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Extension of an Error Estimator Approach Based on Gradient Recovery for the Panel Method

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Summary

The use of computational methods in modern engineering is an irreversible trend, especially in the field of aerodynamics. The validation of these methods is an essential part of their development. In many situations this validation can be made by using experimental results. In other situations, particularly for potential flow methods, the validation can be made by using analytical results (Joukowski, Kármán-Trefftz and Van de Vooren airfoils, for example). In general situations, however, especially when no analytical solutions are available (for example, NACA airfoils series), it is highly desirable to know the errors due to the use of numerical methods in a different but systematic and reliable way. For this reason, estimators for the discretization error are an important tool in the development of computational methods, in order to obtain more general and robust applications. In this work, an extension of a local error estimator approach based on the gradient recovery procedure is presented, for linear elements, and compared with two error estimators, previously presented by the authors, for vortex-based Panel Methods.

Introduction

The Boundary Element Method is an example of a successful computational method, commonly used to obtain numerical solutions for potential flows around airfoils and wings. Earlier versions of the Boundary Element Method for this kind of problems, using the concept of singularities (sources, vortex and dipoles), were developed by the aerodynamic research community under the general denomination of Panel Methods, and are nowadays very disseminated in aerodynamic analysis and design [1]. In the aerodynamic and hydrodynamic fields, these methods have been matured to a level that allows their use as design tools on a routine basis. With suitable adaptations, the Panel Methods became useful and computationally efficient tools for analyzing the flow around bodies of general shape, including bluff bodies and turbomachinery cascades [2].

In the Panel Method, the domain governing equations and the boundary conditions are written in terms of an integral equation on the boundary only. The unknown function of this equation is a singularity distribution on the boundary. In this way, the dimension of the problem is reduced by one, i.e., to the boundary surfaces for tree-dimensional problems, or to the boundary contours for two-dimensional problems.

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The boundary is discretized into panels (flat boundary elements) and the singularity distribution is also discretized in each panel (for example, with constant or linear local distributions). In this way, the contour integral is reduced to a summation of integrals, evaluated in each panel. By enforcing the boundary condition in some collocation points the solution of the integral equation becomes the solution of a system of simultaneous linear equations, for some singularity distribution values.

The Panel Method used for testing the error estimators is based on linear panels and linear vortex distributions, with emphasis on the problem of cusped trailing edges. The method implements a consistent procedure for imposing the Kutta condition [1]. The procedure uses the concept of equality between the vortex density and the tangential velocity on the boundary and is useful for eliminating spurious aerodynamic loadings at the trailing edge region caused by an inconsistent imposition of the Kutta condition. However, the validation of the procedures can be made by using analytical results, using airfoils available in literature, such as the Joukowski airfoils used in this work.

As in other computational methods, the numerical solutions obtained by the Panel Methods are obviously approximated. For airfoil-like aerodynamic shapes, the estimation of the error distribution on the boundary is an important issue, often disregarded in the specialized literature. A technique for this estimation would help to predict and even to correct the numerical results in a selective manner and could be used in adaptive discretization schemes.

Error Estimator Methodologies

In the error estimator based on the gradient recovery procedure, the local error function is obtained, for each element, as a residual from differences between smoothed and non-smoothed rates of change of boundary variables in the local tangential direction. The procedure to obtain these residuals was presented in detail in [3] for a direct approach of the boundary element method applied to potential problems, and is extended in this work for an indirect approach called panel mehthod. Equation (1) presents the final form of an element residual, while Eq.(2) is used to scale the residual with respect to the element size.

$$r^2_{\phi^{(e)}}(\xi) = \left[\left(\frac{\partial \phi}{\partial s}(\xi) \right)^{(e)^*} - \frac{2}{L^{(e)}} \left[N'_1(\xi) N'_2(\xi) \right] \left\{ \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right\}^{(e)} \right]^2 \tag{1}$$

$$\varepsilon_{\phi^{(e)}} = \left[\int_{-1}^{1} r^2_{\phi^{(e)}}(\xi)\, d\xi \right]^{1/2} \frac{L^{(e)}}{L_{total}} \tag{2}$$

