

## **Probabilistic Modeling of Material Variability in Fatigue Crack Growth**

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### **Summary**

This paper presents a probabilistic crack growth model developed for the Holistic Structural Integrity Process (HOLSIP) framework. Statistical data, obtained from testing and fractographic analyses of 2024-T3 test coupons, were used to derive the fatigue crack growth material variability. Results showed the relative impact of material variability in the short and long crack regimes. Monte Carlo simulations showed good agreement between analytical life distributions and test results.

### **Introduction**

The National Research Council Canada (NRC) and other organizations are developing the Holistic Structural Integrity Process (HOLSIP) to augment and enhance traditional safe-life and damage tolerance paradigms in both design and sustainment stages [1]. Two of the fundamental elements in HOLISP are 1) the need for physics-based lifing models that can address both intrinsic and extrinsic factors, such as basic material microstructure, surface integrity, and environment related age degradation, and 2) the need for probabilistic techniques to account for fatigue associated uncertainties. As such, HOLISP models differ from models based on the historical Equivalent Initial Flaw Size (EIFS), which is not a physically measured parameter.

In HOLISP, the life of a component is divided into four distinct phases: nucleation, short crack, long crack, and final instability. This paper presents probabilistic short and long crack growth models developed for the HOLISP framework, and their application to life distribution analysis of 2024-T3 fatigue coupons by Monte Carlo simulation. The models used Microsoft Excel Visual Basic for Applications (VBA) and the AFGROW Component Object Model (COM) server for crack growth analysis.

### **Long Crack Material Behavior Scatter**

In 1979, Virkler [2] performed a series of fatigue tests on 68 identical 2024-T3 coupons with an initial center through (long) crack of 9 mm, subjected to the same loads. The crack growth curves from these tests were used to derive the fatigue crack growth material variability in the present probabilistic crack growth model. The crack growth rates of each coupon were calculated using the secant method described in ASTM E647, and the stress intensity factors were obtained

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from AFGROW, which uses the Glinka's Weight Function solutions [3]. The test data were best-fitted using a piecewise Walker's crack growth curve, where each segment defines a double-loglinear relationship between  $da/dN$  and  $\Delta K$ ,

$$da/dN = 10^C [\Delta K (1 - R)^{(m-1)}]^n \quad (1)$$

where  $da/dN$  is the crack growth rate,  $\Delta K$  is the stress intensity factor range,  $R$  is the stress ratio, and  $C$ ,  $m$ , and  $n$  are the intercept,  $R$  shift, and slope parameters, respectively. Since Virkler used a single stress ratio of  $R = 0.2$ , a plausible arbitrary value of  $m = 0.5$  was assigned to the  $R$  shift parameter. Correlated lognormal distributions of  $C$  and  $n$  were calculated for best-fit 1-segment and 3-segment Walker curves to describe the crack growth rate variability. A comparison of the lives, up to  $a = 49.8$  mm, calculated by a Monte Carlo analysis of 1000 cases with the original Virkler's test result is shown in Figure 1.

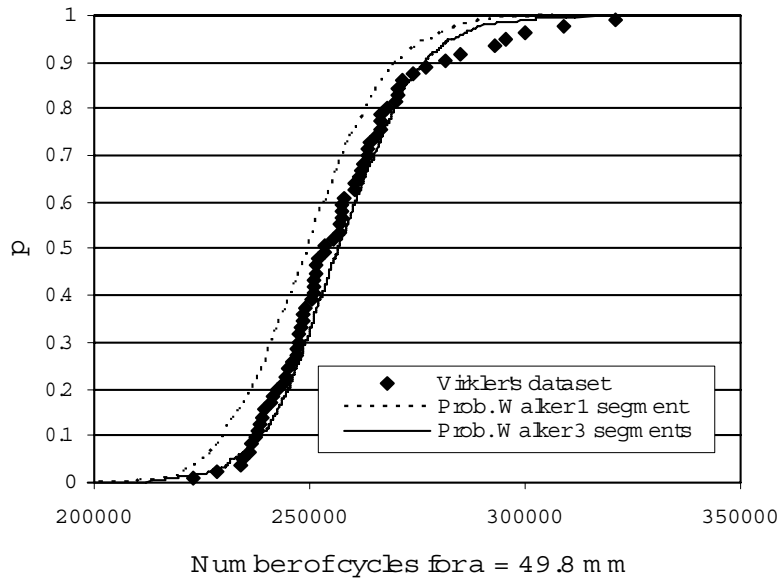


Figure 1: Probabilistic modeling of Virkler's tests.

