise, only *gworst* particle is embedded in the velocity update equation that is called impelled or penalized learning according to the corresponding weight coefficient to balance the exploration and exploitation abilities by changing the flying direction of particles [42]. Subsequent work [43] incorporates both *pworst* and *gworst* into the update scheme to ensure that the extra velocity given from the worst position is always in the direction of the best position as well as the extra velocity given to a particle is maximized when the particle is at its worst position and decreases as the particle moves away.

To increase swarm diversity and to avoid local optima, a fourth part is added to the standard velocity update scheme that enables particles to move away from the *gworst* can disturb particle distribution and make swarm more diversified [20]. In literature [12], the *gworst* particle is updated based on an unequal acceleration factors $c_1 = 3$ and $c_2 = 1$ while the other particles are updated with the equal setting of $c_1 = 2$ and $c_2 = 2$. Unlike the traditional update scheme, the worst replacement strategy is used to update swarm, whereby the *gworst* in swarm is replaced by a better newly generated position in [44]. In more recent work [45], a worst-best example learning strategy is devised, in which the worst particle learns from the *gbest* particle to strengthen itself. In essence, it also belongs to social learning as in PSO, but it is used only on the *gworst* particle, not on all the particles, which will significantly improve not only the worst particle but also the quality of the whole swarm. To sum up, *gworst* particle can enhance the performance of PSO by fighting against premature convergence and avoiding local optima, which is well worth exploring in the field of swarm intelligence optimization.

2.3 Review of the gbest Particle-Based Updating Strategy

In standard particle swarm optimization, it is known that the movement of a particle is usually directed by the velocity and position update formula, in which the personal best position (*pbest*) and global best position (*gbest*) play a critical role in regulating particle search around a region. From the literature, it is observed that the majority of the existing PSO variants adopt *pbest* and *gbest* to guide the search process. Classical work includes the SRPSO [46], in which the self-regulating inertia weight is utilized by the *gbest* particle for a better exploration and the self-perception of the global search direction is used by the rest of particles for intelligent exploitation of the solution space. In literature

[47], IDE-PSO adopts three velocity update schemes for different kinds of particles: good particles, weak particles, and normal particles. Note that except for the normal particle, both the cognitive component associated with *pbest* and the social component associated with *gbest* have been removed from the velocity update formula for the other two cases. After that, the PSOTL [25] is proposed, in which an auxiliary velocity-position update scheme is exclusively used for *gbest* particle to guarantee the convergence of PSO while the other set of the ordinary update scheme is exploited for the updating of other particles.

Despite a great deal of research work dedicated to *gbest* particle for better optimization performance, however, it seems that PSO is somewhat arbitrary to make all the particles get acquainted with the same information from the *gbest*. In this case, no matter how far particle is from the *gbest*, each particle learns information from the global best position. As a result, all of them are quickly attracted to the optimal position and the swarm diversity starts to drastically decrease. Moreover, the search information of *gbest* in standard PSO is merely used to guide the search direction, which will also inevitably impair the diversity of swarm. As a result, it increases the possibility of PSO falling into local optima. Because of this, some PSO variants adopt the concept of an exemplar instead of *gbest* to direct the flying of particles, such as the orthogonal learning PSO [48], in which the exemplar comes from *pbest* or *gbest* resulting from the construction of the experimental orthogonal design. In [49], the leader particle selected from the archive that provides the best aggregation value (*Lbest*) is utilized to replace *gbest* to update the velocity. In literature [50], the non-parametric PSO is developed by discarding the parameters of inertia weight and acceleration coefficients. In particular, each particle uses the best position found by its neighbors (*plbest*) to update its velocity, and the effect of *gbest* is integrated into the position update scheme. Recent work [28] makes use of the *Spbest* and *Mpbest* to replace the personal best position and global best position in standard PSO update scheme, respectively. Specifically, stochastic learning gives particles ability to learn from other excellent individuals in the swarm and moves particles more diverse (*Spbest*). On the other hand, the overemphasis on *gbest* will lead to the quick loss of swarm diversity, thus the mean of the personal best positions (*Mpbest*) of all particles is devised to solve this potential problem by enhancing communication between particles in the dimensional aspect. Similarly, to pull the entire swarm toward the true Pareto-optimal front quickly, MaPSO is developed in [51], in which the leader particle is selected from several historical solutions by using the scalar projections, and a sign function is used to adaptively adjust the search direction for each particle. In more recent work [52], the distance from the particle to its *pbest* and *gbest* are respectively used to replace the cognitive and social components associated with the *pbest* and *gbest* in the velocity update scheme. In a nutshell, it is of utmost importance that the careful incorporation of *gbest* into particle swarm optimization is instrumental in achieving good performance.

3 Standard PSO

Particle swarm optimization is a powerful nature-inspired meta-heuristic optimization technique that emulates the social behavior of bird flocking and fish schooling [1]. In standard PSO (SPSO), each particle denotes a candidate solution in the search space and the method searches for the global optimal solution by updating generations. Given that in an n-dimensional hyperspace, two vectors are related to the ith particle ($i = 1, 2, \ldots, N$), one is the velocity vector $V_i = [v_{i1}, v_{i2}, \ldots, v_{in}]$, and the other is the position vector $X_i = [x_{i1}, x_{i2}, \ldots, x_{in}]$, where N denotes the swarm size. During the search process, each particle dynamically updates its position based on its previous position and new information regarding velocity. Moreover, the ith particle ($i = 1, 2, \ldots, N$) will remember its current personal best location found in the search space so far, usually denoted as $pbest_i = [pbest_{i1}, pbest_{i2}, \ldots, pbest_{in}]$, and the best previous location found by the whole swarm described as $gbest = [gbest_1, gbest_2, \ldots, gbest_n]$.

After these two best values are identified, the velocity and position update as follows:

$$V_i = w \times V_i + c_1 \times rand_1 \times (pbest_i - X_i) + c_2 \times rand_2 \times (gbest - X_i) \quad (1)$$

$$X_i = X_i + V_i \quad (2)$$

where w is the inertia weight used for the balance between the global and local search capabilities of the particle. A large inertia weight, w, at the beginning of the search increases the global exploration ability of swarm. In contrast, a small value of w near the end of the search enhances the local exploitation ability. c_1 and c_2, termed as acceleration coefficients, are positive constants reflecting the weighting of stochastic acceleration terms that pull each particle toward *pbest* and *gbest* positions respectively in the flight process. $rand_1$ and $rand_2$ are two random numbers uniformly distributed in the interval [0, 1]. Besides, the velocity is confined within a range of $[V_{min}, V_{max}]$ to prevent particles from flying out of the search space. Note that if the sum of the accelerations causes the velocity of that dimension to exceed a predefined limit, the velocity of that dimension is clamped to a defined range.

So far, the complete procedure of standard PSO can be described below:

Step 1: Initialization

Initialize the parameters and swarm with random positions and velocities.

Step 2: Evaluation

Evaluate the fitness value (the desired objective function) for each particle.

Step 3: Find *pbest*

If the fitness value of particle i is better than its best fitness value (*pbest*) in history, then set the current fitness value as the new *pbest* of particle i.

Step 4: Find *gbest*

If any updated *pbest* is better than the current *gbest*, then set *gbest* as the current value of the whole swarm.

Step 5: Update

Update the velocity and move to the next position according to Eqs. (1)–(2).

Step 6: Termination

If the termination condition is reached, then stop; otherwise turn to Step 2.

