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Sound wave propagation modeling in a 3D absorbing acoustic dome using the Method of Fundamental Solutions

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Summary

A frequency dependent formulation based on the Method of Fundamental Solutions (MFS) is used to simulate the sound wave propagation in a 3D acoustic space. This solution is approximated by a linear combination of fundamental solutions generated by virtual sources placed outside the domain in order to avoid singularities. The coating materials can be assumed to be absorbent. This is achieved in the model prescribing the impedance that is defined as a function of the absorption coefficient.

The model is first verified against analytical solutions, provided by the image source technique for a parallelepiped room bounded by rigid walls. The applicability of the present model is illustrated by applying it to solve the case of a dome. The fundamental solutions used in this case are defined for a half-space, which avoids the placement of collocation points on the floor.

Introduction

The acoustics of rooms, for speech or music, has been researched for many years since the acoustic behaviour of such rooms needs to be predicted at the design stage. The sound field created inside enclosures depends on their volume, geometry, coating materials, frequency of sound and occupancy. The modeling of the phenomena involved is not simple and different numerical methods of varying complexity have been developed. Among these are: statistical methods [1], methods based on geometric acoustics, such as the image source method [2, 3] and the ray tracing model [4]; hybrid methods combining those two [5]; methods requiring domain discretization, like the finite element method (FEM) and the boundary element method (BEM), and finally some meshless models. The FEM [6] and the BEM can only be used for small spaces and sound sources emitting at low frequencies. The method of fundamental solutions (MFS) [7,8], the plane waves method [9], the element free Galerkin method [9], and the boundary collocation method using radial basis functions [10] are some of the meshless methods used to solve acoustic problems.

In this work, the sound field generated by a 3D sound source inside a 3D enclosure is modeled using the MFS. The model developed allows the boundaries to be rigid or absorbent. The problem is first formulated, the results are then validated using the image source method, and finally an application is presented.

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Problem Formulation

The pressure amplitude generated by a 3D source inside an air-filled three-dimensional enclosed space is calculated by the method of fundamental solutions in the frequency domain (ω). The response inside the domain is found as a linear combination of fundamental solutions for the governing equation. Thus, the scattered pressure (p) wave field is written as

$$p = \sum_{k=1}^{N} \left[a_s G(\mathbf{x}, \mathbf{x_s}, \omega)\right] \tag{1}$$

These solutions represent the sound field generated by a set of virtual sources with amplitude a_s, placed outside the domain on a fictitious boundary in order to avoid singularities. $G(x, x_s, \omega)$ is the 3D Green's function for pressure for a receiver placed at (x, y, z) generated by pressure sources located at (x_s, y_s, z_s).

The 3D Green's function for pressure is well known

$$G(\mathbf{x}, \mathbf{x_s}, \omega) = \mathrm{e}^{-\mathrm{i}\frac{\omega}{\mathrm{c}}r} \Big/ r \tag{2}$$

with $r = \sqrt{(x-x_s)^2 + (y-y_s)^2 (z-z_s)^2}$, c is the sound wave velocity and i=$\sqrt{-1}$. The coefficients a_s are obtained by imposing the required boundary conditions at M collocation points (x_k, y_k, z_k) along the boundary. A system of M equations by N unknowns is then established. In the present case, an equal number of collocation points and sources was defined, leading to a system MxM.

For rigid enclosures, null velocities (incident velocity plus reflected velocity) are ascribed to the boundary, and the Green's function for velocities is then

$$H(\mathbf{x_s}, \mathbf{x_k}, \omega, \mathbf{n}) = (1/-\mathrm{i}\rho\omega)\left(\partial G/\partial r\right)\left(\partial r/\partial n\right) \tag{3}$$

where ρ is the air density and n is the unit outward normal at the collocation point (x_k, y_k, z_k). When the room's coating material exhibits $\hat{Z}$ impedance, this parameter is established by the ratio between pressure and velocity

$$G(\mathbf{x_s}, \mathbf{x_k}, \omega) + H(\mathbf{x_s}, \mathbf{x_k}, \omega, \mathbf{n}) = 0 \tag{4}$$

Consider an enclosure of arbitrary geometry with a horizontal rigid base. In this case there is no need for collocation points at this surface if an appropriate Green's function for half-space is used, written as

$$G(\mathbf{x}, \mathbf{x_k}, \omega) = \mathrm{e}^{-\mathrm{i}\frac{\omega}{\mathrm{c}}r} \Big/ r + \mathrm{e}^{-\mathrm{i}\frac{\omega}{\mathrm{c}}r'} \Big/ r' \tag{5}$$

with $r' = \sqrt{(x-x_s)^2 + (y-y_s)^2 (z+z_s)^2}$. However, absorption may be ascribed to the base by assigning reflection coefficient R_h to it

$$G(\mathbf{x}, \mathbf{x_k}, \omega) = \mathrm{e}^{-\mathrm{i}\frac{\omega}{\mathrm{c}}r} \Big/ r + R_h \mathrm{e}^{-\mathrm{i}\frac{\omega}{\mathrm{c}}r'} \Big/ r' \tag{6}$$

Verification

The MFS model developed in this work is verified against the image source method, applied to a 3D rectangular rigid room 3 m wide, 3 m high and 4 m long.