Both probabilistic models result in life scatters representative of the baseline data. However, the estimated mean lives are too short when a single double-loglinear segment is used to model the material behavior over the entire crack growth range (from 9 to 49.8 mm).

### Short Crack Material Behavior Scatter

In previous projects, considerable fatigue tests and fractographic analyses were carried out at NRC to gather statistical data on the crack-nucleating constituent particles in coupons of bare 2024-T351 sheets [4]. It has been shown that most of these

particles were damaged or debonded during the rolling manufacturing process. As such, a basic assumption was made that the crack nucleation life is very short or negligible for bare 2024-T351 coupons.

The AGARD material model [5], modified over the short crack regime, was used as the average material behavior to correlate the observed crack-nucleating particles with the fatigue lives of bare 2024-T351 fatigue coupons. The particles, referred to as Initial Discontinuity States (IDS) in the HOLSIP terminology, were modeled as initial central semi-elliptical surface cracks. The random variables included the crack-nucleating particle width,  $W$ , and height,  $H$ , as well as the stress intensity factor limit,  $\Delta K_{IDS}$ . The statistical distributions for the particle width and height were derived from scanning electron microscope measurements and the  $\Delta K_{IDS}$  distribution was obtained from reverse-calculation optimization [6]. To ease the integration with long crack material behavior scatter, the modified AGARD was simplified by retaining the six most significant points that define the general curve shape. The original and simplified AGARD models, as well as Virkler’s data points, are shown in Figure 2.

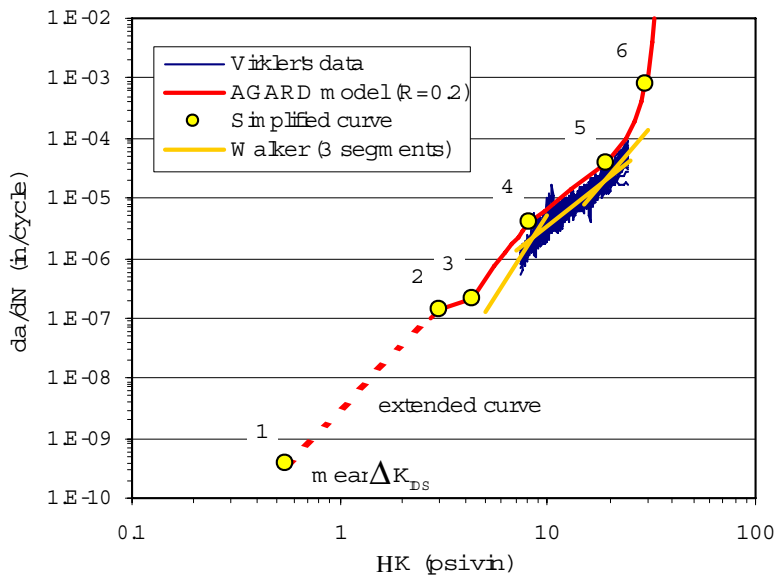


Figure 2: Simplified average material model curve.

### Combined Short-Long Crack Probabilistic Model

Although the two sources of data do not imply the same mean crack growth rates, it was assumed that the Virkler’s scatter could be superimposed onto the simplified AGARD curve to build the short-long crack probabilistic material model. Furthermore, the material scatter was assumed constant for various  $R$  values. Be-

cause Virkler's measurements cover only a portion of the long crack growth regime, the scatter was extended over the entire curve. To do so, Virkler's data was converted to 3-segment Walker curves by best-fitting the portions of the curves delimited by points 4 and 5, which are the only points located within the Virkler's range. The mean curves for the three segments are shown in Figure 2. The correlated log-normal  $da/dN$  distributions at the two points, as well as at the limits of Virkler's range, were derived. To extend the scatter obtained in the Virkler's tests to a wider range, it was assumed that the relative scatter at the ends of the Virkler's set can be used to build the distributions at the AGARD points for lower and higher  $\Delta K$  values.

The tabular lookup format in AFGROW was used to efficiently operate the probabilistic model over the entire crack size range. The approach, shown in Figure 3, consists of generating random  $da/dN$  at each of the selected point using the correlated distributions, except for the first point, for which it is the  $\Delta K_{IDS}$  that is randomly generated. As observed in tests [5], the crack growth rate scatter in the short crack regime is much smaller than that in the long crack regime.

