# Control of Walking Robot by Inverse Dynamics of Link Mechanisms Using FEM 

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This paper presents a control of walking robot by using inverse dynamics of link mechanisms, which has already been proposed and applied in several in-plane motions. In this method, FEM is used for the discretization of equations of motion. This method calculates nodal forces by evaluating equations of motion in a matrix form, and thus information from the entire system can be handled efficiently, and the torques input to each joint of link mechanisms to achieve required motion are calculated easily. This method is suitable to the feed-forward control of closed-loop or continuously link mechanisms. In this paper, this inverse dynamics method is applied to the transverse walking problem of a robot and the control experiment is also conducted. The results illustrate the potential of the proposed method for the control of walking motion of robot.

## Introduction

Fueled by the advances in robotics, humanoid robots composing of complex mechanisms have been aggressively developed in recent years. While a further demand on performance leads to the complication of the mechanism, the necessity for speeding up the motion of the robot is growing. On the other hand, the researches related to the inverse dynamics for the modeling of robot arm have also been carried out aggressively. Inverse dynamics is necessary in order to decide the exact location of a robot arm as well as to achieve fast and accurate motion by feed-forward control.

The conventional approaches by the Newton-Euler method and the Lagrange method for the inverse dynamics of link mechanisms faces great difficulty when applied to the calculation of mechanisms with a large number of degrees of freedom (number of joints), a change of topology when a robot arm grips and releases an object, a closed-loop when it touches a wall, an operation where frequent contact and collision with the environment occurs, and so on, because the equations to be solved changes according to the change in the link systems.

Putting these in the background, the integrated analysis method for inverse dynamics has been proposed ${ }^{(1) \sim(4)}$, which can flexibly correspond to the change of a system. In this method, the finite element method (FEM) is used for the discretization of the equations of motion. This method calculates nodal forces by evaluating the equations of motion in a matrix form. Comparing with the NewtonEuler method or the Lagrange method, information from the entire system can be

[^0]handled efficiently. The torques, which must be inputted to each joint of link mechanisms to achieve required motion, can also be calculated stably and continuously. Therefore, it is expected that this approach can be facilitated to the control of a real mechanism in the demand for faster motion of the robot.

In this paper, inverse dynamics using FEM which can correspond to the change between open and closed-loop systems is applied to the transverse walking problem of a robot. The numerical results illustrate the potential of the proposed method.

## Inverse dynamics of link mechanism using FEM ${ }^{(1) \sim(4)}$

The dynamics equations of link mechanisms are generally derived by the NewtonEuler method or Lagrange Method. In a summarized expression, the equations are written as

$$
\begin{equation*}
\{\tau\}=[M(\theta)]\{\ddot{\theta}\}+\{V(\theta, \dot{\theta})\}+\{G(\theta)\}, \tag{1}
\end{equation*}
$$

where $\{\tau\}$ is the torque vector, $[\mathbf{M}]$ the inertial force matrix, $\{\mathbf{V}\}$ the centrifugal force and Coriolis force term vector, and $\{\mathbf{G}\}$ the gravity force term vector. $\theta$, $\dot{\theta}$, and $\ddot{\theta}$ within the parentheses are the relative variables of the angle, the angular velocity and the angular acceleration between each link, respectively. All of the parameters in the equation relate to each other since they are derived in relative polar coordinates and in the dimension of torque. Therefore most parts of the equations must be revised when the structural configuration of the link mechanism is changed.

On the other hand, torque values are calculated by the following approach in the proposed method. By considering other components, and arranging them into global coordinates in a matrix form, the joint torque vector is expressed as

$$
\begin{equation*}
\left\{\tau^{n}\right\}=\left[L^{n}\right]\left[T^{n}\right]\left\{P^{n}\right\} \tag{2}
\end{equation*}
$$

The superscript on the upper right indicates the total number of links. $\left\{\tau^{n}\right\}$ is the torque vector, $\left\{P^{n}\right\}$ is the vector related to nodal forces, $\left[T^{n}\right]$ is the transformation matrix, and $\left[L^{n}\right]$ is the member length matrix. Information on the $\mathrm{i}+1 \sim \mathrm{n}$ link is summed by multiplying the $\left[L^{n}\right]$ matrix by vector $\left[T^{n}\right]\left\{P^{n}\right\}$, which is the nodal force vector transformed into elemental coordinates. In the case of closed-loop link mechanisms, the above matrix is divided into multiple parts, as shown below, to fix the configuration of passive joints as well as the torque allocation undertaken by active joints.

$$
\left[\mathbf{L}^{n}\right]=\left[\begin{array}{ll}
\mathbf{L}^{a} & 0  \tag{3}\\
0 & \mathbf{L}^{b}
\end{array}\right],
$$

The suffixes $a$ and $b$ are the number of links $(a+b=n)$ when the mechanism is divided into two parts. This is the only process that is different between the algorithms of open- and closed-loop link mechanisms, which of course, can be automatically
alternated in the program. The vector related to the nodal force acting on the i-th link at $t+\Delta t$ is successively calculated using the above vector as follows

$$
\begin{equation*}
\left\{\mathbf{P}_{i}\right\}_{t+\Delta t}=\left\{\mathbf{P}_{i}\right\}_{t}+\left\{\Delta \mathbf{p}_{k}\right\}, \tag{4}
\end{equation*}
$$

The successive values of the $n$-link joint torque are then obtained by substituting eq.(4) into eq.(2). The Newmark's $\beta$ method ( $\beta=1 / 4$ ) is used as the time integration scheme to solve the incremental dynamics equation.

