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Numerical Simulation of Dynamic Nonlinear Fracture based on Various Fracture Path Prediction Theories

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Summary

In this study, applicability of various fracture path prediction theories into dynamic elasto visco-plastic fracture is discussed by the moving finite element analyses. Outline of each fracture path theory and numerical method to predict fracture path are explained. Fracture path predictions are demonstrated in the numerical specimen model under mixed mode loading, and numerical results of fracture path prediction are compared.

Introduction

In some fracture accident, propagating crack tip reaches significant part of structure and catastrophic damages are caused. To prevent serious fracture accidents, establishment of measures to predict crack propagation path is very important in many industrial fields Fractures have been happened in diverse situations and these phenomena have been classified into some types of fracture. Therefore, various fracture path prediction theories have been suggested. Some of these theories were derived based on singular near field of stable crack propagation. Applicability of some theories was also investigated by using numerical simulation[1][2].

Nishioka classifies significant fracture path prediction theories into implicit theories and explicit theories[1]. Path predictions for various dynamic elastic fractures using the local symmetry criterion[3] and the maximum hoop stress criterion[4] were reported by Nishioka and co-workers. Each path prediction theory was introduced into the moving finite element method[5]. In their studies, numerical results were compared with experimental fracture path in three point bending fracture specimen[1], crack bifurcation specimen[2] and etc., and these results indicates the excellent efficiency of numerical path prediction using the moving finite element method.

This study focuses the numerical prediction of crack propagation path for dynamic elasto visco-plastic fracture. Some of the fracture path prediction theories, which are able to apply dynamic nonlinear fracture, are introduced into the moving finite element method. The local symmetry criterion based on the T* integral ($T_2^{*0} = 0$ criterion)[6][7], the maximum T_1^{*0}criterion, the maximum second stress invariant criterion (det. criterion)[8] and maximum hoop stress criterion[4] are introduced into the moving finite element method for dynamic nonlinear fracture[9]. Dynamic elasto visco-plastic fracture path is predicted in numerical specimen under mixed mode dynamic loading.

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The T* integral

In this study, the T* integral[10] is used to evaluate the crack tip condition for dynamic elasto visco-plastic fracture. For calculation of T* value, near and far field domains, V_ε and V_Γ are defined around crack tip, as shown in Fig.1. The components of T* integral based on the global coordinates system, T_k^* $(k = 1,2)$ are calculated from the following path integration:

$$\begin{aligned} T_k^* &= \int_{\Gamma_\varepsilon} \left[(W+K)\,n_k - t_i u_{i,k}\right] dS \\ &= \int_{\Gamma+\Gamma_C} \left[(W+K)\,n_k - t_i u_{i,k}\right] dS + \int_{V_\Gamma - V_\varepsilon} \left[\rho \ddot{u}_i u_{i,k} - \rho \dot{u}_i \dot{u}_{i,k} + \sigma_{ij} u_{i,jk} - W_{,k}\right] dV \end{aligned} \tag{1}$$

where, W and K denote the stress working density and the kinetic energy density, respectively. n_k, t_i and ρ are the components of unit outward normal vector on the integral path, the traction and the mass density, respectively. Γ_ε, Γ and Γ_c mean a near-field path, far-field path and crack surface path, respectively. It is assumed that the near field path Γ_ε and the near-field domain V_ε are extended with the crack propagation process.

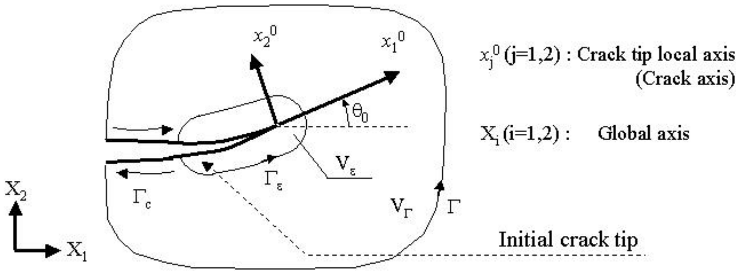


Figure 1: Definition of integral paths around crack tip to evaluate T* values

For the curving crack propagation, the crack-axis components of the T* integral T_ℓ^{*0} $(\ell = 1,2)$ are very useful to evaluate the crack tip condition, because the T* integral has perfect far-field path independence for dynamic nonlinear fracture. T_ℓ^{*0} $(\ell = 1,2)$ can be calculated by following coordinate transformation:

$$T_\ell^{*0} = \alpha_{\ell k}(\theta)\, T_k^* \tag{2}$$

where $\alpha_{\ell k}(\theta)$is the coordinate transformation tensor and θ is the angle between the global axis X_1 and the crack axis x_1^0.