4 Proposed AMS-PSO

4.1 Swarm Initialization

For any swarm intelligence computation, swarm initialization is a crucial step since it affects the convergence rate and the quality of final solutions. From the literature, it can be seen that the most common method is random initialization [24], which is often exploited to generate initial particles without any information about the solution to the problems. However, it is reported that PSO with random initialization cannot be guaranteed to provide stable performance and fast convergence [2]. Based on this recognition, large amounts of research have been devoted to the development of high-quality initial particles for PSO to improve its performance, including chaos based sequence [7], opposition-based learning [12], chaotic opposition-based method [26], and other paradigms based swarm initialization [27]. As one of the most often used chaotic systems, logistic map [2,7,8,25] has been attracting significant research attention in the evolutionary computation area in the most recent

years owing to its properties of randomness, ergodicity, regularity, and non-repeatability, especially in the community of particle swarm optimization. As a result, it is observed that many PSO studies make full use of chaotic systems to enhance their search capabilities without being trapped in local optima from different aspects of chaos-based sequence for swarm initialization [25], chaos-based sequence for parameter setting [8], chaos-based sequence for mutation [26] and so forth. Given this, the classical logistic map is exploited in this work, which can be described as follows:

$$x_{i+1} = \mu \times x_i \times (1 - x_i), i = 0, 1, 2, \ldots \tag{3}$$

where x_i represents the ith chaotic variable in the interval 0 and 1 under the conditions that x_0 is generated randomly for each independent run without being equal to some periodic fixed points 0, 0.25, 0.5, 0.75, and 1. μ is a predefined constant termed as bifurcation coefficient. Note that the value of μ determines the bifurcation of the logistic map. In the case of $\mu < 3$, x_i converges to a single point. When $\mu = 3$, x_i oscillates between two points, and this characteristic change in behavior is called bifurcation. For $\mu > 3$, x_i encounters further bifurcation, eventually resulting in chaotic behavior. Due to the limited space, here we only give a brief introduction to logistic map and for more details please refer to literatures [2,7,8,12,25]. Algorithm 1 illustrates the pseudocode of the chaos-based swarm initialization, which mainly consists of two stages: generating a chaotic variable and mapping the chaotic variable to the search space of the problem to be solved. Note also that x_{min} and x_{max} denote the lower and upper bounds of the optimization variable.

Algorithm 1: Pseudocode of Chaotic Sequence-Based Initialization

Input: μ: bifurcation coefficient, N: swarm size, D: swarm dimension,
$S = \text{NULL}$, X: current swarm.

Process:

1. Randomly initialize chaotic variables.
2. *% Generate chaotic sequence as the initial swarm*
3. **while** *the number of maximal iterations* is not met **do**
4. **if** a chaotic variable falls into fixed points or periodic cycles **then**
5. Execute a very small positive random perturbation.
6. Map them by Eq. (3).
7. **else**
8. Update chaotic variables by Eq. (3) directly.
9. **end if**
10. **end while**
11. Save the chaotic variables in set X.
12. *% Map chaotic variables in X to the solution space*
13. **for** $i = 1$ **to** N **do**
14. **for** $j = 1$ **to** D **do**
15. $x_{ij} = x_{min} + (x_{max} - x_{min}) * x_{ij}$
16. **end for**
17. **end for**
18. Save the initial variables in set X.

Output: the N particles as the initial swarm.

4.2 Update for gbest Particle

As discussed in Section 2, the movement of particles heavily depends on *pbest* and *gbest*, and the acceleration coefficients stand for the weighting of *pbest* and *gbest*. Hence, the proper control of these two components is very important in finding the optimal solution accurately and efficiently. Moreover, they are capable of achieving an effective balance between exploration and exploitation during the evolution process under the conditions of proper pertinent parameters, especially the cognitive weighting factor c_1 and the social weighting factor c_2. Because of this, the velocity update equation of particles is devised by varying the value of acceleration factors to control the influence of cognitive information and social information, respectively. On one side, instead of introducing complex learning strategies adopted by most existing PSOs, the ultimate goal of this work is to explore how to obtain much better PSO performance through the use of learning strategies as simple as possible. On the other side, since all of the particles may carry useful information and the performance of swarm may be enhanced by updating all particles, including *pbest*, *pworst*, and normal particles. Hence, different updating strategies are adopted for different particles in different evolution stages. Specifically, to guarantee the convergence of AMS-PSO, an auxiliary update strategy is exclusively used to keep the *gbest* particle moving until it has reached a local minimum under the assumption of minimization [53], which can be expressed as below:

$$v_{\xi d}(t+1) = -x_{\xi d}(t) + p_{gd}(t) + wv_{\xi d}(t) + \rho(t)(1 - 2r_{2d}(t)) \tag{4}$$

$$x_{\xi d}(t+1) = x_{\xi d}(t) + v_{\xi d}(t+1) = p_{gd}(t) + wv_{\xi d}(t) + \rho(t)(1 - 2r_{2d}(t)) \tag{5}$$

where ζ denotes the index of the *gbest* particle, $-x_{\zeta d}(t)$ resets the particle's position to the *gbest* $p_{gd}(t)$, $wv_{\zeta d}(t)$ indicates the current search direction, $\rho(t)(1-2r_{2d}(t))$ generates a random sample from a sample space with side lengths $2\rho(t)$. Note that ρ is a scaling factor defined below. The addition of the ρ term enables PSO to perform a random search in an area surrounding the *gbest*. It determines the diameter of the search area to proceed with exploring.

$$\rho(t+1) = \begin{cases} 2\rho(t), & if\ \#\ successes\ > s_c \\ 0.5\rho(t), & if\ \#\ failures\ > f_c \\ \rho(t), & otherwise \end{cases} \tag{6}$$

where #*successes* and #*failures* denote the numbers of consecutive successes and failures, respectively. Here, a failure is defined as $f(p_g(t)) = f(p_g(t-1))$ whereas success is just the opposite. s_c and f_c are preset thresholds, the optimal choice of their values depends on the objective function. In common cases a default initial value $\rho(0) = 1.0$ has been found empirically to produce encouraging results.

4.3 Update for gworst and Other Particles

Note that compared to the studies of the *gbest* particle, almost none of the researchers pay attention to the *gworst* particle, thus unable to discover the potential information concealed around it. However, the *gworst* particle does not mean it does not contain any useful information at all. On the contrary, just as the *cask effect* says that the weak part often determines the level of the whole organization. So the effect of the *gworst* particle will be fully considered in this work. At the same time, it must be emphasized that both exploration and exploitation are equally important and should be explored cleverly during the evolution process. Based on this scenario, it is expected to realize the twin goals of balancing the global and local search abilities as well as decreasing the risk of falling into local optima by adjusting parameters c_1 and c_2 for PSO. Without loss of generality, a large cognitive weighting factor c_1 and a small social weighting factor c_2 are applied to guide the search for *gworst*

particle to avoid possible stagnation [12]. In addition, for other normal particles in swarm, three sets of different acceleration coefficients are utilized to direct the search process. To be specific, the equal weightage is applied to c_1 and c_2 in the early evolution stage to enhance the exploration ability when the fitness of particle is less than or equal to the average fitness of the whole swarm, otherwise, a small c_1 and a large c_2 is adopted to speed up the convergence rate. Accordingly, a large c_1 and a small c_2 are exploited in the velocity update formula to improve the exploitation capability when the fitness of particle is greater than the average fitness of the entire swarm, otherwise, Gaussian mutation [26] is timely applied to mutate *pbest* of particles to sustain the swarm diversity in the later evolution stage for fine-tuning the search process.

$$pbest'_{ij} = pbest_{ij} + gaussian_j() \tag{7}$$

where $gaussian_j()$ is a random number generated by Gaussian distribution.