In this approach, the nodal forces are evaluated in the absolute Cartesian coordinate system, and in the dimension of force. The equation is completely separated into terms of different parameters, and each matrix has a clear physical meaning related to the modeled link mechanism. The member length matrix $\left[L^{n}\right]$, for example, contains components in the dimension of length, which relates force to torque. The configuration of the components in the matrix expresses the structural connectivity of the link mechanism. The separation of the parameters makes the equation highly expansible and flexible, and thus the calculation scheme becomes applicable to complex link mechanisms without difficulty.

## Walking Simulation

This chapter shows the result of the walking simulation. Figure 1(a)-(e) shows dividing the robot model and the process of walking in five steps. Two cases were simulated. Case 1 shows the robot walking on a level ground surface. Case 2 shows the robot walking uphill with an inclination of 5 degrees from level ground. The first step is to shift the center of gravity backward from the basic stance shown in (a). This leads to the second step shown in (b) where the forefoot is raised. The forefoot is then made to land further in the direction of motion as shown in (c). The fourth step (d) shifts the center of gravity to the front, causing the hind leg to be raised. The fifth and final step (e) results in the landing of the hind leg on the ground. The process of walking is the result of continuous iteration of these five steps.

The length of a long link drawn in the heavy line is 10 cm , the length of the short link drawn in the thin line is 2 cm , and the gravitational acceleration of $9.8 \mathrm{~m} / \mathrm{s}^{2}$ was given to the material of density $3.6 \mathrm{~g} / \mathrm{cm}^{3}$. The four circular points represent the joints of the link mechanisms. The angle of the joints changes by adding the joint torques obtained through inverse dynamics calculation at each point, causing the robot model to walk.

The walking motion is simulated with ABAQUS/Standard Ver. 6.4 by adding the torques of 20 increments every two seconds to each joint. Figure 2 compares the track of the robot's forefoot and the target orbit. In case 1, the horizontal displacement by one cycle of each joint is the range of $6.59 \sim 6.60 \mathrm{~cm}$ and agrees well with the analytical result 6.59 cm . In case 2 , the horizontal displacement by one cycle of

Case 1


Case 2


Figure 1: Walking Process


Figure 2: Target orbit and Tracks of forefoot
each joint is the range of $3.56 \sim 3.59 \mathrm{~cm}$ and agrees well with the analytical result of 3.59 cm . The vertical displacement by one cycle of each joint is the range of $0.313 \sim 0.314 \mathrm{~cm}$ and also agrees well with the analytical result of 0.314 cm . This section shows the possibility that using FEM in the inverse dynamics calculation can generate appropriate input data for the feed forward control of walking robot.

## Torque Control Experiment by Calculation of Inverse Dynamics

The purpose of the experiment is to further improve the control performance of the robot by applying feed-forward control. This includes comparing control performance to when only general feed-back control is applied. Due to the limitation of space, this section describes only the design of the experiment, and the result of experiment is described in the presentatiion.

The controlled object is two-legged robot "KHR-1" on sale from Kondo Kagaku Co., Ltd. Joint torques are controlled by outputting the PWM signal that corresponds to the calculated torques from SH7045F microcomputer to DC motors through H bridge circuits. The links of the robot are composed of the combination of some parts, the servo motor, the aluminum frames and etc. Therefore, it is necessary to obtain information such as the position of a link's center of gravity, to calculate inverse dynamics. Before simulating the walking motion of the robot, it is first necessary to simulate the 2-link mechanism shown in Fig. 3 in a control experiment. The results from this control experiment are then used to verify the validity of the link mechanism's model and to determine the position of its center of gravity.


Figure 3: 2-link mechanism

Fig. 3 shows the raising movement of the 2 -link mechanism that fixes the position of Joint1. In the analysis, each joint rotates $\pi / 2(\mathrm{rad})$ in 1 second at a constant speed. The torques calculated by the proposed method using FEM are almost corresponding to the torques calculated by the Newton-Euler method. Furthermore, the orbit of simulation that the point of the link drew is corresponding to its target orbit. The information of the link mechanism is considered from this problem's simulation and experiment results, and is used for two walking problems of the robot.

## Concluding Remarks

In this report, a method for the inverse dynamics of link mechanisms using FEM is implemented in the transverse walking problem of a robot. A control experiment is further conducted. The results illustrate the potential of the proposed method for controlling the walking motion of a robot. The overall objective of the proposed method is to achieve fast and accurate motion of a robot by controlling its link mechanisms. Future work includes improving the accuracy of the method by combining general feedback and feed-forward control.

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