

## **High-precision Path Prediction Simulation of Non-straight and High-speed Propagating Crack**

Nishioka T.<sup>1</sup>, Fujita N.<sup>1</sup>, Fujimoto T.<sup>1</sup> and Kogame M.<sup>1</sup>

### **Summary**

In this study, numerical simulations of mixed-mode fracture paths in dynamic fracture phenomenon are carried out by using moving finite element method based on Delaunay automatic mesh generation. In addition, the experiments under same condition was carried out, and the both results were compared. The calculated paths by the simulation agree well with the fracture paths of the experiments.

### **Introduction**

There are fatigue cracks and small cracks arising from the factory in material that composes structure. When the impact load is given to them then the destruction starts, dynamic destruction phenomenon that crack pass on at high speed is generated. In addition, it is overwhelming majority that stress field around crack tip became multi-axial stress field. It is very difficult that fracture behavior under the mixed-mode impact load is simulated, because the mechanism of dynamic fracture phenomenon is understood and the crack passed on in an arbitrary direction.

In this study, we simulated dynamic fracture path structures subject to eccentric impact loading moving finite element method based on Delaunay automatic mesh generation[1][2][3]. In past times, we did computer simulations of path prediction at load eccentricity ratio “e”=0.1, 0.2, 0.3 in our laboratory[4][5]. However, in case the ratio is 0.2 and 0.3, the face of primary crack made contacts. For highly accurate path prediction up to the edge to be possible, it is necessary to developed the program that considered this contact phenomenon. We did a more highly accurate numerical analysis by using the program that consider the contact phenomenon.

Moreover, we carried out the experiment the impact destruction under the same condition that used optical interferometrical observation[6] together, the fracture path obtained by the numerical simulation is compared with the experiment route, then the validity of this method was verified. The calculated path by the simulation agree well with the results of the experiments.

### **Moving Finite Element Method Based on Delaunay Automatic Mesh Generation**

Momentarily modifying the mesh pattern is necessary for the crack propagation phenomenon is analyzed with a high degree of accuracy. Using conventional moving finite element method is difficult in case that the crack curve in an arbitrary direction. The moving finite element method based on Delaunay automatic mesh

---

<sup>1</sup>Structural Strength Simulation Engineering Laboratory, Faculty of Maritime Sciences, Kobe University, 5-1-1 Fukae Minamimachi, Higashinada-ku, Kobe, 658-0022, Japan

generation developed in our laboratory is well suited for such non-direct propagation cases.

First, we describe Delaunay automatic mesh generation method. Delaunay triangulation automatically generates triangles that cover the entire area within an arbitrarily specified convex region. The generated triangular elements do not contain any other nodes within their circumscribed circles. This feature is also preferable to the triangular finite elements from an accuracy point of view. Taniguchi developed the modified Delaunay triangulation method that can treat convex regions such as a cracked body. He also developed a program for automatic mesh generation on the basis of the modified Delaunay triangulation. In the modified Delaunay triangulation, only exterior and interior boundaries points, and specified interior points (if they are necessary), are required for automatic mesh generation. The mesh pattern is automatically generated using the exterior boundary points and the specified interior points. Figure 1 portrays an example of mesh generation for a plate with an edge crack.