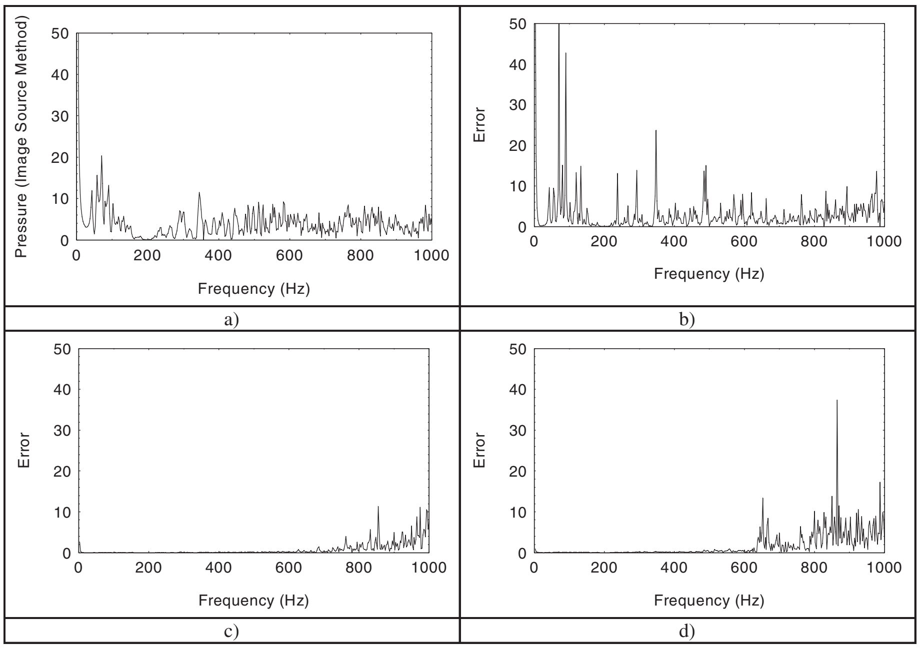

Figure 1: a) Pressure response obtained using the image source method; error obtained when the distances between the fictitious sources and the boundary, in the MFS model, are: b) 0.1 m; c) 0.2 m; d) 0.3 m.

The sound source is placed at (0.5 m, 0.5 m, 0.5 m) and the pressure is registered at a receiver placed at (2.5 m, 3.5 m, 2.5 m). In the MFS model the response is calculated for different fixed distances between the fictitious and the real boundary. Three distances are chosen for display: 0.1 m, 0.2 m and 0.3 m. The reference responses are those obtained with the image source model. In the MFS model the number of virtual sources used for this verification is 1996. Figure 1 shows the pressure amplitude obtained using the Image Source Method (Figure 1a) and the error found with the MFS method. The error exhibits a significant variation with the distance between boundaries. It can be observed that the error is greater at high frequencies, which may signify that the number of sources and collocation points is insufficient at those frequencies. The best results are obtained when the distance between the fictitious and the real boundaries is set at 0.20 m.

Applications

The algorithm described above is used to simulate the 3D wave field, generated inside a dome (with an oblate semi-ellipsoid shape) where the lengths of the three semi-axes are 10 m, 10 m and 8 m in the *x*, *y* and *z* directions respectively, as in Figure 2. The air filling the 3D space has density 1.22 kg/m^3, allowing the propagation of sound waves whose velocity is 340 m/s. A 3D pressure source located at (0.0 m, 0.0 m, 4.0 m) disturbs the medium, whose the pressure fluctuation is registered at a vertical grid of receivers placed at *x*=0.0 m. The receivers are 0.05 m apart in both directions (*y* and *z*).

