

## Recent Discovery for the Governing Condition of Dynamic Crack Bifurcation Phenomena

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### Summary

This paper summarizes recent discovery for the Governing Condition of Dynamic Crack Bifurcation Phenomena, which were found by the author's laboratory.

### Introduction

Branched cracks are often observed in brittle materials and structures. Many researchers [1-8] However, the mechanism or the governing condition of dynamic crack branching has not been fully clarified. Thus, for humankind, the dynamic crack bifurcation problems has remained important unsolved problems for long time, until recent our study [1].

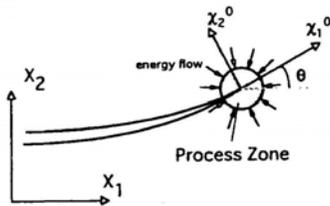


Figure 1: energy flow to a process zone  
Energy flow to a process zone

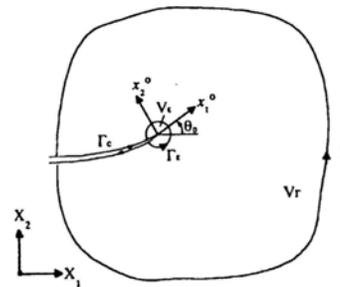


Figure 2: the dynamic J integral paths

### Energy Concepts for Dynamic Crack Branching[1]

#### Dynamic J Integral and Energy Flow Rate into a Propagating Crack Tip

For dynamic fracture mechanics, Nishioka and Atluri [2] have derived the dynamic J integral ( $J'$ ) which has the following salient features: (i) it physically represents the dynamic energy release rate; (ii) if a process zone is considered (see Fig.1), it has the meaning of the energy flow rate to the process zone around the propagating crack tip. (iii) it has the property of the path-independent integral, which gives a unique value for an arbitrary integral path surrounding the crack tip; (iv) It can be related to the stress intensity factors by arbitrarily shrinking the integral path to the crack tip.

$$J' = J'_1 \cos \theta_0 + J'_2 \sin \theta_0 \quad (1)$$

$$J'_k = \lim_{\epsilon \rightarrow 0} \int_{\Gamma_\epsilon} [(W + K)n_k - t_i u_{i,k}] dS \quad (2.a)$$

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For a crack propagating at a  $\theta_0$  measured from the global  $X_1$  axis (see Fig.1), the dynamic J integral ( $J'$ ) was derived by Nishioka and Atluri [2,3] in a path-independent form:

$$J'_k = \lim_{\varepsilon \rightarrow 0} \left\{ \int_{\Gamma + \Gamma_c} [(W + K) n_k - t_i u_{i,k}] dS + \int_{V_\Gamma - V_\varepsilon} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV \right\} \quad (2.b)$$

,where  $u_i$ ,  $t_i$ ,  $n_k$  and  $\rho$  denote the displacement, stress, outward direction cosine, and mass density, respectively.  $W$  and  $K$  are the strain and kinetic energy densities, respectively, and  $(\ )_{,k} = \partial(\ ) / \partial X_k$ . Integral paths are defined in Fig.2. Physically, the near-tip region  $V_\varepsilon$  can be considered as the process zone in which micro-processes associated with fracture occur.

The crack-axis components of the dynamic J integral can be evaluated by the coordinate transformation. Thus the dynamic energy release rate  $G$  can be expressed by

$$G = J' = J'_1{}^0 \quad (3)$$

where  $J'$  is the magnitude of the vector of the dynamic J integral, and  $J'_1{}^0$  is the tangential component of the dynamic J integral along the crack axis. The dynamic J integral can be related to the instantaneous stress intensity factors  $K_I$ ,  $K_{II}$  and  $K_{III}$  for the elastodynamically propagating crack with velocity  $C$ , as in [2],

$$J'_1{}^0 = \frac{1}{2\mu} \left\{ A_I(C) K_I^2 + A_{II}(C) K_{II}^2 + A_{III}(C) K_{III}^2 \right\} \quad (4)$$

$$J'_2{}^0 = -\frac{A_{IV}(C)}{\mu} K_I K_{II} \quad (5)$$

Where  $A_M$  ( $M=I \sim IV$ ) are functions of crack velocity  $C$ .

#### Various Criteria for Dynamic Crack Branching [1]

In general, the governing equation for the onset of crack branching may be expressed in the following form:

$$A_b \geq R_b \quad (6)$$

Where  $A_b$  is an applied driving force at the crack tip or a measure of the intensity, and

$R_b$  is a material resistance against crack branching.  $A_b$  may also be called as fracture parameter.