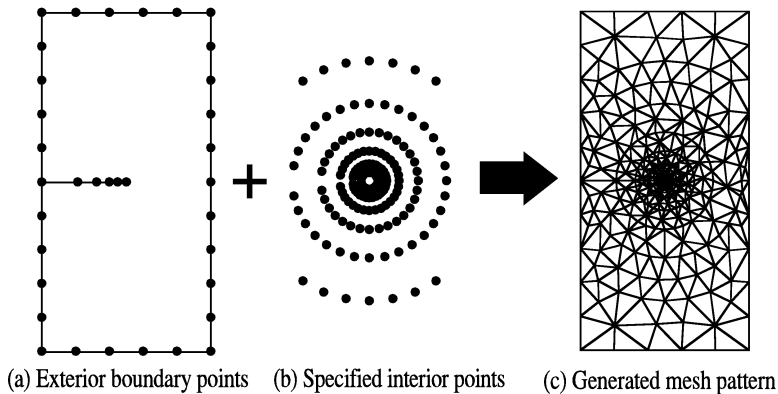


Figure 1: An example of mesh generation

Then, we describe the element breakdown in the crack propagating. In the moving finite element method based on Delaunay automatic mesh generation, a crack is advanced as shown Figure 3. As can be seen from the figure, the group of specified interior points is accompanied the crack tip propagating. In addition, the specified interior points is arranged in a concentric pattern and thickly toward the center. Thereby, the crack tip always remains at the center of the group of moving elements, even for complicated crack propagation, then the stress singularity at the crack tip is analyzed in high accuracy. Figure 2 is conceptual diagram of the moving finite elements around propagating crack tip.

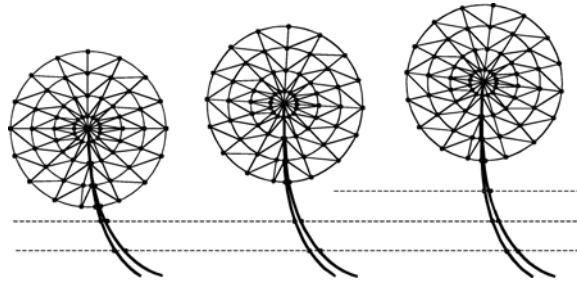


Figure 2: Moving elements around a propagating crack tip

### Dynamic J integral

The path independent dynamic J integral derived by Nishioaka and Atluri is effective to the crack that receives the impact load and the crack high-speed propagating.[7] The path independent dynamic J integral ( $J'$ ) in the case with a whole coordinate system like figure 3 is expressed by following equation 1.

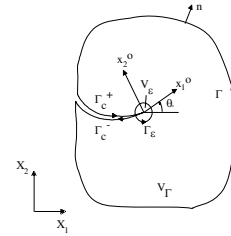


Figure 3: Definition of integral paths

$$\begin{aligned}
 J'_k &= \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} [(W + K) n_k - t_i u_{i,k}] dS \\
 &= \lim_{\varepsilon \rightarrow 0} \left\{ \int_{\Gamma + \Gamma_c} [(W + K) n_k - t_i u_{i,k}] dS + \int_{V_\Gamma - V_\varepsilon} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV \right\}. \quad (1)
 \end{aligned}$$

Where  $W$  and  $K$  are the strain and kinetic energy densities, respectively, and  $(\cdot)_{,k} = \partial(\cdot)/\partial X_k$ . The displacement, traction are denoted by  $u_i, t_i$  respectively. The integral paths are defined in figure 3. In most numerical analyses, far-field integrals are used to evaluate the value of the dynamic J integral. In most numerical analyses, the dynamic J integral ( $J'$ ) is evaluated by

$$J'_k = \int_{\Gamma + \Gamma_c} [(W + K) n_k - t_i u_{i,k}] dS + \int_{V_\Gamma} [\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k}] dV. \quad (2)$$

The crack-axis components of the dynamic J integral can be expressed by the following coordinate transformation:

$$J'_k{}^0 = \alpha_{kl} J'_l. \quad (3)$$

The in-plane mixed-mode stress intensity factor is evaluated is evaluated in high accuracy by using the aforementioned expression[8]. Its name is the component separation method, and the formulae used in this method are

$$K_I = \delta_I \left\{ \frac{(2\mu J_1^0 \beta_2)}{A_I (\delta_I^2 \beta_2 + \delta_{II}^2 \beta_1)} \right\}^{\frac{1}{2}} = \delta_I \left\{ \frac{2\mu \beta_2 (J'_1 \cos \theta_0 + J'_2 \sin \theta_0)}{A_I (\delta_I^2 \beta_2 + \delta_{II}^2 \beta_1)} \right\}^{\frac{1}{2}} \quad (4)$$

$$K_{II} = \delta_{II} \left\{ \frac{(2\mu J_1^0 \beta_1)}{A_{II} (\delta_I^2 \beta_2 + \delta_{II}^2 \beta_1)} \right\}^{\frac{1}{2}} = \delta_{II} \left\{ \frac{2\mu \beta_1 (J_1' \cos \theta_0 + J_2' \sin \theta_0)}{A_{II} (\delta_I^2 \beta_2 + \delta_{II}^2 \beta_1)} \right\}^{\frac{1}{2}} \quad (5)$$

where  $\delta_I$  and  $\delta_{II}$  are the mode-I and mode-II crack opening displacement, and  $A_I(C)$ ,  $A_{II}(C)$  are function of crack velocity.

