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# Effect of the Reynolds Number on the Flow Pattern in a Stenotic Right Coronary Artery

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## Summary

It is well known that the Reynolds number has a significant effect on the blood flow in human arteries. We developed a three dimensional model with simplified geometry for a diseased right coronary artery segment to study the influence of the Reynolds number on the flow pattern in a stenotic coronary artery. Computations were carried out under physiological flow conditions to examine how the characteristics of the flow, such as the flow velocity and the pressure drop along the inner wall, change corresponding to the varying of the blood viscosity or to the varying of the mean inlet flow rate. We found that some characteristics, such as the maximum flow shift and the area of flow separation zone, increase as the Reynolds number increases, no matter whether the increase of the Reynolds number is due to a decrease of the blood viscosity or an increase of the inlet flow rate. However the total pressure drop reacts differently as the Reynolds number increases depending on whether it is caused by the change of the blood viscosity or the inlet flow rate.

## Introduction

The blood flow in stenotic arteries has been extensively investigated by many researchers via clinical observations, experimental investigations and numerical simulations [1-11]. It is now widely believed that hemodynamics plays an important role in the formation and progression of atherosclerosis. The characteristics of the blood flow in a stenotic artery are affected by many factors, such as the geometry of the artery, pulsatile velocity waveform, velocity profile at the inlet boundary, the Reynolds number, etc. Among these, the Reynolds number has a significant effect on the blood flow pattern in stenotic arteries, especially on the flow downstream of the stenosis.

Many experimental and mathematical researches on hemodynamics in stenotic arteries have been reported to investigate the influence of the Reynolds number. Back et al. [1] conducted an in-vitro flow study on the effect of mild atherosclerosis on flow resistance in a coronary artery and found the variation of the pressure coefficient with the Reynolds number is significant. Gach et al. [3] presented a characteristic relationship between the reattachment length in the downstream of a stenosis and the stenotic Reynolds number using MRI. Talukder et al. [10] experimentally studied the effects of the number of stenosis and the distance between consecutive stenoses on the total pressure drop across a series of stenoses. They

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also examined the variation in the dimensionless pressure drop coefficient with the Reynolds number. However, most of these studies of the effect of the Reynolds number on the blood flow in arteries were focused on the varying of the Reynolds number due to the change of the flow rate. Not much attention has been paid to the influence of the Reynolds number on the flow characteristics corresponding to the varying of the blood viscosity.

The viscosity of blood depends on the concentration of protein, the formed blood element percentage (haematocrit, Ht), the pH value of the plasma and temperature [5]. The kinematic viscosity of blood can vary between 0.016 and 0.096 $cm^2/s$ [7]. A variation in the blood viscosity will result in a change in the Reynolds number. Therefore it is very important to understand the influence of the blood viscosity, in addition to the influence of the flow rate, on the stenotic blood flow when investigating the effect of the Reynolds number.

The purpose of this work is to investigate the difference in the correlation of the flow pattern to the decrease of the blood viscosity or the increase of the mean inlet flow rate, though either of these two changes can result in the increase of the Reynolds number.

## Modeling

In this study, the blood is assumed to be laminar, incompressible, and Newtonian. The wall is assumed rigid. These assumptions have been shown to be reasonable by many researchers [2, 8]. Flow motion is described by the unsteady three dimensional Navier-Stokes equations. The geometry model of a segment of the atherosclerotic right coronary artery is a curved artery with a stenosis at the inner wall, as shown in Fig. 1a), constructed based on the picture of a diseased artery in Fig. 5 in [9] in regard to the location, size and the shape of the stenosis. The artery has circular cross sections with a 0.45cm diameter at both inlet and outlet and with varying diameters in the region of stenosis. The bend has an angle of $72^0$ with a 1.8cm radius of curvature at the central line of the tube. The neck of the stenosis is at the end of the bend with the area of the cross section reduced by 51%. The straight inlet and outlet tubes are 1.0cm and 4.0cm long, respectively. The system of Navier-Stokes equations is imposed with the following boundary conditions: no-slip conditions are applied to the velocity components at the rigid artery wall; a surface traction free condition is imposed on the outlet boundary; at the inlet boundary, a blunt inlet velocity profile with a time varying waveform is assumed for the axial velocity u = Qf(r)w(t), where w(t), as shown in Fig. 1b), is a physiological pulsatile coronary velocity waveform [2, 6]; r is the radial distance of a point to the center of the cross section circle; f(r) is 1 for $r < 0.17$cm and continuously decreases from 1 to 0 for $0.17 < r \leq 0.225$cm; Q is a scalar chosen as different values in the computation such that Qf(r)w(t) yields different mean flow rates when examining the effect of the Reynolds number.