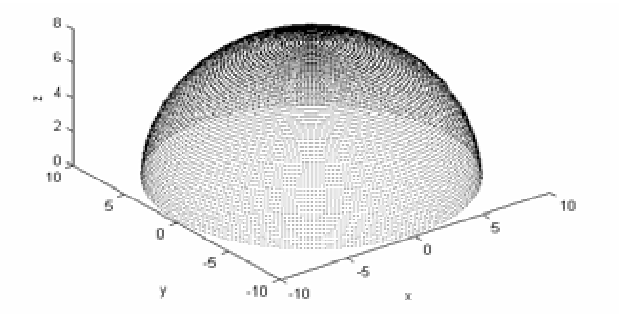

Figure 2: Geometry of the problem.

Computations have been performed in the frequency domain from 2.5 Hz to 1280 Hz, with an increment of 2.5 Hz. Pressure amplitudes in the frequency domain are submitted to an inverse Fourier transform in order to obtain responses in the time domain. The source is modeled as a Ricker pulse with a characteristic frequency of 350 Hz. The number of sources and collocation points used is 16141. Two situations are selected to illustrate the applicability of the model: case A - all the boundaries are rigid; case B - the pavement is rigid and the dome is absorbent.

A sequence of snapshots that displays the pressure wave field along the grid of receivers at different instants is presented. The pressure amplitude is displayed in a gray scale which ranges from black to white as the amplitude increases. Figure 3 compares case A (left column) with case B (right column), where the absorption coefficient ascribed to the dome is α=0.7. Both examples reveal a similar wave field pattern. However, when the dome is absorbent the wave amplitude suffers consecutive attenuations each time the waves reach the dome.

Conclusions

An MFS algorithm using fundamental solutions for an acoustic half-space has been implemented in order to model wave propagation inside a room with a dome configuration. This model permits absorption to be prescribed at the boundaries. The accuracy of the results depends on the distance between the fictitious sources

Rigid case (A) Dome with absorption (B)

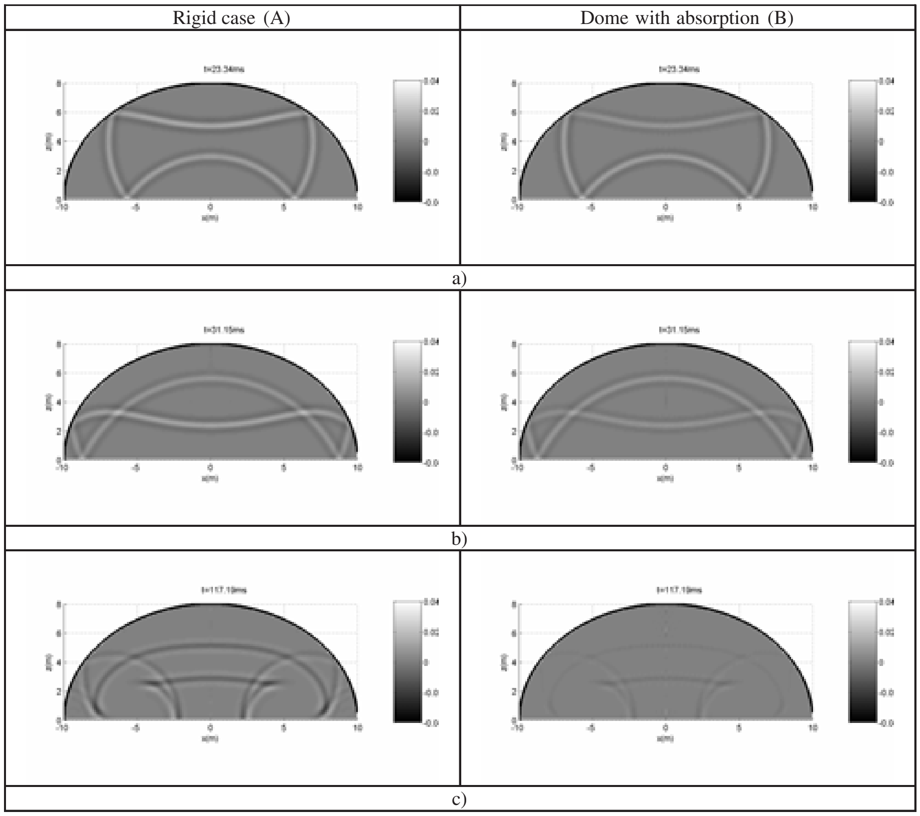

Figure 3: Time displacements for a characteristic frequency of 350 Hz, at a grid of receivers when the boundaries are rigid (left) and when the dome exhibits absorption (right): a) t=23.34 ms; b) t=31.15 ms; c) t=117.19 ms.

and boundaries and on the number of nodal points and sources. The adequate choice of these two parameters yields reliable results.

In a 3D problem, the dimensions of the room and the computation frequency range are limited by the number of nodal points and sources required since they define the dimension of the system to be solved.

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