In contrast to most existing PSO variants that formulate the velocity and position updates based on large amounts of complex mechanisms, such as various devised parameters [2], topology structure [30], hybrid learning strategy [32], elite and dimensional learning strategies [33], enhanced learning strategy [34] and so on. Different from the existing update strategies, however, AMS-PSO takes advantage of distinct update schemes for different particles based on the simple yet very effective constant acceleration factors to enhance the performance of PSO. In essence, the idea behind AMS-PSO is somewhat identical to the multi-swarm particle swarm optimization [13]. That is to say, the larger swarm is first divided into several small-sized sub-swarms that can be regrouped after a predefined period during the evolution process, whose main aim is to explore different sub-regions of the solution space with different search strategies and to take advantage of the experience gathered by others through sharing information until the end of the search process.

Besides that, the classical linear decreasing inertia weight is leveraged in AMS-PSO method as follows [23]:

$$w(t) = (w_{max} - w_{min}) \times \frac{(iter_{max} - iter)}{iter_{max}} + w_{min} \tag{8}$$

where $w_{\max}$ and $w_{\min}$ are the initial and final values of inertia weight, *iter*, and $iter_{\max}$ denote the current iteration and the maximal allowed the number of iterations, respectively.

Up to this point, the succinct procedure of AMS-PSO can be described as follows:

Algorithm 2: Pseudocode of the proposed AMS-PSO algorithm

Input: w_0: inertia weight, N: swarm size, D: swarm dimension, $iter_{\max}$: the maximal allowable number of iterations, β: control factor.

Process:

1. Initialize the particles of swarm by **Algorithm 1**.
2. Calculate w by Eq. (8).
3. **while** the maximum number of iterations (iter $\leq$ iter$_{\max}$) is not met **do**
4. Calculate f_i and f_{avg}.
5. **switch** (f_i)

(Continued)

Algorithm 2 (continued)

{
case *gbest*:
Update velocity and position by Eqs. (4)–(5) with equal c_1 and c_2.
case *gworst*:
Update velocity and position by Eqs. (1)–(2) with unequal c_1 and c_2.
default
if $iter \leq \beta iter_{max}$ **then**
% *Enhance exploration ability*
if $f_i \leq f_{avg}$ **then**
Update velocity and position by Eqs. (1)–(2) with equal c_1 and c_2.
else
Update velocity and position by Eqs. (1)–(2) with unequal c_1 and c_2.
end if
else
% *Enhance exploitation ability*
if $f_i \leq f_{avg}$ **then**
Perturb *pbest* of particles by Gaussian mutation with Eq. (7).
else
Update velocity and position by Eqs. (1)–(2) with unequal c_1 and c_2.
end if
end if
}
Calculate the fitness values of the new particle X_i.
if X_i *is better than* $pbest_i$ **then**
Set X_i to be $pbest_i$.
end if
if X_i *is better than gbest* **then**
Set X_i to be *gbest*.
end if
end while

Output: *gbest* particle as the final optimal solution.

5 Experimental Results and Analysis

5.1 Basic Test Functions

To demonstrate the effectiveness of AMS-PSO proposed in this paper, six well-known benchmark functions are adopted for simulation, they are expressed as follows:

(1) Sphere function

$$\min f_1(x) = \sum_{i=1}^{d} x_i^2$$

where the global optimum $x^* = 0$ and $f(x^*) = 0$ for $-10 \leq x_i \leq 10$.

(2) Schwefel function

$$\min f_2(x) = \sum_{i=1}^{d} |x_i| + \prod_{i=1}^{d} |x_i|$$

where the global optimum $x^* = 0$ and $f(x^*) = 0$ for $-100 \le x_i \le 100$.

(3) Rosenbrock function

$$\min f_3(x) = \sum_{i=1}^{d-1} \left[100(x_i^2 - x_{i+1})^2 + (x_i - 1)^2\right]$$

where the global optimum $x^* = (1, 1, \ldots, 1)$ and $f(x^*) = 0$ for $-30 \le x_i \le 30$.

(4) Rastrigin function

$$\min f_4(x) = \sum_{i=1}^{d} \left[x_i{}^2 - 10\cos(2\pi x_i) + 10\right]$$

where the global optimum $x^* = 0$ and $f(x^*) = 0$ for $-5.12 \le x_i \le 5.12$.

(5) Griewank function

$$\min f_5(x) = \frac{1}{4000}\sum_{i=1}^{d}(x_i)^2 - \prod_{i=1}^{d}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$$

where the global optimum $x^* = 0$ and $f(x^*) = 0$ for $-600 \le x_i \le 600$.