Fracture Path Prediction Theories

In fracture path prediction theories based on mechanics, crack-propagating direction is derived from the near-field condition of crack tip. In some of the theories, the specific crack tip condition is prescribed, and this condition has to be always satisfied during crack propagation. These theories have been classified as the implicit fracture path prediction theories[1]. In the finite element analyses to predict fracture path with the implicit theory, iterative calculation is required to detect the crack propagation direction. In the iterative calculation at each time step, crack tip propagation to tentative direction are assumed. Final crack propagating direction is obtained from convergence of the iterative calculations.

In the local symmetry criterion[3], local mode I condition near propagating crack tip is assumed. For linear fracture, the stress intensity factor K_{II}=0 condition is used as parameter of local symmetry condition. For nonlinear fracture with large deformation, conventional linear fracture mechanics parameters cannot be used as the criterion. Some researchers[6][7] suggest the $T_2^{*0} = 0$ criterion as the local symmetry criterion for nonlinear fracture.

The T* integral component T_1^{*0} means the energy flow rate into the domain V_ε. Fracture work is supplied by this energy flow. For V_ε domain of propagating crack tip, maximum energy flow rate is caused in the maximum T_1^{*0} criterion. The $T_2^{*0} = 0$ criterion and the maximum T_1^{*0} criterion are classified into the implicit fracture path prediction theories.

In the explicit fracture path prediction theories[1], crack-propagating direction is determined from the near field distribution of specific parameter at time t. For the time step increment Δt in finite element analyses, the crack propagating direction during Δt (from t to $t+\Delta t$) can be evaluated based on the near field deformation at time t. Therefore, iterative calculation is not required in numerical prediction with the explicit theories.

The maximum second stress invariant criterion (det. criterion) was proposed by Papadopoulos[8]. In the numerical prediction based on this theory, distribution of second stress invariant I_2 is estimated on the concentric circle line. The concentric circle is defined around propagating crack tip, and center point of the circle is identical with crack tip position. Occurrence of maximum I_2 value on the concentric circle line indicates propagation direction from current crack tip.

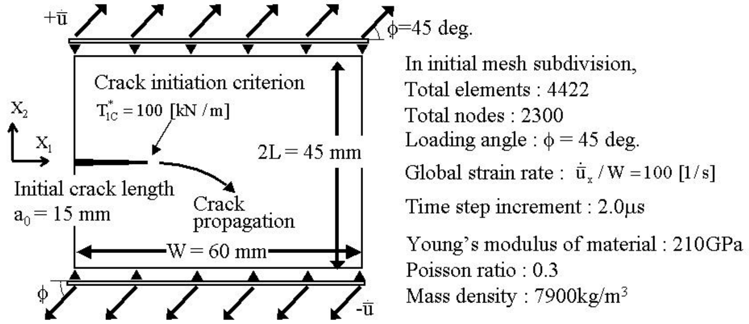
In the maximum hoop stress criterion[5], hoop stress $\sigma_{\theta\theta}$ is used instead of I_2 in the above mention. The hoop stress $\sigma_{\theta\theta}$ is the stress component, which depends on positions of the concentric circle and the current crack tip ‘A’. $\sigma_{\theta\theta}$ can be calculated at a points ‘S’ on the concentric circle by the following equation:

$$\sigma_{\theta\theta} = \sigma_x \sin^2\theta + \sigma_y \sin^2\theta - \tau_{xy} \sin\theta\cos\theta \tag{3}$$

where σ_x, σ_y and τ_{xy} are stress components based on global coordinates. Angle θ is prescribed by the global axis X_1 and line $\overline{AS}$.

