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The Theory of Critical Distances and the estimation of notch fatigue limits: L, a_0 and open notches

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Summary

This paper investigates some practical aspects related to the use of the Theory of Critical Distances (TCD) when employed to estimate notch fatigue limits. The accuracy of different formalisations of the theory was checked by using experimental data taken from the literature. This exercise allowed us to confirm that the simplest formalisation of the TCD, in which both critical distance and critical stress are material constants [1], is also the most accurate one, giving predictions falling within an error interval of about ±20%. The TCD is also accurate when applied to notches having large opening angles.

Different Formalisations of the TCD

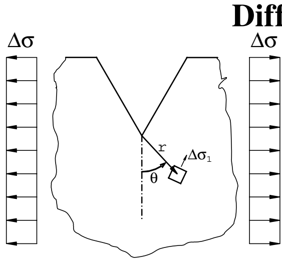

Figure 1: Notched specimen under a remote uniaxial fatigue loading.

The TCD postulates that notched components are in the fatigue limit condition when the effective stress, $\Delta\sigma_{eff}$, calculated using the linear-elastic stress field in the fatigue process zone, is equal to the material's plain fatigue limit, $\Delta\sigma_0$. In more detail, and according to the most modern formalisations of the TCD, $\Delta\sigma_{eff}$ can be calculated variously, as follows (Fig. 1):

$$\Delta\sigma_{eff} = \Delta\sigma_1 \left(\theta = 0, r = D_{PM}\right) = \Delta\sigma_0 \tag{1}$$

$$\Delta\sigma_{eff} = \frac{1}{D_{LM}} \int_0^{D_{LM}} \Delta\sigma_1 \left(\theta = 0, r\right) dr = \Delta\sigma_0 \tag{2}$$

$$\Delta\sigma_{eff} = \frac{4}{\pi D_{AM}} \int_{-\pi/2}^{\pi/2} \int_0^{D_{AM}} \Delta\sigma_1 \left(\theta, r\right) \cdot dr \cdot d\theta \cong \Delta\sigma_0 \tag{3}$$

Eq. (1) formalises the Point Method (PM) [1, 2], Eq. (2) the Line Method (LM) [1, 2, 3]; and Eq. (3) the Area Method (AM) [1]. In these equations D_{PM}, D_{LM} and D_{AM} are the critical lengths to be used to apply the PM, the LM and the AM, respectively.

According to suggestions by Tanaka [2], Lazzarin and co-workers [3] and Taylor [1], these critical lengths take the following values: D_{PM}=L/2, D_{LM}=2L and D_{AM}=L, where L is the material characteristic length, which is defined as:

$$L = \frac{1}{\pi} \left(\frac{\Delta K_{th}}{\Delta\sigma_0} \right)^2 \tag{4}$$

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Here ΔK_{th} is the threshold value of the stress intensity factor range and $\Delta\sigma_0$ is the plain fatigue limit, both determined at the appropriate load ratio, R.

Previously, the accuracy of the above formalisations of the TCD was systematically checked, considering both standard notches [4] and real components [5]: these investigations allowed us to prove that the TCD is successful in estimating notch fatigue limits, giving predictions falling within an error interval of about ±20%. This raises the simple question: "Why does the TCD work?". Even though it is very difficult to answer this question coming to a definitive conclusion, we have noted that the TCD may work because it is capable of predicting the propagation (or non-propagation) of cracks initiating at the tip of the stress raiser and having length equal to 2L [6]. In other words, according to this idea, non-propagating cracks (NPCs) should have a length equal to 2L when initiated at the apex of crack-like notches. Though this justification is appealing, it does not explain the reason why the TCD is successful also in predicting fatigue limits of blunt notches [4, 6].

In any case, assuming that the TCD's accuracy is due to its capability of predicting NPC length, one might argue that, according to Yates and Brown's idea [7], the critical lengths in Eqs. (1), (2) and (3) are not related to L but to the ElHaddad parameter a_0, which is defined as follows [8]:

$$a_0 = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{F \cdot \Delta\sigma_0}\right)^2 \quad (5)$$

Here F is the geometrical correction factor for the LEFM stress intensity factor, which depends on notch geometry and other factors, thus a_o is not a material constant. An alternative formulation of the TCD would use the following three critical distances $D_{PM}=a_0/2$, $D_{LM}=2a_0$ and $D_{AM}=a_0$. Now the question is: "Are the predictions made using a_0 more accurate than the ones obtained using L?". In order to answer this question, in the next section the accuracy of these two different ways of using the TCD will be checked and compared by using experimental data taken from the literature.

In 1997, Lazzarin, Tovo and Meneghetti [3] argued that, in order to correctly apply the PM, the range of the maximum principal stress at the point having coordinates (θ=0, r=L) must be corrected by using an adimensional function depending on both L and the notch root radius, r_n. Their LTM-PM was defined:

$$\Delta\sigma_{eff} = \Delta\sigma_1\left(\theta = 0, r = L\right)\frac{1+\sqrt{2}\frac{L}{r_n}}{1+\frac{L}{r_n}} = \Delta\sigma_0 \quad (6)$$

Thus, for these workers, the critical stress range is not a material constant. In the next section, the selected data will be used also to discover what formalisation of the PM is the most accurate one. Furthermore, the accuracy of the TCD will be checked in the presence of notches characterised by large values of the notch opening angle; this is an important check because most workers studying notches have confined themselves to opening angles of less than 90^o.