Two existing error estimators specific for the panel method are used in this work for comparison purposes. The Panel Method uses the concept of zero internal velocity as shown in [4] and [5]. The method for calculation of these velocities, however, is based on the *a posteriori* evaluation of the Cauchy Principal Value of the tangential velocity V_j in the collocation points j. A local error estimator, also called an "exterior" error, can be obtained as in Eq.(3a), as the difference between this Cauchy Principal Value and half of the vortex density $\bar{\gamma}_j$, previously calculated by the Panel Method, and given by Eq.(3b). This error estimator was implemented in [5], and used in this work for comparison purposes.

$$\text{(a)}\ \ \varepsilon_{ext_j} = V_j - \frac{\bar{\gamma}_j}{2} \quad \text{(b)}\ \ \bar{\gamma}_j = \frac{\gamma_j + \gamma_{j+1}}{2} \tag{3}$$

As discussed in [4] and [5], this error estimator is based on a constant potential inside the airfoil, wherein the correspondent velocity (potential derivative) must be zero. Thus, a non-zero velocity distribution calculated inside the airfoil is an error estimator. The proposed calculation, however, must be made at a certain distance of the collocation points, and not directly at these points. In this way, computational difficulties are found in the region near very sharp (cusped) trailing edges, such as those of Joukowski airfoils. Another aspect to be considered is the error increase produced by some classical panel methods at the region of a cusped trailing edge. Special treatments to impose the Kutta condition in a consistent manner were implemented in [2], and are used in the panel method code used in this work.

Another error estimator used for comparison purposes is given in Eq.(4), and is based on a curvature effect calculation known as a corrector for low order Panel Methods using vortex distributions [6]

$$\varepsilon_{curv_j} = \frac{\theta_j}{4\pi}\bar{\gamma}_j \tag{4}$$

where θ_j is the curvature angle covered by the panel j, which is also equal to the slope variation of the actual airfoil contour between the panel node points j and $j+1$. Details for the derivation of Eq.(4) can be found in [6].

Results and Conclusions

An exact local error is obtained comparing the numerical solution using the Panel Method to the analytical solution. The error estimators results for the gradient recovery approach is compared with the error results for the other two error estimators, which are specific for the panel method, and also with this exact error, as shown in Figures (1) and (2), where s is the natural coordinate of the airfoil (where $s = 0$ and $s = 1$ indicates the airfoil trailing edge). As shown in these figures, the error estimator plots indicate correctly the airfoil regions with higher errors, for all cases evaluated.

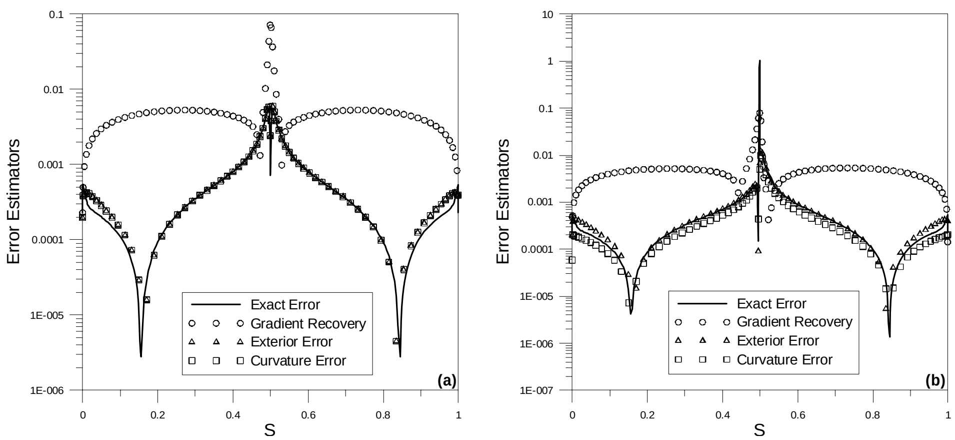

Figure 1: Joukowski airfoil without camber, for *n* = 300 linear panels, and for two different attack angles: (a) $\alpha = 0°$ and (b) $\alpha = 3°$.