### Dynamic Fracture Experiment under Eccentric Impact Loading

The specimen configuration used in three-point bending test under an impact load with loading eccentricity is shown in figure 4. The specimen material is PMMA, and its solid state properties: Young's modulus  $E=2.948\text{GPa}$ , poisson's ratio  $\nu=0.329$ , density  $\rho=1190\text{kg/m}^3$ . The specimen size: height  $W=100\text{mm}$ , thickness  $t=10\text{mm}$ , span of the supports  $S=400\text{mm}$ , initial crack length  $a_0=50\text{mm}$ . The load eccentricity ratio is defined as  $e = l(S/2)$ , where  $l$  is the distance between the center of the specimen and the loading point. The impactor of  $1\text{kg}$  in weight is dropped at speed about  $5\text{m/s}$ , and this is assumed to be compulsion displacement. In addition, C.G.S.(Coherent Gradient Sensing ) method that can be able to observe principal stress slope distribution is used at this experiment. The crack velocity propagates in PMMA is much high. Therefore it is so difficult to obtain sharp crack propagation films using usual high speed video camera. Hence we used ultrahigh-speed video camera.

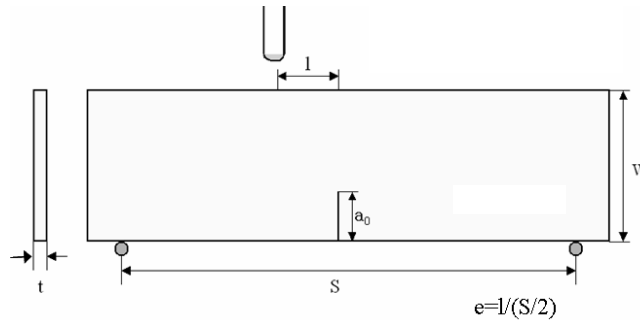


Figure 4: Specimen configuration

### The how to simulation

This simulation is composed of to combine the crack propagation velocity history obtained from the experiment and the crack propagation direction prediction theory. The local symmetry criterion is used for the crack propagation direction prediction theory in this simulation. The local symmetry criterion is assumed theory that the crack tip is moved on the direction to become  $K_{II}$ (Stress intensity factor of mode-II) value zero.

### Numerical Simulation Results

Figure 5 gives that the crack-tip coordinate history obtained from the experiment and the simulation is plotted. In each load eccentricity ratio, the simulation results is in good agreement with the experiment results. Thus this crack propagation prediction method was proven the validity. In addition, the increase in crack growth angle with increasing load eccentricity ratio is also well modeled. It is seen that the crack propagation angle swells as the load eccentricity ratio rises from this figure.