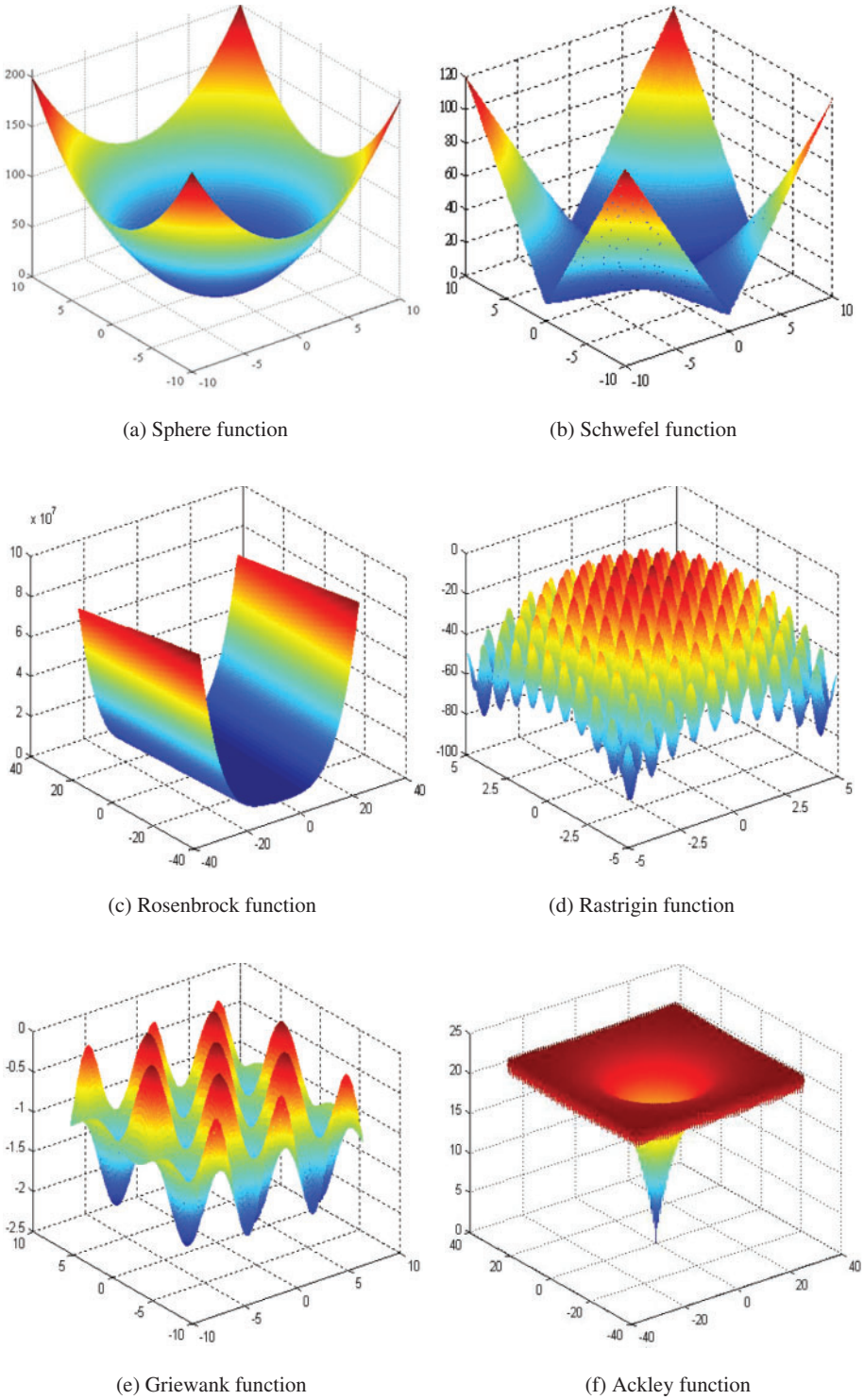
(6) Ackley function

$$\min f_6(x) = -20\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d} x_i^2}\right) - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_i)\right) + 20 + e$$

where the global optimum $x^* = 0$ and $f(x^*) = 0$ for $-32 \le x_i \le 32$.

Fig. 1 depicts the 2-dimensional graphical shows of the six test functions.

Table 1 shows the search range, optimal x^*, global optimum and property of test functions. Note that all of the test functions are to be minimized. The first three functions are unimodal whereas the last three are multimodal. Specifically, f_1 is a continuous and convex function that has several local minima except for the global one. f_2 is complex, with many local minima. It is deceptive in that the global minimum is geometrically distant, over the parameter space, from the next best local minima. Hence the search method is potentially prone to convergence in the wrong direction. Note that f_3 is also referred to as the valley or banana function, which is a popular test problem for the gradient-based optimization algorithms and the global minimum lies in a narrow, parabolic valley. However, even if this valley is easy to find, convergence to the minimum is difficult. f_4 has several local minima, but locations of the minima are regularly distributed. Similarly, f_5 has many widespread local minima that are regularly distributed in the allowable search space. As for function f_6, it is widely used for testing optimization algorithms. In its two-dimensional form as shown in the plot above, it is characterized by a nearly flat outer region and a large hole at the center, this function poses a risk for the optimization algorithm, particularly hill climbing, to be trapped in one of its many local minima.



(a) Sphere function (b) Schwefel function

(c) Rosenbrock function (d) Rastrigin function

(e) Griewank function (f) Ackley function

Figure 1: Graphical shows of the test functions

Table 1: Search range, optimal x^*, global optimum, and property of test functions

Function	Search range	Optimal x^*	Global optimum	Property
f_1	$[-100, 100]^D$	$\{0\}^D$	0	Unimodal
f_2	$[-10, 10]^D$	$\{0\}^D$	0	Unimodal
f_3	$[-30, 30]^D$	$\{1\}^D$	0	Unimodal
f_4	$[-5.12, 5.12]^D$	$\{0\}^D$	0	Multimodal
f_5	$[-600, 600]^D$	$\{0\}^D$	0	Multimodal
f_6	$[-32, 32]^D$	$\{0\}^D$	0	Multimodal

5.2 Parameter Settings

The parameter settings can be described as follows: the bifurcation coefficient μ is set to be 4, swarm size $N = 40$, time-varying inertia weight starts at 0.9 and ends at 0.4 according to Eq. (8), viz. $w_{\max} = 0.9$ and $w_{\min} = 0.4$, the control factor β is set as 0.5 by trial and error by taking into account the tradeoff between convergence rate and computational accuracy. In particular, the equal acceleration coefficients are set to be 2 ($c_1 = c_2 = 2$) whereas the unequal setting of $c_1 = 3$ and $c_2 = 1$ is exclusively used for *gworst* particle. At the same time, the first group of the unbalanced setting of $c_1 = 1.5$ and $c_2 = 2.5$ is leveraged in the case of the fitness of particle is greater than the average of swarm in the early evolution process while the second group of the unequal setting of $c_1 = 2.5$ and $c_2 = 1.5$ is applied under the conditions of the fitness of particle is greater than the average of the entire swarm in the later search process. In such a case, a well-balanced exploration and exploitation can be achieved. For each test function, 30 independent runs are performed by AMS-PSO, and each run is with 1000 iterations. Without loss of generality, the most often used metrics, including best solution, average solution, and standard deviation, are leveraged to demonstrate the performance of AMS-PSO proposed in this paper.

5.3 Results and Analysis

Note that since the superiority of chaos-based swarm initialization has already been validated in our previous studies [2,7,25], so we will not detail it anymore here. Instead, to illustrate the effect of different update schemes for PSO, AMS-PSO is first compared with its three variants in this section, involving AMS-PSO-gb, which denotes the acronym of AMS-PSO without the auxiliary update strategy that is exclusively applied to *gbest* particle. Likewise, AMS-PSO-gw is the acronym of AMS-PSO without an update strategy based on the unequal acceleration factors that are applied to *gworst* particle. Besides, AMS-PSO-no is the acronym of AMS-PSO without an update strategy based on unequal acceleration factors that are applied to normal particles. In other words, except for the different update schemes based on different acceleration factors, all the experiments are carried out under the same conditions described in Subsection 5.2.

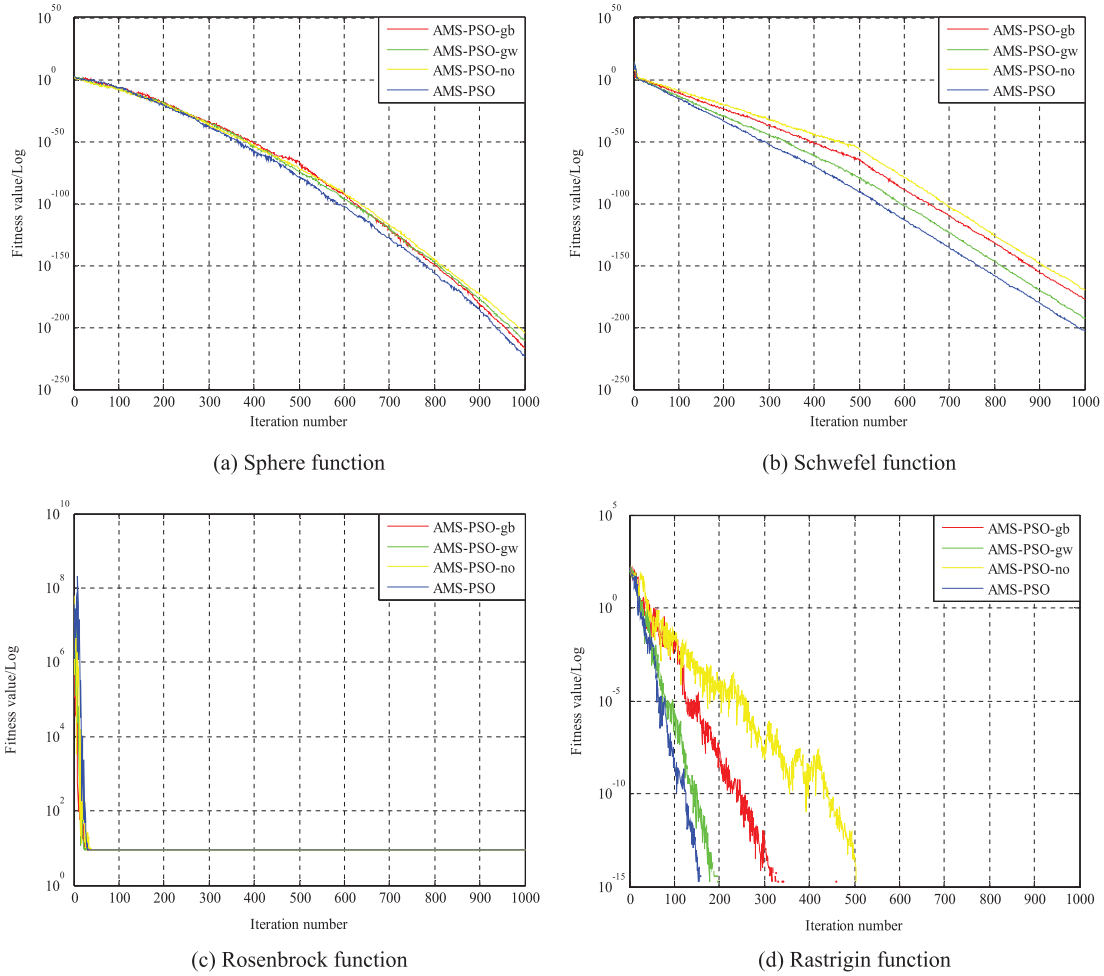
From Table 2, it is observed that AMS-PSO markedly outperforms its three variants on all the test functions except on f_3 and f_4 with the same optimization results. For the first two unimodal functions, AMS-PSO achieves an advantage of 6, 8, and 13 along with 29, 14, and 36 orders of magnitude over the other corresponding variants in terms of best solution, respectively. As for the third unimodal function, since the global minimum of f_3 lies in a narrow and parabolic valley, it is rather difficult to converge to the minimum even if the valley is easy to be found. Hence, the optimized result on function f_3 is relatively poor compared to the best solutions of other functions achieved by our proposal. What

