

Calculation of Energy Release Rate in Mode I Delamination of Angle Ply Laminated Composites

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Summary

The compliance equation is used to calculate the energy release rate for angle ply laminated double cantilever composite beam specimen. Instead of the traditional approach of a beam on an elastic foundation, a second order shear thickness deformation beam theory (SSTDBT) has been considered.

keywords: Double Cantilever Beam, Energy Release Rate, Fracture Energy

Introduction

Fiber-reinforced composite materials are widely used in all kinds of engineering structures owing to their high strengths and low densities. Laminated composite structures are made up of layers of orthotropic materials that are bonded together. The layers may be of different materials, or of the same orthotropic material, with the principal material directions of each layer oriented at different angles to the reference axes. By altering material or orientation, or both, of each layer, a structural designer can tailor the strength and other properties of a laminate to the requirements of a given application. Because of the low stiffness in the transverse direction, when out-of-plane loading exists, separation between plies occurs, i.e. delamination. The presence and growth of delamination under static or fatigue loadings will lead to safety problems by reduction of structural stiffness or initiating catastrophic fracture [1–4]. Hence in any engineering design of laminated composite structures, the delamination mode of failure should be taken into account [5,6]. Usually the delamination modes in composite laminates contain Mode I (opening), Mode II (in plane shearing) and Mode III (out of plane tearing) fractures. Therefore, it is important to characterize these fracture modes to prevent delamination damage.

The overwhelming literature on delamination is mainly related to unidirectional composites. A feature that complicates the analysis of delamination growth in angle ply laminates is that delamination exists between angle-ply (e.g. $\pm\theta$). In an optimum laminate design, the objective will be to minimize the crack driving force and/or the crack-induced interfacial principal tensile stress in the angle-ply laminate under transverse shear loading. This problem is cast as a single- or multi-criterion optimization problem. The design variables are the ply angle θ and the relative ply thicknesses of the sublaminates. It is recognized that a delamination between angle-ply has the feature of a crack between dissimilar anisotropic materials which substantially complicates the fracture mechanics analysis.

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The double cantilever beam (DCB) test [7] is the most widely used method for determining the Mode I toughness of composite materials. The test specimen, the applied load, P , to the two arms and the corresponding load line displacement, δ , are schematically illustrated in Fig. 1. A model based on an Euler-Bernoulli beam on an elastic Winkler foundation for analysing the DCB specimen was originally developed by Kanninen [8,9] for isotropic materials and by Williams [10] for transversely isotropic materials. This model is extended to angle-ply laminates by Ozdil and Carlsson [11]. The beam displacements derived from this model are used to calculate the compliance and strain energy release rate of the DCB specimen. Recently, Hamed *et al.* [12], introduced an improved analytical model for delamination in composite beams under general edge loading which also takes into account the shear-thickness deformation.

In the present study, Hamed model together with the compliance equation has been used to develop a method of determining the Mode I interlaminar fracture toughness in thick angle ply laminated composites.

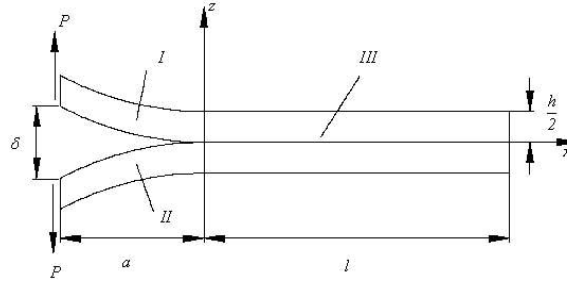


Figure 1: Angle ply DCB specimen divided into three regions

Theoretical Analysis

Consider the DCB specimen in Fig. 1 made of an angle ply laminate and divided into three regions *I*, *II* and *III*. Assuming a second order shear thickness deformation beam theory (SSTDBT), the displacement field is

$$\begin{aligned} u_1(x, y, z) &= u(x) + z\psi_x(x) + z^2\eta_x(x), \\ u_2(x, y, z) &= 0, \\ u_3(x, y, z) &= w(x) + z\psi_z(x), \end{aligned} \quad (1)$$

