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Extension of Fictitious Crack Model to Self-Affine Cracks

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Summary

The concepts associated with concrete crack formation and propagation is discussed in the light of fractal and probability theories. These concepts are employed to model the tortuous crack path in a highly heterogeneous medium of concrete. Fictitious Crack Model (FCM)[1] is extended to analyze plain concrete beams with such tortuous cracks. The parametric study has been carried out in order to obtain the influence of various fracture parameters on the beam response.

Introduction

The quantitative description of rough fracture surfaces of concrete has been an important challenge for many years. Looking at the fracture surface of a concrete specimen, one realizes that the self-affine geometry of crack faces results from the stochastic nature of crack growth. This is due to the heterogeneous nature of concrete that makes the crack tortuous leading its way through weak bonds, voids, mortar and getting arrested on encountering a hard aggregate forming crack face bridges. These mechanisms contribute to the tendency of the main crack to follow a tortuous path. Therefore one can easily treat fracture surface of concrete to be an ideal candidate to be considered as a fractal. Further, the softening response itself can be treated as a fractal and the very process of cracking and the microcracking, which could be considered very close to the stick and slip process as a fractal. Therefore modeling a crack as a fractal and characterizing it by a fractal dimension has become the focus of research in recent years.

The fractal dimension

Precise definition of the fractal dimension [2] is almost elusive as the curious properties of fractal geometry itself. From a rigorous mathematical perspective, a set of points is said to exhibit fractal behaviour when the Hausdorff (or “fractal”) dimension is strictly a non-integer, having fractional dimension between those of the intuitive topological dimensions of 0,1,2 etc. With a view to developing a test for the fractal character of naturally occurring systems, a prominent property is the scale–dependent behaviour of many observed physical properties. A classic example is Richardson’s measurements of the length of continental coastlines where the length measured depends on the measuring scale; the finer the scale, the longer the length. An important additional phenomena is which self-similarity does provides a necessary and sufficient condition for the Hausdorff dimension to be uniquely

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defined. Naturally occurring fractals may lack the regularity of geometrically constructed ones; nevertheless, they are most often self-similar in a statistical sense. For eg. Shown in fig. 1., the two curves are characterized by the same value of D

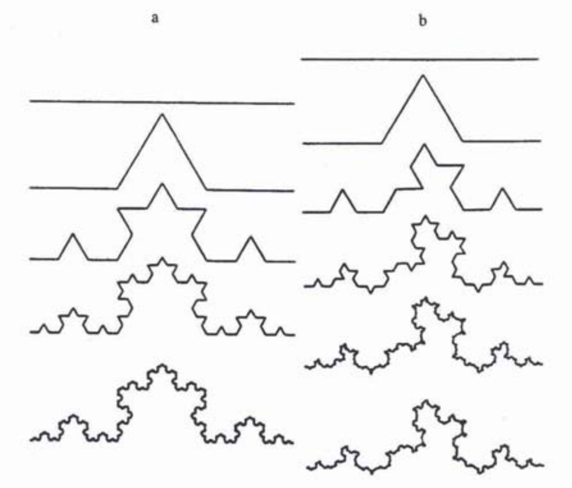

Figure 1: (a) Regular Self-Similar (b) Random Statistically Self-Similar versions of the triadic Koch curve. Note that these curves are characterized by the same fractal dimension D = ln 4/ln 3 = 1.26

Fractal geometry has found particular application in the characterization of fracture surfaces of concrete since 1990 [3,4], where the fractal dimension has been used as qualitative indicators of the roughness of a fracture surface or its profile. Also singular fractal functions were used to simulate the reduction of peak stress under tensile load for concrete [5,6]. The release of elastic energy stored in the structure as the loading process progresses for quasi-brittle materials like concrete was interpreted as fractional release of strains following a pattern of fractal curve. During slow fracture of concrete materials, the process consists of numerous phases of intense microcracking without collapse (stick) and some intermittent phases of coalescence of some microcracks (slip). It was concluded that those qualitative interpretations confirm the great potential of application of fractal geometric concepts to answer the size effect in failure of cementitious material

Actual Fracture Surfaces and Fracture Energy

It is assumed that fracture surface is smooth and straight in case of concrete and the fracture energy depends upon the area of the fracture surface