Results of Numerical Fracture Path Prediction

The moving finite element method is already developed for dynamic elasto visco-plastic fracture in author's previous study[9]. Aforementioned fracture path prediction theories are introduced into the moving finite element method[6]. At each time step, the fracture path direction is founded by a fracture path prediction theory. The moving finite element technique achieves the modeling of newly predicted crack elongation. Mesh subdivision is re-generated at each time step by the Delaunay automatic triangulation[11]. In order to simulate dynamic elasto visco-plastic fracture, the finite element formulation is derived based on the large deformation theory[12] and the special variational principle for nonlinear fracture[13].



In initial mesh subdivision,
Total elements : 4422
Total nodes : 2300
Loading angle : $\phi = 45$ deg.
Global strain rate : $\dot{\bar{u}}_x / W = 100$ [1/s]
Time step increment : 2.0μs
Young's modulus of material : 210GPa
Poisson ratio : 0.3
Mass density : 7900kg/m^3

Figure 2: Numerical fracture specimen under mixed mode loading

Fracture path prediction is demonstrated in the numerical specimen under mixed mode loading, as shown in Fig.2. It is assumed that the specimen consists of elasto visco-plastic material and the equivalent stress – visco-plastic strain relation is interpolated by the Malvern type constitutive equation[14] :

$$\dot{\bar{\varepsilon}}^{vp} = 1000\left(\frac{\bar{\sigma}}{\sigma_f} - 1\right), \quad \sigma_f = 300 \times 10^6 \left(\bar{\varepsilon}^{p}\right)^{0.2} \tag{4}$$

where σ_f, $\bar{\varepsilon}^{vp}$ and $\bar{\varepsilon}^{p}$ are quasi-static equivalent stress, equivalent visco-plastic strain and quasi-static equivalent plastic strain, respectively. Elastic parameters of the material are shown in Fig.2.

Figure 3(a) shows the mesh subdivision near the fracture path, which is predicted by the local symmetry criterion. Non-straight fracture path can be expressed by the moving element technique. To evaluate residual permanent strain distribution, fine mesh subdivision occupies in post crack propagation area. For other fracture path prediction theories, similar mesh subdivisions are constructed, and the equivalent visco-plastic strain distributions are shown in Fig. 3(b)-3(d). Distinct plastic strain are remained near the post crack propagation line. Under the condition in this study, all theories indicate similar predicted fracture path. Other

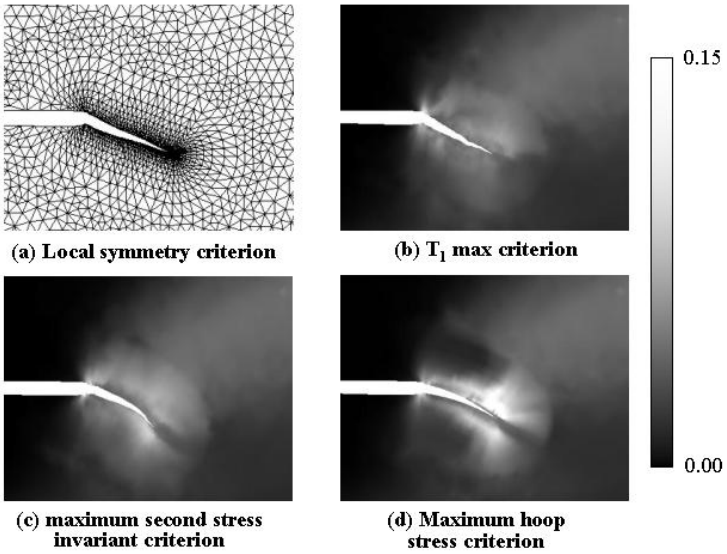


Figure 3: Mesh subdivision and equivalent visco-plastic strain distributions

results will be presented in the conference.

Conclusions

In this study, some fracture path prediction theories are considered for dynamic elasto visco-plastic fracture. These theories are introduced into the moving finite element method. Each predicted fracture path are similar in the numerical specimen under mixed mode loading. This research was supported by the grant from Hyogo Science and Technology Association (17W114).

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