in all three regions, where u_1 , u_2 and u_3 are the displacement components of a point in the x -, y - and z -directions, respectively. Note that the superscripts *I*, *II* and *III* will be given to all of the displacement components to distinct them for the regions *I*, *II* and *III*. Substituting Eq. 1 into the strain-displacement relations of elasticity we obtain

$$\varepsilon_x = \varepsilon_1^0 + z\kappa_0^1 + z^2\kappa_1^1, \quad \varepsilon_y = 0, \quad \varepsilon_z = \varepsilon_3^0, \quad \gamma_{yz} = 0, \quad \gamma_{xz} = \varepsilon_5^0 + z\kappa_5^0, \quad \gamma_{xy} = 0 \quad (2)$$

where

$$\varepsilon_1^0 = u', \quad \kappa_0^1 = \psi'_x, \quad \kappa_1^1 = \eta'_x, \quad \varepsilon_3^0 = \psi_z, \quad \varepsilon_5^0 = \psi_x + w', \quad \kappa_5^0 = 2\eta_x + \psi'_z \quad (3)$$

and a prime denotes differentiation with respect to x . Since we are developing a beam theory, a state of plane stress is presented and therefore

$$\sigma_y = \sigma_{yz} = 0. \quad (4)$$

Next by imposing Eq. 4 on three dimensional Hooke's law of elasticity, the plane stress constitutive law for the k th layer of each region is

$$\begin{Bmatrix} \sigma_x \\ \sigma_z \\ \sigma_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} \bar{\bar{C}}_{11} & \bar{\bar{C}}_{13} & \bar{\bar{C}}_{16} \\ \bar{\bar{C}}_{13} & \bar{\bar{C}}_{33} & \bar{\bar{C}}_{36} \\ \bar{\bar{C}}_{16} & \bar{\bar{C}}_{36} & \bar{\bar{C}}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xy} \end{Bmatrix}^{(k)}, \quad \sigma_{xz}^{(k)} = \bar{\bar{C}}_{55}^{(k)} \gamma_{xz}^{(k)}, \quad (5)$$

where

$$\bar{\bar{C}}_{ij} = \bar{C}_{ij} - \frac{\bar{C}_{i2}\bar{C}_{j2}}{\bar{C}_{22}} \quad i, j = 1, 3, 6 \quad \text{and} \quad \bar{\bar{C}}_{55} = \bar{C}_{55} - \frac{\bar{C}_{45}^2}{\bar{C}_{44}}. \quad (6)$$

In Eq. 6, $\bar{\bar{C}}_{ij}$ denote off-axis stiffness coefficients of the k th layer [13]. Using the principle of minimum total potential energy, and considering the displacement field in Eq. 1, the equilibrium equations together with the boundary conditions (B.C.'s) at boundaries and continuity conditions (C.C.'s) and the equilibrium conditions (E.C.'s) at the intersection of three regions will be obtained. Further by substituting the laminate constitutive relations into the equilibrium equations, the governing equilibrium equations in terms of displacement components for the three regions will be found as

$$\begin{aligned} -A_{11}u'' - A_{13}\psi'_z - B_{11}\psi''_x - D_{11}\eta''_x &= 0, \\ -B_{11}u'' - B_{13}\psi'_z - D_{11}\psi''_x - E_{11}\eta''_x + A_{55}(\psi_x + w') + B_{55}(2\eta_x + \psi'_z) &= 0, \\ -D_{11}u'' - D_{13}\psi'_z - E_{11}\psi''_x - F_{11}\eta''_x + 2B_{55}(\psi_x + w') + 2D_{55}(2\eta_x + \psi'_z) &= 0, \\ -A_{55}(\psi'_x + w'') - B_{55}(2\eta'_x + \psi''_z) &= 0, \\ A_{13}u' + A_{33}\psi_z + B_{13}\psi'_x + D_{13}\eta'_x - B_{55}(\psi'_x + w'') - D_{55}(2\eta'_x + \psi''_z) &= 0, \end{aligned} \quad (7)$$

where $A_{ij}, B_{ij}, D_{ij}, E_{ij}$ and F_{ij} are laminate stiffness coefficients [12]. Giving the superscripts *I, II* and *III* to all of the displacement components and also the stiffness coefficients in Eq. 7, a system of ordinary differential equations (ODE) of order 30 will be resulted. This system of ODEs together with the B.C.'s, C.C.'s and E.C.'s can be solved and all displacement components for the three regions can be obtained.