$G_F = Wf/B(D-a)$ = Work of fracture / Area of the uncracked ligament

This area of the uncracked ligament was assumed to be straight, however in practice it is not so. The actual length of the crack is much more than the assumed straight length

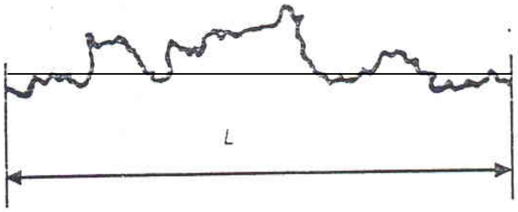

Figure 2: Tortuous crack approximated to a straight crack

The total area of the fracture surface should include the entire fractured surface that has been created during the process of the fracture of concrete. Hence assuming a tortuous curve to be a fractal curve, one can characterize a crack with its fractal dimension from which the actual fracture energy can be computed. In this paper the effect of random nature of the microcracks in the fracture process zone of concrete is investigated using FCM developed by Hillerborg et al (1976). It is the most used model to describe damage localization in materials with disordered microstructure. According to this model the material is characterized by a stress-strain relationship $(\sigma - \varepsilon)$, valid for the undamaged zones, and by a stress-crack opening displacement relationship $(\sigma - \omega)$, describing how the stress decreases from its maximum value σ_t to zero as the crack opens up from zero to the critical displacement W_C. The area below the $\sigma - \omega$ curve represents the energy G_f spent to create the unit crack surface.

Plain concrete beams with tortuous cracks

In normal strength concrete, lesser as tortuosity increases, brittleness decreases. The distribution of coarse-size aggregates in matrix is quite random in nature and leads to random deviations of the crack path and the extent of deviation depends on the maximum aggregate sizes. Tortuosity depends on the coarse aggregates. A number of random crack trajectories are generated assuming the crack deviations as random variable based on the maximum aggregate size. Sets of such simulated trajectories are as shown in fig. 3

Analysis

Plain concrete beams with such tortuous crack are analyzed using FCM to study the influence of important fracture parameters on the beam response. A typical sketch of three point bend beam with a tortuous crack where the stresses are replaced by nodal forces are shown in fig. 4

The analysis reveals the influence of maximum aggregate size upon the pre-peak and post peak behaviour in support of experimental findings. The load-deformation responses for a set of curves are shown in fig 5. As d_a is increased,

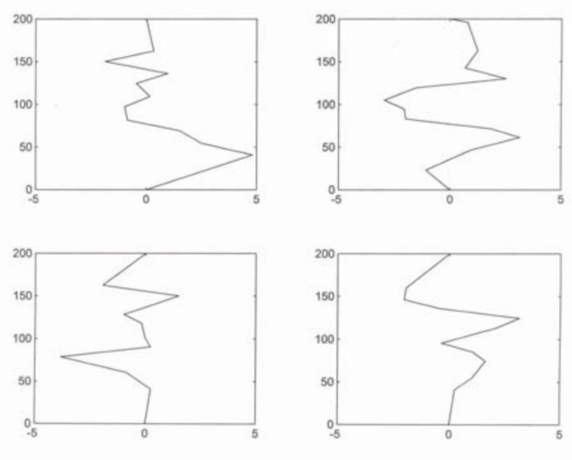

Figure 3: A set of simulated tortuous cracks

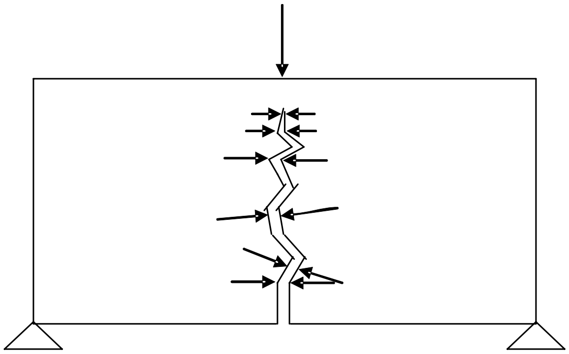

Figure 4: Stresses are replaced by nodal forces in FEM

the pre-peak behavior is more nonlinear and the post-peak behavior less abrupt and its tail more extensive. It is also observed that as a/D increases, the load carrying capacity is considerably reduced and for higher values of a/D the material becomes more brittle as the softening is considerably reduced.