In this paper, we consider the following fracture parameter as the condition for the governing parameter for dynamic crack branching.

- (1)  $J'_{total}$  (the total energy release rate or total energy flow due to a unit crack extension) This parameter is nothing other than the dynamic J integral at the current propagating crack tip.

$$J'_{total} = J'. \quad (7)$$

Thus, this can be evaluated by using Eq.(2) in a numerical analysis, or Eq. (18) (see section 4 in [1]) in an experiment with the method of caustics.

- (2)  $J'_{excess}$  (excess energy flow due to a unit crack extension)

$$J'_{excess} = J' - J'_{arrest} \quad (8)$$

where  $J'_{arrest}$  is the dynamic J value for an arresting crack tip. Therefore,  $J'_{excess}$  can be interpreted as the excess energy to overcome the energy of crack arrest.  $\Phi_{total}$  has the physical meaning of the flux of total energy flow into the propagating crack tip, or into a process zone.

- (3)  $\Phi_{total}$  (total energy flow flux into a propagating crack tip, per a unit time ) This parameter can be defined by

$$\Phi_{excess} = (J' - J'_{arrest}) \times C. \quad (9)$$

where  $J'_{arrest}$  is the dynamic J value for an arresting crack tip. Therefore,  $J'_{excess}$  can be interpreted as the excess energy to overcome the energy of crack arrest.  $\Phi_{total}$  has the physical meaning of the flux of total energy flow into the propagating crack tip, or into a process zone surrounded by  $\Gamma\epsilon$ .

- (4)  $\Phi_{excess}$  (excess energy flow rate or total energy inflow flux into a propagating crack tip, per unit time) The combination of the ideas in the second and the third parameters as

$$\Phi_{excess} = (J' - J'_{arrest}) \times C. \quad (10)$$

### Experimental Discoveries

#### World-most Advanced Ultrahigh-Speed Video Camera System

Since the dynamic crack branching phenomena are very fast transient, in order to clarify the mechanism the phenomena, high-speed photography is mandatory.

In this paper, two types of high-speed photography system were used. Especially, the world most advanced ultrahigh-speed video camera (one million frames per second, 100 frames, 1 frame: 312x260 pixels, 10 nano exposure time) was utilized to record various types of dynamic crack bifurcation phenomena.

#### Experimental Evidences of the $\Phi_{total}$ criterion for Various Dynamic Crack Bifurcation Phenomena



139.6  $\mu$ s

Figure 3: Highspeed photograph of dynamically branching crack

**Two Crack Branching in HOMALITE 991**

A high speed photograph of dynamically branching crack at 139.6 $\mu$ s is shown in fig3. The size of caustic patterns were converted to the dynamic J integral values, using the theory of caustics developed in [4] and Eq. (4).

The dynamic J integral values at dynamic crack branching are plotted against in fig.4.

It is seen that dynamic crack branching does not occur at a particular value of the dynamic J Integral or at a particular crack velocity.

The total energy flux at crack branching are evaluated by Eq. (9), and plotted in Fig.5, against crack velocity. The value of the total energy flux into the tip of branching crack are almost constant for various crack velocities at the instance of dynamic crack branching.

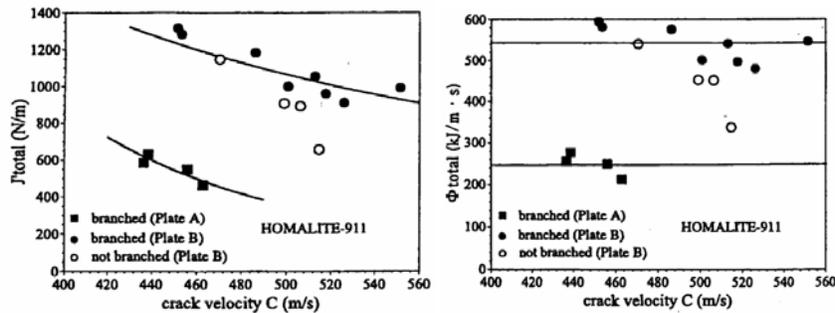


Figure 4: Energy flow rates at crack branching Energy flux at crack branching

The  $\Phi_{total}$  values at crack branching for various specimens are summarized in figure 5.

The  $\Phi_{total}$  value is almost constant, and 4800n/s for PMMMA.