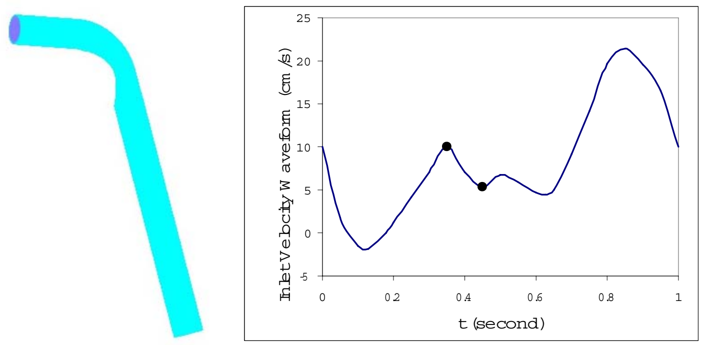


Figure 1: a) Three dimensional geometry model of a stenotic curved artery; b) Pulsatile velocity waveform w(t)

Numerical computations were performed using Comsol Multiphysics. Navier-Stokes equations were solved using the finite element method with piecewise quadratic functions for velocity and piecewise linear functions for pressure over a tetrahedral mesh. Computations were repeated over different meshes to ensure that the numerical solutions were mesh independent. The initial conditions for velocity and pressure were obtained by solving the system of steady state Navier-Stokes equations. Four cycles were simulated to ensure that the flow was truly periodic.

## Observations

Computations were carried out under various physiologic flow conditions to investigate the effect of the Reynolds number on the blood flow in a curved artery with a stenosis at the inner wall. The results were analyzed to compare the change in blood flow corresponding to the increase of the Reynolds number caused by decreasing the blood viscosity and by increasing the inlet flow rate. The Reynolds number was calculated by $R_e = \bar{U}D/\nu$, where $\bar{U}$ is the mean inlet velocity at the peak time, D is the diameter of the inlet tube, $\nu = \eta/\rho$ is kinematic viscosity, $\eta$ is the dynamic viscosity of blood and $\rho = 1.05 \text{g/cm}^3$. When examining the effect of the blood viscosity, $\eta$ was chosen as 0.0245, 0.0295 and 0.0345 dynes/cm$^2$, while the scalar Q for the inlet flow rate was fixed as 1. It resulted in the Reynolds number Re = 491, 407 and 348 respectively. On the other hand, when examining the effect of the inlet flow rate, the scalar Q was chosen as 1, 0.8305 and 0.7101, while $\eta$ was fixed as 0.0245 dynes/cm$^2$. It also resulted in Re = 491, 407 and 348, respectively.

From the numerical results we observed that some flow characteristics, such as the maximum flow shift, area of flow separation and magnitude of the reverse flow patterns, correlate to the change of the Reynolds number in a similar manner, no matter whether the change of the Reynolds number is caused by the varying of the

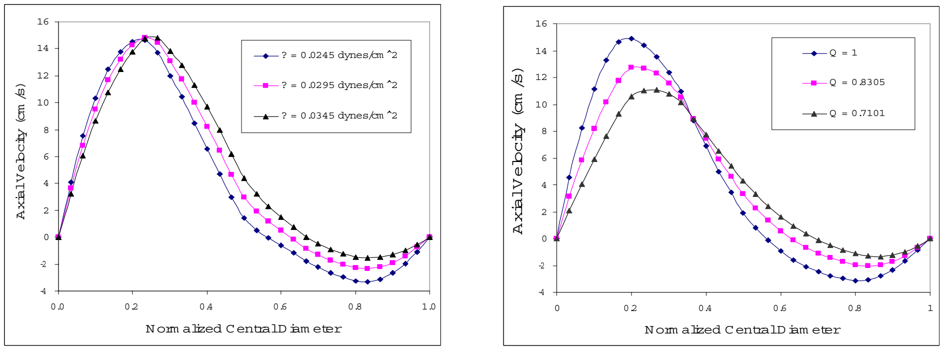


Figure 2: Axial velocity along the central diameter at a cross section in the post stenonsis region, t = 0.45s, Q = 100. a) Effect of viscosity; b) Effect of flow rate.