Material	**Ref.**	**R**	$\Delta\sigma_0$ [MPa]	$\Delta\mathbf{K}_{th}$ [MPa m$^{1/2}$]	$\mathbf{F}_{min}$	$\mathbf{F}_{max}$	**Spec. type**	**Load type**
Mild Steel	[9]	-1	420	12.8	1.12	1.12	DENP	AX
					1.308	1.308	CNB	AX
C45	[10]	-1	582	8.1	1.09	1.239	CNB	RB
C36	[10]	-1	446	7.1	1.09	1.193	CNB	RB
6060-T6	[11]	0.1	110	6.1	1.12	1.13	DENP	AX
AISI 304	[12]	-1	720	12.0	1.308	1.308	CNB	AX
EN-GJS-800-8	[13]	0.1	440	8.1	1.13	1.126	DENP	AX
Grey Iron	[14]	-1	155	15.9	1.28	1.28	CNB	AX
		0.1	99	11.2				
		0.5	68	8.0				
		0.7	48	5.2				
AA356-T6	[15]	-1	231	4.4	1.141	2.78	CNB	RB
Ni-Cr Steel	[12]	-1	1000	12.8	1.001	1.045	CNP	AX
Steel 15313	[16]	-1	440	12	1.13	1.4	CNB	AX

CNB= circumferential notch cylindrical bar, **CNP**= center notch in plate,
DENP= double edge notch in plate, **RB**= rotating bending, **AX**= push-pull

Table 1: Summary of the experimental data.

Systematic Comparison Using Experimental Data

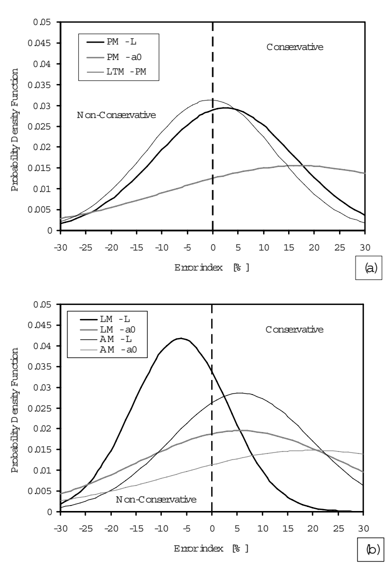

Figure 2: Accuracy of the different formalisations of the TCD.

In order to answer the questions arising in the previous section, experimental data were selected from the technical literature. Table 1 summarises the sources of data used. Figure 2a reports the Probability Density Function vs. Error diagram obtained by applying the three different formulations of the PM considered in the present study, where the error was calculated as follows: $Error[\%] = \left(\frac{\Delta\sigma_{eff}-\Delta\sigma_0}{\Delta\sigma_0}\right)\cdot 100$ This clearly shows that poor accuracy was obtained when using a_o; the other two methods (using L with either a constant or varying critical stress range) gave good accuracy.

Fig.2b compares the LM and AM: again the use of the material constant L is seen to be preferable to the use of a_o in both cases. This confirmation that the critical distance is a material constant, unaffected by notch geometry, is a very useful result as it allows the TCD to be applied with confidence to real stress concentration features on components, which often have very complex geometry.

Next, the TCD was applied to predict fatigue limits generated by testing samples having notch opening angles larger than 90° (Tab. 2). As an example, the $\Delta\sigma_{eff}$ vs. $\Delta\sigma_0$ diagram obtained by applying the L based PM is reported in Figure 3: again the different formalisations of the L-based TCD, that is, the PM, the LM and the AM, were seen to be successful, giving predictions falling within an error interval of ±20%, this accuracy being independent of the notch opening angle value.

Material	**Ref.**	**R**	$\Delta\sigma_0$ [MPa]	ΔK_{th} [MPa m$^{1/2}$]	**Opening Angle**	**Spec. type**	**Load type**
FeP04	[17]	0.1	247	10.0	45°, 135°, 160°	DENP	AX
HT 60 (1)	[18]	0	580	13.0	90°, 135°, 165°	DENP	AX
					135°	BT	AX
					135°	CT	AX
SS41	[19]	0.05	231	6.4	90°, 120°	DENP	AX
					120°	SENP	AX
					135°	CT	AX
HT 60 (2)	[20]	0.05	425	6.6	90°, 120°	DENP	AX
					120°	SENP	AX
					135°, 150°	CT	AX
BT= butt type, **CT**= cruciform type, **DENP**= double edge notch in plate **AX**= push-pull							

Table 2: Summary of the experimental data generated testing open notches.

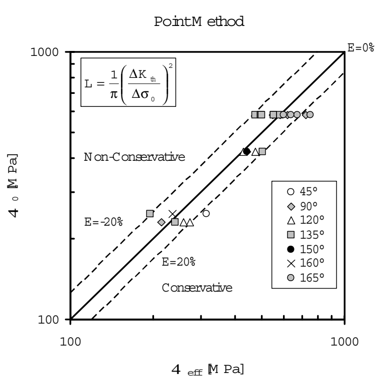

Figure 3: PM and open notches.

Conclusions

The use of the TCD with material-constant values of critical stress ($\Delta\sigma_o$) and critical distance (L) is advocated, as it demonstrates high predictive accuracy with wide applicability. The use of a geometry-dependant critical distance (a_o) gives lower accuracy; the use of a geometry-dependant critical stress (the LTM method) gives similar accuracy but is more difficult to apply. Notches with a wide range of opening angles can be satisfactorily analysed using the TCD.

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