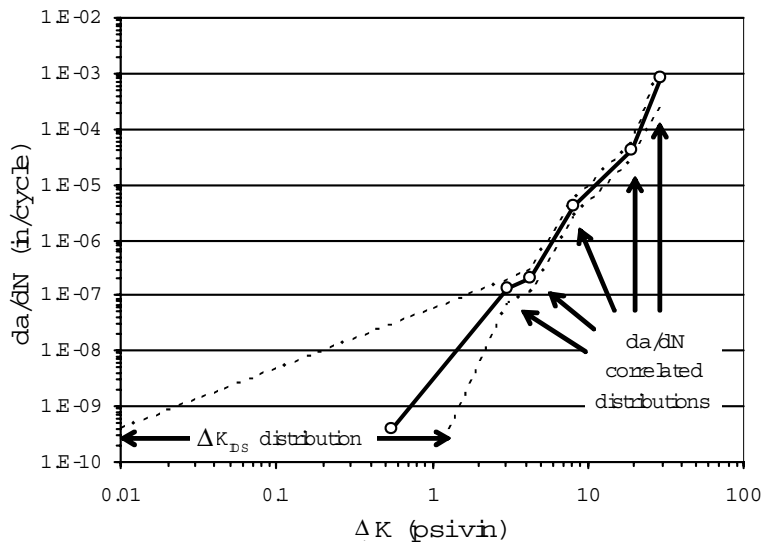


Figure 3: Sort-long crack probabilistic material model.

### Monte Carlo Simulations of 2024-T351 Fatigue Test Coupons

Monte Carlo simulations were carried out using the probabilistic crack growth model to calculate the life distribution of 38 coupons tested in previous NRC projects. The thicknesses of these specimens were 0.063, 0.16 and 0.5 inches, and they were loaded using load ratios of 0.05 and 0.1 and maximum stresses of 40, 44 and 48 ksi. Three levels of increasing complexity were used for the model:

the short crack model with  $W$  and  $H$  variability only, the short crack model including the variability in  $W$ ,  $H$ , and  $\Delta K_{IDS}$ , and the combined short-long model. For each model, six Monte Carlo analyses of 1000 cases each, differentiated to account for the various thicknesses, stress levels and stress ratios of the coupons, were performed. The combined results of these analyses are presented in Figure 4.

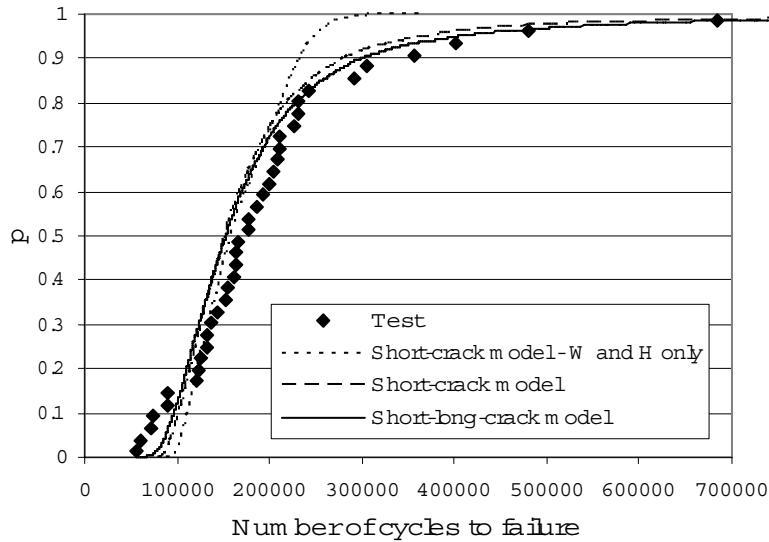


Figure 4: Simulated and tested life distributions of 2024-T351 test coupons.

These results show a good agreement between the analytical and experimental lives of the coupons. A more detailed analysis of the results showed that the various simulations subsets adequately generated life distributions for the coupons with common thickness or applied stress. It is seen that the short-crack growth material variability, seen in the present model as the  $\Delta K_{IDS}$  distribution, must be considered in the fatigue life distribution estimation of the 2024-T3 coupons with no prior damage other than an IDS. The addition of long-crack material variability, which is relatively smaller, has a limited effect on the analyzed life distributions.

### Conclusion

A probabilistic model for crack growth in 2024-T3 aluminum alloys was developed for the HOLSIP framework. Statistical data obtained from fatigue tests and fractographic analyses were used to address the crack growth material variability in the short and long crack growth regimes. Results showed that the material behavior scatter in the short crack regime is needed to model coupons with cracks nucleated from particles. Monte Carlo simulations showed that the model is able to estimate the fatigue life scatter and distributions for test coupons with different thickness and applied stress characteristics. As such, it is now being extended to

include external random factors, such as loading and corrosion, for the HOLSIP based fatigue risk assessments [7].

### References

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