Hamed [12] calculated the energy release rate using the J-integral. Here the classical Irwin-Kies expressions [14] will be used to obtain the strain energy release rate (SERR) from the beam theory solution, i.e.,

$$G = \frac{P^2}{2b} \frac{\delta C}{\delta a}. \quad (8)$$

where in Eq. 8, P is the load, b is the specimen width, a is the crack length and C the compliance is defined as

$$C = \frac{\delta}{P}, \quad (9)$$

To obtain the G , we must first express C in terms of the crack length a . From Fig. 1, it is obvious that

$$\delta = u_3^I \Big|_{x=-a, z=0} - u_3^{II} \Big|_{x=-a, z=0} \quad (10)$$

Because of complicated computations, however, the problem cannot be solved explicitly in terms of a . Therefore the DCB problem has been solved for a range of crack length and then a least-square regression was fitted to obtain C as a function of crack length, a

$$C(a) = C_3 a^3 + C_2 a^2 + C_1 a + C_0 \quad (11)$$

The above equation can be differentiated with respect to crack length a and G is calculated from Eq. 8.

Numerical Examples and Discussions

Glass/polyester DCB specimens consisting of anti-symmetric angle ply laminates of the form $[\pm 30^\circ]_5$ and $[\pm 45^\circ]_5$ was chosen as those in [11] where $h=7.3 \text{ mm}$, $b=20 \text{ mm}$, $a=35 \text{ mm}$ and $l=100 \text{ mm}$. Also $E_1 = 34.7 \text{ GPa}$, $E_2 = 8.5 \text{ GPa}$, $\nu_{12} = \nu_{13} = 0.27$, $\nu_{23} = 0.5$, $G_{12} = G_{13} = 4.34 \text{ GPa}$, $G_{23} = 2.83 \text{ GPa}$. Further for $\theta = 45^\circ$, $E_3 = 9.85 \text{ GPa}$ and for $\theta = 30^\circ$, $E_3 = 9.37 \text{ GPa}$. Considering these properties, the corresponding C_i are presented in Table 1.

Table 1: Polynomial coefficients of compliance in Eq. 11.

| Lay up | C_0 | C_1 | C_2 | C_3 |
|--------------------|--------------------|-----------------|----------------|----------------|
| $[\pm 30^\circ]_5$ | $0.1525402534e-7$ | 0.0000108485 | 0.0027207675 | 0.3901954340 |
| $[\pm 45^\circ]_5$ | $-0.1547188457e-7$ | -0.0001224524 | 0.0036116202 | 0.6069910987 |

Fig. 2a compares SERR obtained from the compliance equation with those obtained from J-Integral [12] for two anti-symmetric angle ply laminates of $\theta = 30^\circ$ and $\theta = 45^\circ$, and for various values of crack length at constant applied load of $P=40 \text{ N}$. The results are in good agreement with each other. In similar crack lengths,

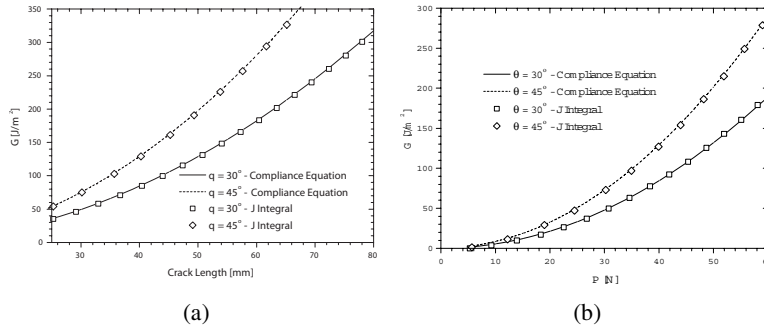


Figure 2: Comparison of pure mode I strain energy release rate, G , obtained from J-Integral and compliance equation for $[\pm\theta]_5$ specimens versus (a) Crack length at constant applied load of $P=40$ N, (b) Applied load P at constant crack length of $a=40$ mm.