is more, all the AMS-PSOs converge to the same global minimum with dimension 10. In comparison, f_4 is multimodal, which is the widely used test function in the research of evolutionary computation. Likewise, it is prone to be trapped in a local minimum on its way to the global minimum. However, by achieving an effective balance between the exploration and exploitation search ability, AMS-PSO gets the global optima on each dimension on f_4. This is largely attributed to the different update schemes applied to regulate the particle search process at different evolution stages. Meanwhile, the stability of PSO is improved obviously due to the high-quality initial particles generated by the chaotic map. Concerning the rest two multimodal functions f_5 and f_6, AMS-PSO can also get promising results in terms of best solution, average solution, and standard deviation in comparison with its three variants, which further validates the respective merit of the auxiliary update strategy for the global best particle, the unequal weightage for the global worst particle as well as equal weightage and Gaussian mutation for other particles, and all of these strategies should be used together for good performance based on their complementary information.

Table 2: Results of AMS-PSO with different update schemes under $D = 10$

Function	Method	Best solution	Average solution	Standard deviation
f_1	AMS-PSO-gb	3.4026e−218	4.2943e−213	0.0000e−000
	AMS-PSO-gw	1.8100e−216	1.0681e−208	0.0000e−000
	AMS-PSO-no	1.5718e−211	7.5912e−205	0.0000e−000
	AMS-PSO	1.9420e−224	4.6632e−216	0.0000e−000
f_2	AMS-PSO-gb	5.1099e−178	3.5904e−175	0.0000e−000
	AMS-PSO-gw	2.4412e−193	1.4239e−189	0.0000e−000
	AMS-PSO-no	8.2301e−171	6.6005e−169	0.0000e−000
	AMS-PSO	1.9292e−207	7.2248e−204	0.0000e−000
f_3	AMS-PSO-gb	9.0000e−000	9.0000e−000	0.0000e−000
	AMS-PSO-gw	9.0000e−000	9.0000e−000	0.0000e−000
	AMS-PSO-no	9.0000e−000	9.0000e−000	0.0000e−000
	AMS-PSO	9.0000e−000	9.0000e−000	0.0000e−000
f_4	AMS-PSO-gb	0.0000e−000	0.0000e−000	0.0000e−000
	AMS-PSO-gw	0.0000e−000	0.0000e−000	0.0000e−000
	AMS-PSO-no	0.0000e−000	0.0000e−000	0.0000e−000
	AMS-PSO	0.0000e−000	0.0000e−000	0.0000e−000
f_5	AMS-PSO-gb	1.3096e−001	1.4483e−001	1.6133e−002
	AMS-PSO-gw	1.1633e−001	1.2357e−001	6.3108e−003
	AMS-PSO-no	1.4467e−001	1.5742e−001	1.6036e−002
	AMS-PSO	7.3165e−003	8.8363e−003	4.8173e−003
f_6	AMS-PSO-gb	2.4631e−013	3.4386e−012	3.5168e−012
	AMS-PSO-gw	9.0821e−013	4.5243e−012	6.4578e−012
	AMS-PSO-no	4.2772e−012	2.9177e−011	1.7719e−011
	AMS-PSO	2.9747e−014	3.8837e−012	3.7115e−012

To illustrate the convergence process of different AMS-PSOs, Fig. 2 depicts the evolution curves of PSOs with different update schemes for all the test functions. Taking Fig. 2a for example, the evolution curves of the four AMS-PSOs decline at almost the same rate during the whole search process, and they are very close to each other. It is worth noting that the evolution trends for function f_2 are somewhat similar to Fig. 2a except for obvious differences. Note that an interesting observation comes from Fig. 2c, all of the convergence curves decrease sharply in the first 50 iterations. After that, they converge very slowly and flatly until the end of the search. That is to say, all of them get stuck in local optimal at this point. This shows that the multiple update strategies formulated in this work have a marginal effect on the optimization performance for function f_3. Similarly, one can see that all the PSOs evolve very fast on f_5 and f_6 at the early 100 iterations, and subsequently evolve very slowly and flatly without obvious improvement except for AMS-PSO tested by function f_6. Besides, it can be seen that AMS-PSO quickly converges to the optimal solution before 200 iterations on f_4. In contrast, AMS-PSO-no evolves with a slow convergence rate even though it converges to the optimal solution at the end. Note that the convergence curves of AMS-PSO-gb and AMS-PSO-gw lie between the above two cases, and AMS-PSO-gw evolves much faster than AMS-PSO-gb. That is, the global best particle plays an important role in guaranteeing the convergence of PSO.