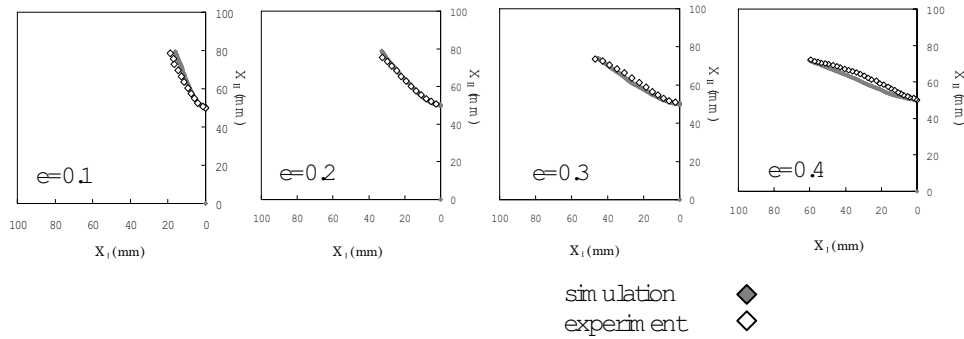


Figure 5: Crack-tip coordinate history

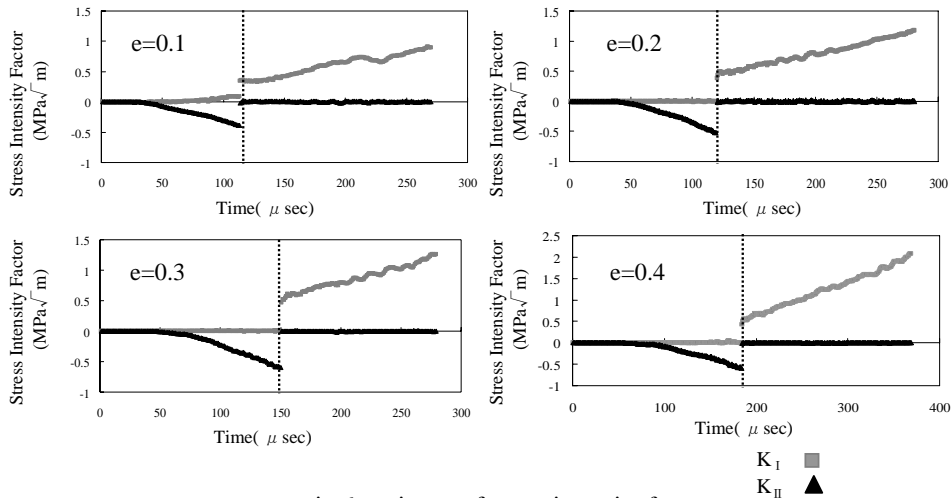


Figure 6: History of stress intensity factor

Lastly, the history of the stress intensity factor of both load eccentricity ratio are shown in figure 6. These results show that mode-II is upper hand before propagation, however after propagation, mode-I is upper hand in contradiction to before propagation. Moreover, from the  $K_{II}$  value is zero all of the time, It see that the propagation direction prediction is done with propriety along the local symmetry

criterion.

### **Conclusions**

Numerical simulations of mixed-mode fracture paths in dynamic fracture phenomenon are carried out by using moving finite element method based on Delaunay automatic mesh generation. The results obtained from the computer simulation well accorded with experiments results.

As a future challenge that lies ahead, We will do the simulation and the experiment by increased load eccentricity ratio, we want to verify what behavior.

### **References**

1. Taniguchi, T. (1992): Automatic Mesh Generation for FEM: Use of Delaunay Triangulation, Morikita Publishing.
2. Nishioka, T., JSME Int. J.,37-4, A(1994), pp.313-333.
3. Nishioka, T., Int. J. Fract., 86-1/2 (1997), pp.127-159.
4. Nishioka, T., Tokudome, H. and Kinoshita M., "Dynamic Fracture-Path Prediction in Impact Fracture Phenomena Using Moving Finite Element Method Based on Delaunay Automatic Mesh Generation", International Journal of Solids and Structures, Vol. 38, No.30-31, (2001-7), pp.5273-5301.
5. Nishioka, T., Hashimoto, M. and Fujimoto, T. Numerical Simulation of Mixed-mode Three Points Bending Fracture under Impact Load, The Japan Society of Materials Science(2001), pp.359-360.
6. Nishioka, T., Kogame, M. and Fujimoto, T., Sakakura, K., C.G.S. Observations of Three points Bending Fracture under Impact Load, The Japan Society of Mechanical Engineers, The Japanese Society for Experimental Mechanics Publications, (2006)
7. Nishioka, T. and Atluri, S. N., Eng. Fract. Mech., 18-1, (1983), pp.1-22.
8. Nishioka, T., Murakami, R. and Matsuo, S., Computational Mechanics '91, ICES Publications, (1991), pp.750-775.