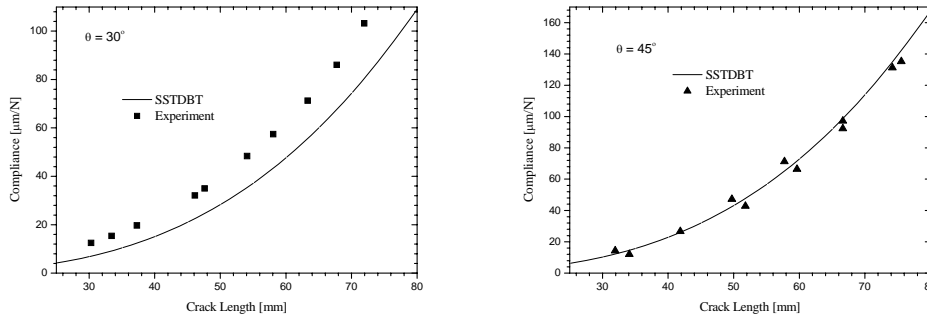


Figure 3: Comparison of Calculated and measured Compliance

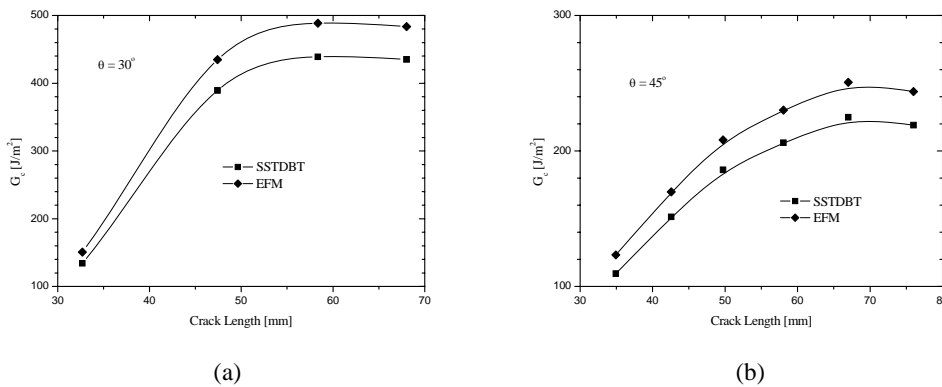


Figure 4: Comparison of pure mode I fracture energy, G_c , obtained from elastic foundation model, EFM [11] and SSTD BT using compliance equation for $[\pm\theta]_5$ specimens (a) $\theta = 30^\circ$ (b) $\theta = 45^\circ$.

SERR for $\theta = 45^\circ$ are greater than when $\theta = 30^\circ$. In Fig. 2b, SERR are plotted versus applied load for two angle ply lay-ups at constant crack length of $a=40$ mm.

Similar to Fig. 2a, the results obtained from compliance equation are in accordance with the results from J-Integral.

In Figs. 3 the calculated compliances from SSTDBT theory are compared with the experimental results obtained by Ozdil and Carlsson [11]. It is observed that when $\theta = 30^\circ$, SSTDBT underestimates the compliance value but for $\theta = 45^\circ$ the results are very close. In Figs. 4 pure mode I fracture energy, G_c , obtained from elastic foundation model, EFM [11] and SSTDBT using compliance equation for $[\pm\theta]_5$ specimens at two angle-ply of $\theta = 30^\circ$ and $\theta = 45^\circ$ are compared.

It is shown and verified that the second order shear thickness deformation beam theory (SSTDBT) together with compliance equation accurately estimates the mode I fracture toughness and strain energy release rate for any angle-ply laminates. The method is robust and general and other mode of fracture can be modelled using this method.

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