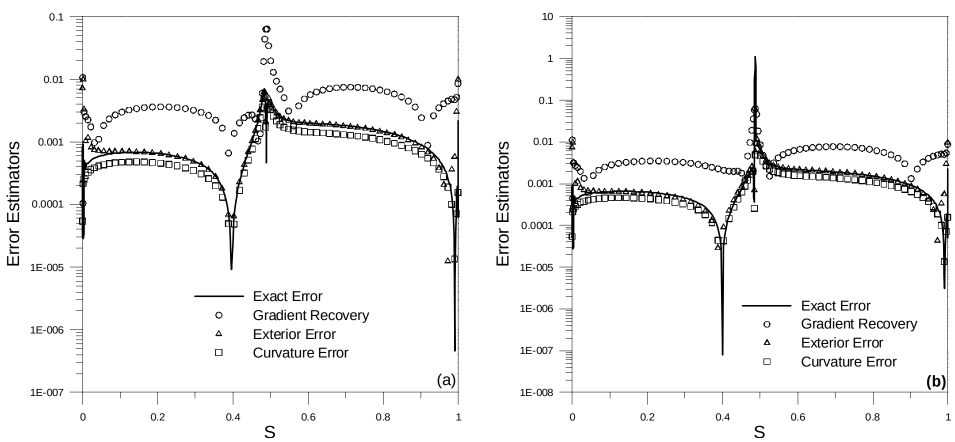

Figure 2: Joukowski airfoil cambered, panel number, for *n* = 300 linear panels, and for two different attack angles: (a) $\alpha = 0°$ and (b) $\alpha = 3°$.

In this work, several test cases were evaluated for Joukowski airfoils, discretized with 80, 150 and 300 panels, in order to test the influence of the number of panels in the discretization on the response of the error estimators. The influence of the angle of attack and cambering was also evaluated.

Figures (1) and (2) show the influence of a change in the angle of attack, for airfoils without and with camber, for the 300-panel mesh. The same cases were tested for different meshes, are not plotted here, as in all the cases tested, the local errors simply decreased with mesh refinement, while the error plots presented similar trends as shown in these figures. Thus, an increase in the panel number produces a somewhat uniform decrease of the error magnitudes around the airfoil contour, without essentially modifying the shape of the error distributions.

The error estimators based on the CPV and curvature effects, used in this work for comparison purposes, present results that are very similar and accurate for symmetrical airfoils, with a slight advantage for the curvature error estimator near the trailing edge, for cambered airfoils. These error estimators are specific for the panel method, while the gradient recovery error estimator, although not as accurate in the regions of small errors, presents the advantage to be a more general error estimator, that can be used regardless the order of the interpolation polynomial, or the panel method code being implemented.

In general, all error estimators were able to reasonably capture the shape and the magnitude of the exact error, particularly in the regions of maximum and minimum errors. On the other hand, the gradient recovery error estimator, in the airfoil regions with small errors, was not as efficient as the other error estimators analyzed. This behavior may be explained by the fact that the error estimators based on the Cauchy Principal Value of tangential velocities and on the curvature effect were developed specifically for panel method implement in this work, while the error estimator was based on a more general principle, and not tuned to this problem in particular.

The error estimators that were compared are well suited for the case of airfoils with cusped trailing edges, evaluated in this work. As a matter of fact, the region of higher errors was correctly found by all error estimators, in all test cases carried out. Also, the error estimators based on the panel method have correctly captured the shape and the magnitude of the actual error distribution around all the airfoil, particularly at the regions with small errors, where the gradient recovery approach was not as efficient, indicating a need for improvement in this procedure, to better capture these small errors.

An increase in the panel order and/or in the vortex distribution order is expected to give more accurate results and consequently to reduce the local error obtained with the methods implemented. The implementation of higher order panel methods, and their corresponding error estimators, is an ongoing research, currently in the test and validation phase.

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