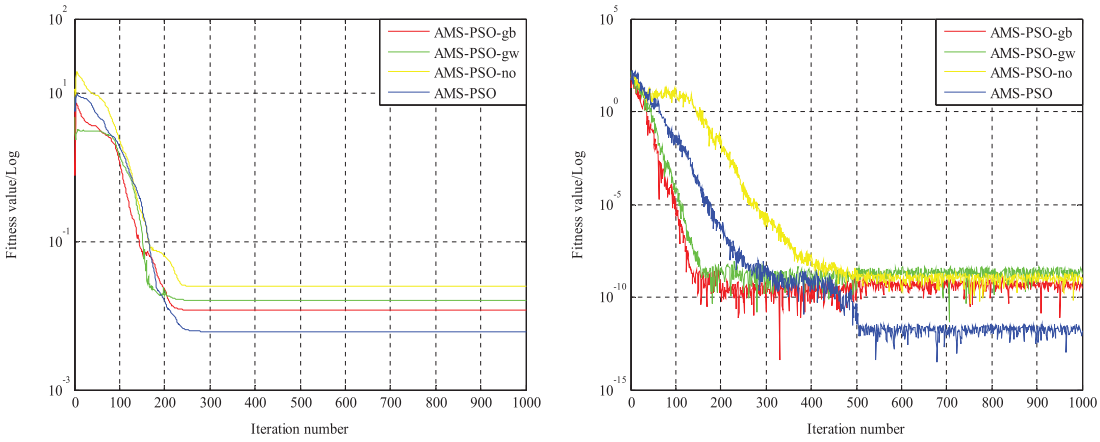
(a) Sphere function

(b) Schwefel function

(c) Rosenbrock function

(d) Rastrigin function

Figure 2: (Continued)



(e) Griewank function (f) Ackley function

Figure 2: Evolution curves of different AMS-PSO variants

5.4 CEC'17 Test Suite

To further illustrate the effect of the proposed AMS-PSO algorithm, experiments are also conducted on the well-known CEC 2017 test suite [54] that has been widely used by researchers [55–58]. It includes 30 functions, which can be roughly classified into four groups. Specifically, the first group involves three unimodal functions (f_1–f_3), in which function f_2 has been excluded here because it shows unstable behavior especially for higher dimensions. Functions f_4–f_{10} belongs to the second group, and all of them are multimodal functions. The third group comprises ten hybrid functions (f_{11}–f_{20}), and the last group includes ten composition functions (f_{21}–f_{30}). Thus the total number of test functions is twenty-nine. Note that Table 3 lists the name, search range, global optimum, and property of CEC'17 test functions.

Table 3: Name, search range, global optimum and property of CEC'17 test functions

Func.	Name	Search range	Global optimum	Property
f_1	Shifted and rotated bent cigar	$[-100, 100]^D$	100	Unimodal
f_3	Shifted and rotated zakharov	$[-100, 100]^D$	300	Unimodal
f_4	Shifted and rotated rosenbrock's	$[-100, 100]^D$	400	Multimodal
f_5	Shifted and rotated rastrigin's	$[-100, 100]^D$	500	Multimodal
f_6	Shifted and rotated expanded scaffer's F6	$[-100, 100]^D$	600	Multimodal
f_7	Shifted and rotated lunacek bi_rastrigin	$[-100, 100]^D$	700	Multimodal
f_8	Shifted and rotated non-continuous rastrigin's	$[-100, 100]^D$	800	Multimodal
f_9	Shifted and rotated levy	$[-100, 100]^D$	900	Multimodal
f_{10}	Shifted and rotated schwefel's	$[-100, 100]^D$	1000	Multimodal
f_{11}	Hybrid function 1 (N = 3)	$[-100, 100]^D$	1100	Hybrid

(Continued)

Table 3 (continued)

Func.	Name	Search range	Global optimum	Property
f_{12}	Hybrid function 2 (N = 3)	$[-100, 100]^D$	1200	Hybrid
f_{13}	Hybrid function 3 (N = 3)	$[-100, 100]^D$	1300	Hybrid
f_{14}	Hybrid function 4 (N = 4)	$[-100, 100]^D$	1400	Hybrid
f_{15}	Hybrid function 5 (N = 4)	$[-100, 100]^D$	1500	Hybrid
f_{16}	Hybrid function 6 (N = 4)	$[-100, 100]^D$	1600	Hybrid
f_{17}	Hybrid function 6 (N = 5)	$[-100, 100]^D$	1700	Hybrid
f_{18}	Hybrid function 6 (N = 5)	$[-100, 100]^D$	1800	Hybrid
f_{19}	Hybrid function 6 (N = 5)	$[-100, 100]^D$	1900	Hybrid
f_{20}	Hybrid function 6 (N = 6)	$[-100, 100]^D$	2000	Hybrid
f_{21}	Composition function 1 (N = 3)	$[-100, 100]^D$	2100	Composition
f_{22}	Composition function 2 (N = 3)	$[-100, 100]^D$	2200	Composition
f_{23}	Composition function 3 (N = 4)	$[-100, 100]^D$	2300	Composition
f_{24}	Composition function 4 (N = 4)	$[-100, 100]^D$	2400	Composition
f_{25}	Composition function 5 (N = 5)	$[-100, 100]^D$	2500	Composition
f_{26}	Composition function 6 (N = 5)	$[-100, 100]^D$	2600	Composition
f_{27}	Composition function 7 (N = 6)	$[-100, 100]^D$	2700	Composition
f_{28}	Composition function 8 (N = 6)	$[-100, 100]^D$	2800	Composition
f_{29}	Composition function 9 (N = 3)	$[-100, 100]^D$	2900	Composition
f_{30}	Composition function 10 (N = 3)	$[-100, 100]^D$	3000	Composition

To fully demonstrate the advantages of our proposal, AMS-PSO is conducted with dimension 30 and is compared with seven state-of-the-art PSO variants, including MPSO [28], IH-PSO [59], LIPS [60], CDWPSO [61], EPSO [62], HCLPSO [31] and CLPSO [63]. Among the compared PSOs, LIPS, EPSO, HCLPSO and CLPSO[1] as well as MPSO[2] have been reproduced and the experimental results are used to evaluate their performance. As for IH-PSO and CDWPSO, for the sake of fair comparison, their experimental results are directly taken from [59,61,64] to be compared because it is almost impossible to reproduce these two PSO variants exactly as they were in their original literature, even including the running environment of program. Without loss of generality, the mean best solution (Mean) and standard deviation (Std.) are utilized to verify the performance, and the best result in a comparison is emphasized as bold font. Table 4 lists the parameter settings employed in the experiment. Likewise, the swarm size is 50, each PSO variant is run 30 times on every test function with 2000 iterations for each run, and the stopping criterion is set as reaching the total number of iterations.

[1] https://github.com/P-N-Suganthan/CODES
[2] https://github.com/lhustl/MPSO

Table 4: Parameter settings of the compared PSO variants

PSO variant	Inertia weight	Acceleration coefficients	Other parameters
MPSO	w: 0.9∼0.4	$c_1 = c_2 = 2$	err = 1e−8
LIPS	$w = 0.7298$	$c_1 = c_2 = 2.05$	$nsize = 3$
EPSO	/	/	$\varepsilon = 0.01$, $\chi = 0.729$, $nsize = 3$, $g_1 = 8$, $g_2 = 12$
HCLPSO	w: 0.99∼0.2	$c_1 = 2.5∼0.5$, $c_2 = 0.5∼2.5$, $c = 3∼1.5$	K: 3∼1.5, $a = 0.05$, $b = 0.25$
CLPSO	w: 0.9∼0.4	$c_1 = 3∼1.5$	$m = 7$, $c_{4\_1} = 2.5∼0.5$, $c_{4\_2} = 0.5∼2.5$, $P_c = 0.5$
AMS-PSO	w: 0.9∼0.4	$c_1 = 3$, $c_2 = 1$; $c_1 = 1.5$, $c_2 = 2.5$; $c_1 = 2.5$, $c_2 = 1.5$; $c_1 = c_2 = 2$	$\mu = 4$, $\beta = 0.5$