**Computational Verifications**

The author and coworkers[5] have developed the moving finite element method based on Delaunay automatic triangulation, this method made it possible to sim-

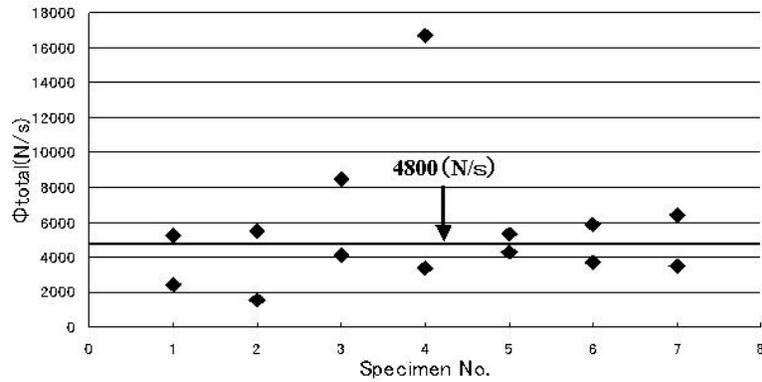


Figure 5: Experimental results of critical  $\Phi_{total}$  value for PMMA specimen

ulate very complicated dynamic fracture phenomena. Figure 6 shows one of the frames of ultrahigh-speed video recoding of multiple dynamic crack branching phenomena. Figure 7 shows the simulation result obtained by the moving finite element method based on Delaunay automatic numerical simulation was carried out using moving finite element method based on Delaunay automatic triangulation. Numerical result agrees excellently with the experiment (see figure 6 ). It is extremely interesting to see in Fig 8 that the  $\Phi_{total}$  criterion is repeatedly satisfied in the multiple dynamic crack branching phenomena.

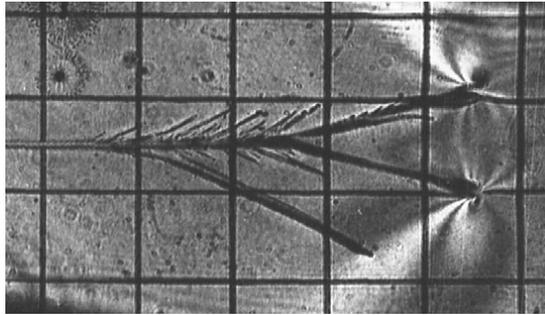


Figure 6: A ultra-high speed photograph of multiple crack branching phenomena

### Conclusions

Recent discoveries for the governing condition on of dynamic crack bifurcation phenomena were summarized. We found that the dynamic crack bifurcation occurs when the total energy flux  $\Phi_{total}$  in to the process zone or a propagating crack tip rack a critical material resistance [the  $\Phi_{total}$  criterion].

Furthermore, using the moving finite element method based on Delaunay automatic triangulation [MFEMBOADAT], the explicabilities of the  $\Phi_{total}$  criterion

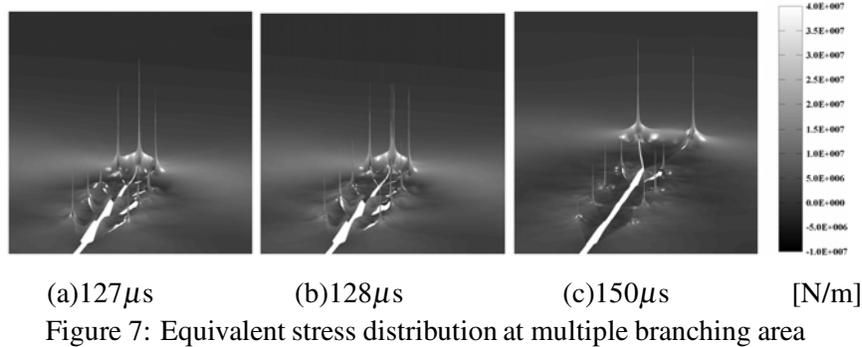
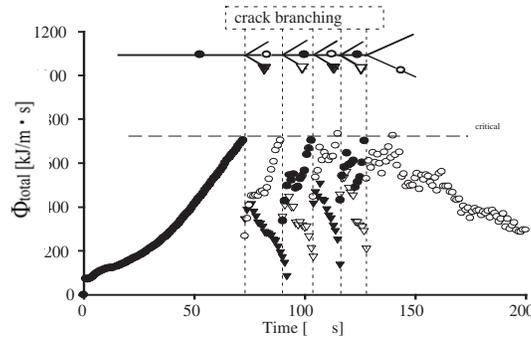


Figure 7: Equivalent stress distribution at multiple branching area

Figure 8:  $\Phi_{total}$  in multiple crack branching

were repeatedly verified. Utilizing the world most advanced ultrahigh-speed video camera various types of dynamic crack bifurcation phenomena were recorded. From the experimental evidences, the  $\Phi_{total}$  criterion is valid for various specimen types, and various materials.

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