blood viscosity or the inlet flow rate. Fig. 2 is a plot of the axial velocity along the normalized central diameter at a cross section in the post stenosis region when t = 0.45s. A central diameter is the diameter that connects the top point (outer wall) and the bottom point (inner wall) of the circle. It is in the plane of curvature. x = 0 is at the outer wall, and x = 1 is at the inner wall. Fig. 2a) and 2b) show the effects of the blood viscosity and the inlet flow rate on the flow respectively. The curves show a maximum flow shift towards the outer wall. In addition, they demonstrate a reverse flow near the inner wall. We can see that as the Reynolds number increases, the maximum flow shifts further towards the outer wall and the reverse flow at the inner wall is stronger, regardless if the increase of the Reynolds number is due to the change of the blood viscosity or the inlet flow rate. However, comparing Fig. 2a) and 2b), we can find that as the Reynolds number changes from 348 to 491, the maximum axial flow velocity in Fig. 2b) varies more significantly than that in Fig. 2a), which indicates that the axial velocity at the post stenosis region is more sensitive to the inlet flow rate.

The influence of the Reynolds number on the pressure drop along the stenotic curved artery is shown in Fig. 3. It plots the pressure drop p-$p_e$ along the inner wall in the plane of curvature when t = 0.35s. $p_e$ is a reference pressure selected as the pressure at the outlet. By comparing Fig.3a) and 3b) we can see that at a higher Reynolds number corresponding to a smaller viscosity, the total pressure drop is smaller. Contrarily, at a higher Reynolds number corresponding to a larger inlet flow rate, the total pressure drop is larger.

We then examined the dimensionless pressure drop in order to find a direct correlation to the Reynolds number. Fig. 4 plots the dimensionless pressure drop, (p-$p_e$)/($0.5\rho u_{avg}^2$), along the inner wall in the plane of curvature. Here $u_{avg}$ is a time-averaged flow velocity at the inlet boundary. Fig. 4 shows that the dimensionless

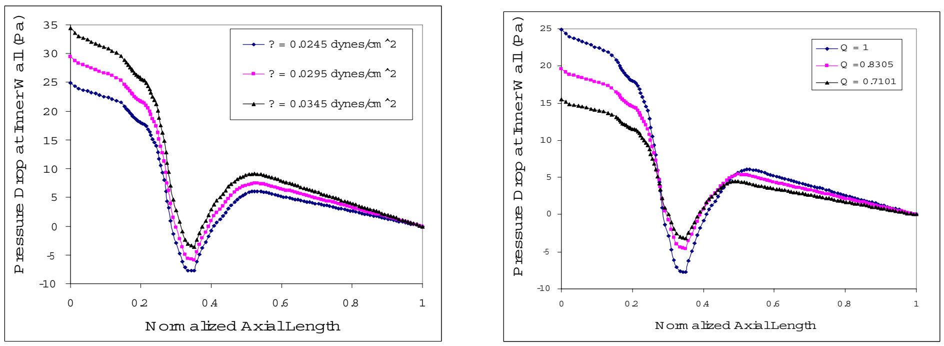


Figure 3: Pressure drop at the inner wall along the artery from inlet to outlet. a) Effect of viscosity; b) Effect of flow rate

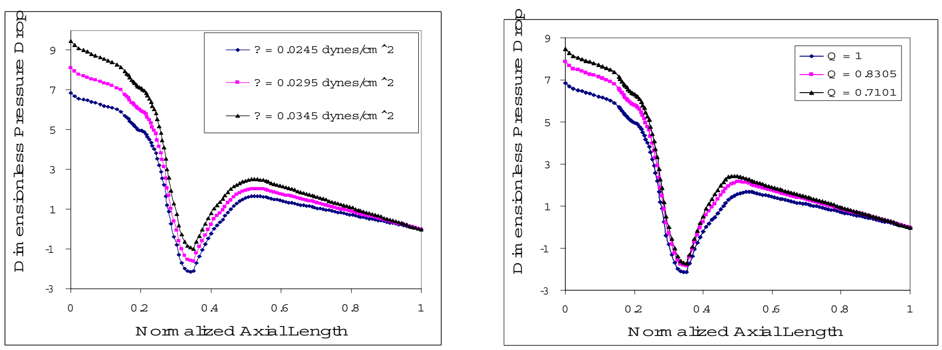


Figure 4: Dimensionless pressure drop, $(p-p_e)/(0.5\rho u^2_{avg})$ a) Effect of viscosity; b) Effect of flow rate

pressure drop decreases as the Reynolds number increases, no matter whether due to the decreasing of the blood viscosity or the increasing of the inlet flow rate. However, we can see that as the Reynolds number changes from 348 to 491, the variation of the dimensionless pressure drop in Fig. 4a) is more significant than that in Fig 4b), which indicates that the dimensionless pressure drop is more sensitive to the blood viscosity. This behavior is very important since the rate of work done by the ventricle in pumping blood is closely related to the pressure drop and thus flow resistance.

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