From the results reported in Table 5, it can be clearly observed that AMS-PSO achieves better performance than other PSO variants on most of the functions. To be specific, AMS-PSO obtains the best result on functions $f_3, f_5, f_7, f_8, f_{11}, f_{22}, f_{24}, f_{25}$ and f_{28} respectively in terms of the mean value. At the same time, it is worth noting that our proposal obtains the same mean as EPSO, HCLPSO and CLPSO on functions f_6 and f_{25}, EPSO and CLPSO on function f_{17}, MPSO and IH-PSO on function f_{20}, HCLPSO on function f_{23} and CDWPSO on function f_{27}, respectively. Particularly, it achieves the global optimum on f_6. Another interesting observation is that even if AMS-PSO performs slightly worse than EPSO on f_9 and f_{21}, CLPSO on f_{16} and HCLPSO on f_{29} and f_{30} according to the mean, it achieves much better standard deviation on these functions, especially on functions f_9, f_{29} and f_{30}, it outperforms EPSO and HCLPSO with 1, 1 and 2 orders of magnitude difference. This can be largely attributed to the high-quality initial particles generated by the logistic map. In addition, note that AMS-PSO ranks first 14 times, second 9 times, third twice, fourth twice and fifth twice among the twenty-nine test functions. In contrast, MPSO, IH-PSO, LIPS, CDWPSO, EPSO, HCLPSO and CLPSO achieve 3, 1, 3, 1, 8, 6 and 5 first-place ranks, respectively. The average rank of AMS-PSO is 1.93, much lower than that of other PSO variants listed in Table 5. As a result, its overall rank is the first. In sum, all the experimental results demonstrate that AMS-PSO possesses promising optimization performance, at least for the functions of CEC'17 test suite employed here.

Table 5: Comparison results of AMS-PSO with other PSO variants

	Metric	MPSO	IH-PSO	LIPS	CDWPSO	EPSO	HCLPSO	CLPSO	AMS-PSO
f_1	Mean	4.63e+07	5.96e+09	**7.61e+02**	1.46e+10	3.61e+03	4.46e+03	3.63e+04	2.92e+03
	Std	7.31e+08	2.94e+09	**7.98e+02**	3.82e+09	4.41e+03	5.03e+03	1.22e+04	3.16e+03
f_3	Mean	7.43e+02	5.28e+04	6.93e+04	7.21e+04	4.83e+02	5.68e+03	6.47e+04	**3.55e+02**
	Std	2.72e+02	1.12e+04	1.88e+04	3.83e+03	**2.11e+02**	3.44e+03	1.16e+04	3.02e+02
f_4	Mean	5.09e+02	8.83e+02	6.04e+02	3.06e+03	**4.55e+02**	4.83e+02	5.15e+02	4.82e+02
	Std	1.30e+02	4.78e+02	8.30e+01	1.14e+03	3.44e+01	3.51e+01	**1.15e+01**	1.63e+01
f_5	Mean	5.67e+02	6.99e+02	5.61e+02	8.38e+02	5.55e+02	5.55e+02	5.96e+02	**5.42e+02**
	Std	3.28e+01	3.18e+01	1.62e+01	3.10e+01	1.61e+01	1.62e+01	1.20e+01	**1.15e+01**
f_6	Mean	6.02e+02	6.46e+02	6.07e+02	6.72e+02	**6.00e+02**	**6.00e+02**	**6.00e+02**	**6.00e+02**
	Std	8.61e+00	7.99e+00	4.31e+00	9.59e+00	2.18e−01	**1.16e−03**	8.90e−03	4.92e−01
f_7	Mean	7.96e+02	1.09e+03	7.86e+02	1.23e+03	7.82e+02	7.94e+02	8.66e+02	**7.70e+02**
	Std	2.27e+01	8.64e+01	1.66e+01	4.49e+01	1.74e+01	1.81e+01	1.43e+01	**1.37e+01**

(Continued)

Table 5 (continued)

	Metric	MPSO	IH-PSO	LIPS	CDWPSO	EPSO	HCLPSO	CLPSO	AMS-PSO
f_8	Mean	9.05e+02	1.02e+03	8.63e+02	1.08e+03	8.66e+02	8.58e+02	9.01e+02	**8.50e+02**
	Std	1.71e+01	3.40e+01	1.33e+01	2.37e+01	1.82e+01	3.05e+01	**1.03e+01**	1.12e+01
f_9	Mean	1.28e+03	5.30e+03	1.23e+03	9.19e+03	**9.30e+02**	9.36e+02	1.31e+03	9.31e+02
	Std	5.93e+02	1.47e+03	2.36e+02	1.12e+03	1.14e+02	6.90e+01	1.52e+02	**4.78e+01**
f_{10}	Mean	4.40e+03	7.58e+03	3.86e+03	7.76e+03	3.88e+03	**3.83e+03**	5.13e+03	4.11e+03
	Std	6.71e+02	5.38e+02	2.63e+02	5.15e+02	5.88e+02	6.83e+02	**2.57e+02**	4.06e+02
f_{11}	Mean	1.23e+03	1.86e+03	1.26e+03	4.73e+03	1.21e+03	1.18e+03	1.27e+03	**1.17e+03**
	Std	3.69e+02	1.02e+03	9.53e+01	1.08e+03	3.72e+01	5.17e+01	**2.98e+01**	3.29e+01
f_{12}	Mean	1.23e+06	2.34e+08	9.71+05	1.32e+09	**5.18e+04**	1.12e+05	1.88e+06	1.10e+05
	Std	1.34e+06	3.98e+08	1.69e+06	6.81e+08	**2.91e+04**	8.08e+04	5.71e+05	8.91e+04
f_{13}	Mean	1.80e+04	3.19e+07	**4.12e+03**	1.11e+09	1.62e+04	8.95e+03	2.37e+04	1.85e+04
	Std	5.04e+04	4.18e+07	**2.61e+03**	6.81e+08	3.51e+04	6.56e+03	1.32e+04	3.77e+04
f_{14}	Mean	**3.12e+03**	4.85e+04	2.62e+04	1.88e+06	1.12e+04	2.63e+04	3.11e+04	5.89e+03
	Std	**3.06e+03**	4.18e+04	3.13e+04	1.42e+06	1.29e+04	2.14e+04	2.29e+04	5.26e+03
f_{15}	Mean	3.30e+03	9.48e+03	**2.60e+03**	9.89e+03	3.93e+03	7.10e+03	3.33e+03	5.13e+03
	Std	3.23e+03	1.02e+03	1.96e+03	2.99e+03	3.13e+03	6.44e+03	**1.15e+03**	4.08e+03
f_{16}	Mean	2.61e+03	3.23e+03	2.29e+03	4.29e+03	2.30e+03	2.43e+03	**2.18e+03**	2.19e+03
	Std	3.02e+02	4.40e+02	2.27e+02	3.79e+02	1.85e+02	3.52e+02	1.43e+02	**1.38e+02**
f_{17}	Mean	2.05e+03	2.34e+03	1.93e+03	2.81e+03	**1.88e+03**	1.94e+03	**1.88e+03**	**1.88e+03**
	Std	1.93e+02	2.22e+02	9.46e+01	2.60e+02	8.91e+01	1.42e+02	6.62e+01	**4.99e+01**
f_{18}	Mean	**6.00e+04**	8.17e+05	2.05e+05	1.46e+07	1.82e+05	2.38e+05	2.55e+05	2.00e+05
	Std	**3.46e+04**	9.74e+05	3.20e+05	9.48e+06	3.19e+05	1.93e+05	1.13e+05	2.31e+05
f_{19}	Mean	5.21e+03	5.82e+03	3.01e+03	6.23e+03	2.91e+03	5.74e+03	**3.04e+02**	4.78e+03
	Std	2.19e+03	9.93e+03	1.53e+03	4.11e+03	2.74e+03	5.52e+03	**1.26e+03**	3.82e+03
f_{20}	Mean	**2.24e+03**	**2.24e+03**	2.46e+03	2.59e+03	2.30e+03	2.27e+03	2.80e+03	**2.24e+03**
	Std	8.31e+01	5.85e+01	1.97e+02	1.94e+02	2.49e+02	9.28e+01	2.03e+02	**4.39e+01**
f_{21}	Mean	2.37e+03	2.50e+03	2.36e+03	2.61e+03	**2.35e+03**	2.36e+03	2.39e+03	2.37e+03
	Std	2.35e+01	2.44e+01	1.77e+01	2.64e+01	3.98e+01	1.74e+01	2.86e+01	**1.52e+01**
f_{22}	Mean	2.31e+03	5.43e+03	2.32e+03	4.01e+03	2.31e+03	2.44e+03	2.43e+03	**2.29e+03**
	Std	8.21e+02	2.00e+03	3.03e+01	7.30e+02	**1.26e+01**	8.03e+02	9.08e+01	3.33e+01
f_{23}	Mean	2.74e+03	2.94e+03	2.75e+03	3.23e+03	2.73e+03	**2.72e+03**	2.75e+03	**2.72e+03**
	Std	7.50e+01	6.13e+01	2.66e+01	8.49e+01	2.33e+01	1.81e+01	4.26e+01	**1.67e+01**
f_{24}	Mean	2.89e+03	3.11e+03	2.91e+03	3.45e+03	2.88e+03	2.90e+03	2.92e+03	**2.87e+03**
	Std	1.12e+02	7.85e+01	2.92e+01	6.28e+01	3.82e+01	2.48e+01	5.37e+01	**2.12e+01**
f_{25}	Mean	2.99e+03	3.47e+03	2.92e+03	3.50e+03	**2.89e+03**	**2.89e+03**	**2.89e+03**	**2.89e+03**
	Std	2.77e+01	1.69e+02	4.13e+01	1.94e+02	1.15e+01	1.33e+00	1.40e+00	**1.29e+00**
f_{26}	Mean	4.46e+03	6.31e+03	3.72e+03	8.42e+03	**3.10e+03**	4.04e+03	3.98e+03	3.46e+03
	Std	2.48e+03	4.95e+02	8.33e+02	7.27e+02	6.21e+02	5.91e+02	4.31e+02	**3.99e+02**
f_{27}	Mean	3.24e+03	3.38e+03	3.31e+03	**3.20e+03**	3.22e+03	3.23e+03	3.22e+03	**3.20e+03**
	Std	3.77e+01	8.14e+01	2.64e+01	**2.27e−04**	8.77e+00	1.21e+01	3.85e+00	2.71e+00
f_{28}	Mean	3.33e+03	4.07e+03	3.30e+03	3.30e+03	3.21e+03	3.21e+03	3.25e+03	**3.20e+03**
	Std	5.23e+01	4.56e+02	8.91e+01	**5.75e−02**	4.70e+01	2.12e+01	1.05e+01	1.84e+01
f_{29}	Mean	3.60e+03	4.53e+03	3.83e+03	5.38e+03	3.60e+03	**3.52e+03**	3.53e+03	3.53e+03
	Std	2.20e+02	3.74e+02	1.50e+02	5.53e+02	2.94e+02	3.23e+02	6.74e+01	**2.90e+01**
f_{30}	Mean	1.63e+04	7.66e+06	1.03e+05	7.69e+07	9.81e+03	**8.84e+03**	2.18e+04	8.86e+03
	Std	8.03e+03	6.46e+03	2.00e+04	2.31e+07	5.15e+03	4.22e+05	4.88e+07	**4.79e+03**
Avg. rank		3.76	6.14	3.48	6.93	2.21	3.03	4.21	1.93
Final rank		5	7	4	8	2	3	6	1