Fracture energy versus tortuousity of the crack

The effects of the maximum aggregate on the fracture energy for various tortuous curves are shown in fig. 6. From fig. 6, it is observed that the fracture energy increases with increase in maximum aggregate size. This is due to the reason that in case of tortuous cracks of different aggregates, the crack pattern is a meandering path as the crack deflects around the aggregates, which tends to raise the energy consumption and affect the apparent toughness. Similar results were obtained by Issa et al [7,8] and they came to the conclusion that fracture energy increases with aggregate size.

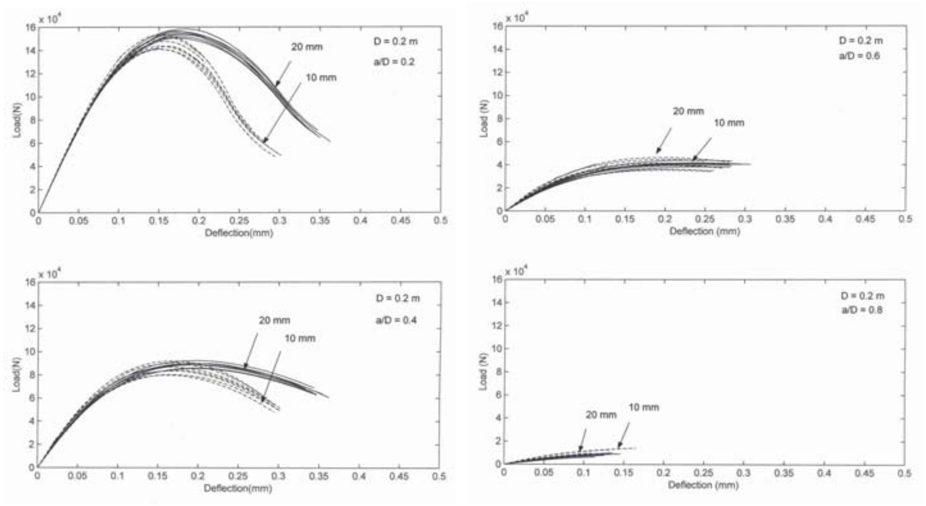

Figure 5: Load deformation response for plain concrete beams of 0.2 m depth and differing only by the maximum size of the aggregate

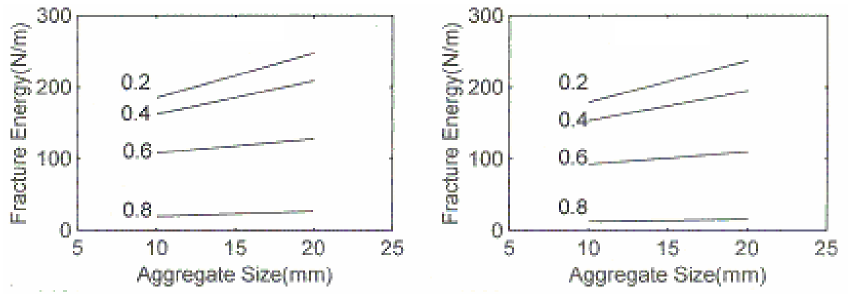

Figure 6: Variation of fracture energy with maximum aggregate size for beams of 0.2 m depth

Conclusions

The well-known FCM has been extended to plain concrete beams with tortuous cracks. The influence of the maximum aggregate size upon the pre and post peak behaviour of the load-deformation responses are in support of experimental findings as found in literature. Further, fracture energy increases with increase in size of the aggregates. The reason is that the presence of larger size of the coarse aggregates forces the crack path to meander around them, increasing the actual fracture area, and increasing the work of fracture W_f and consequently of G_f. Also considering a tortuous crack as a fractal crack, the total fracture area is computed to obtain the actual fracture energy. However, although much data have been generated to quantify concrete fracture surfaces, an understanding of the true relevance and limitations of the fractal concepts on well-characterized fracture modes is yet to be explored.

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