Fig. 3 illustrates the histogram of mean ranks for AMS-PSO and the compared PSOs mentioned in this section, which is mainly used to indicate the number of times each PSO has acquired the ranks in the range of 1 to 8. Our proposal achieves the top ranks compared to other PSO variants, which further bears out the effect of AMS-PSO for numerical function optimization.

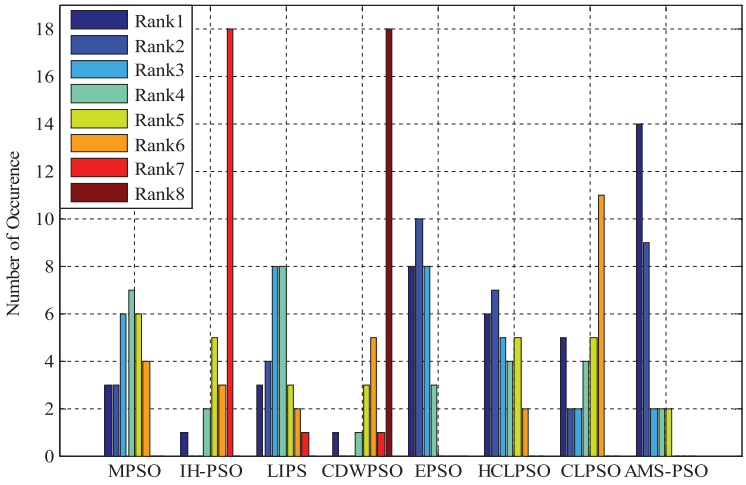


Figure 3: Histogram of the mean rank

In addition, to thoroughly and fairly compare AMS-PSO with other methods, we have validated it from the perspective of statistical analysis [65,66] by trial and error. To be specific, a pairwise statistical test named Wilcoxon signed-rank test is conducted between the results of the proposed AMS-PSO and the results of other PSO variants with the significance level of 5%, which is a non-parametric test that can be used to check for the statistical significance difference between two algorithms. As shown in Table 6, the number of test functions showing that AMS-PSO is significantly better than the compared PSO (Better), almost the same as the compared PSO (Same), and significantly worse than the compared PSO (Worse), respectively. Note that the "Merit" score is calculated by subtracting the "worse" score from the "better" score. From Table 6, it can be seen that AMS-PSO has achieved excellent performance and performed significantly better than the other seven PSOs on the majority of the test functions.

Table 6: Statistical analysis of Wilcoxon test between AMS-PSO and its competitors

Item	MPSO	IH-PSO	LIPS	CDWPSO	EPSO	HCLPSO	CLPSO
Better	19	25	20	25	13	15	18
Same	7	4	5	4	9	11	8
Worse	3	0	4	0	7	3	3
Merit	16	25	16	25	6	12	15

6 Conclusions and Future Work

This paper presents an adaptive multi-updating strategy based particle swarm optimization, which takes advantage of the chaotic sequence to generate high-quality initial particles to increase the

stability and accelerate the convergence rate of the PSO, the different update schemes to regulate the particle search process at different evolution stages, the unequal weightage of acceleration coefficients to guide the search for the global worst particle, and an auxiliary update strategy for the global best particle to ensure convergence of the PSO. Conducted experiments demonstrate that the proposed PSO outperforms several state-of-the-art PSOs in terms of solution accuracy and effectiveness.

As for future work, AMS-PSO will be compared with more competitive PSOs on more complex functions and in some real-world applications such as image processing, feature selection, optimization scheduling, and robot path planning, etc. More importantly, we intend to delve deeper into the use of different update schemes in different scenarios, especially for adequate acceleration coefficients tuning in a wide range of applications. Last but not the least, we will probe into the relationship between different acceleration coefficients and convergence of PSO as well as the relationship between stability and well-distributed initial particles generated by the logistic map from a computational efficiency perspective.

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