 ICCES, vol.4, no.3, pp.171-177, 2007

Identification Problems in Metal Forming

Iwona Pokorska[1] and Andrzej Sluzalec[2]

Summary

This work has been focused on a unified approach for parameter identification in metal forming processes of poroplastic materials. In order to solve the associated inverse problem a chosen functional is minimized by the use of gradient based methods and a sensitivity analysis. Several numerical and experimental results were presented. These are: the direct problem of simple compression, re-identification of flow stress and identification of loading functions.

keywords: Metal Forming, Sensitivity, Rigid-Poroplasticity

Introduction

The purpose of this work is to present a unified strategy for parameter identification of poroplastic materials in the context of the finite-element method. Considering the solution of the inverse problem, various strategies exist in the literature. A common-classical-approach for solution of the inverse problem is to consider parameter identification as an optimization problem. In this respect a least-squares functional is minimized in order to provide the best agreement between experimental data and simulated data in a specific norm. In order to stabilize the numerical results it may be necessary to amend this basis function by a regularization term. In the context of identification for poroplastic material models we use model information of the specific material law. If we consider the parameter identification in the context of the finite element method, this approach is similar to procedures in shape optimization. In the corresponding terminology the material parameters are the design variables of the optimization problem.

Algorithms for solution of the resulting optimization problem may be classified into two classes i.e. methods which only need the value of the least-squares function and descent methods which require also the gradient of the least-squares function. This paper is structured as follows: In the first section the basic equations for the direct problem for modeling poroplastic material behavior are summarized. Next a short review for solution of the discretized direct problem in the context of the finite element method is presented. Finally a solution of inverse problem is described. The numerical examples of identification of material parameters in powder forming of poroplastic materials are presented in the last section.

Solution of the Inverse Problem

Inverse Problem

Denote by κ such material parameters as parameters describing the power law

[1]Technical University of Czestochowa, Poland. e-mail: pokorska@bud.pcz.czest.pl
[2]Technical University of Czestochowa, Poland. e-mail: sluzalec@k2.pcz.pl

of matrix material, parameters characterizing the relation between apparent yield stress of matrix material and base material and parameters characterizing the loading function for porous material which have to be found. Let $\bar{U}$ be the observation space. Since only incomplete data are available from the experiment, we introduce an observation operator W:$U \to \bar{U}$ mapping the displacement field $\mathbf{u}(\cdot,\kappa)$ to points of the observation space $\bar{U}$. The inverse problem of poroplasticity can be expressed to find κ such that

$$W\mathbf{u}(\cdot,\kappa) = \bar{\mathbf{u}} \tag{1}$$

for given $\bar{\mathbf{u}} \in \bar{U}$.

In solution of inverse problems in poroplasticity it is convenient to analyze the density field. Note that there exists a mapping W_2: $U_\rho \to \bar{U}_{\bar{\tilde{\rho}}}$ where $\bar{U}_{\bar{\tilde{\rho}}}$ is the observation space of relative densities and $\bar{U}_{\tilde{\rho}}$ is the space of relative densities and there exists W_3: $U_\rho \to \bar{U}_{\tilde{\rho}}$. Consider the following optimization problem

$$\begin{aligned} J(\kappa) &= \frac{1}{2}\left\|W\mathbf{u}(\cdot,\kappa) - \bar{\mathbf{u}}\right\|_{\bar{U}}^2 \to \min_{\kappa} \\ J_2(\kappa) &= \frac{1}{2}\left\|W_2\tilde{\rho}(\cdot,\kappa) - \bar{\tilde{\rho}}\right\|^2 \to \min_{\kappa} \end{aligned} \tag{2}$$

Denote the set of n_{mp} points, where experimental data are available by $\{x_i\}_{i=1}^{n_{mp}}$ at n_{tdat} time steps $\{t_j\}_{j=1}^{n_{tdat}}$. The total number of experimental data is $n_{dat} = n_{mp}n_{dim}n_{tdat}$, where dim is the dimension of space. The observation operators W and W_2 are defined by

$$\begin{aligned} W\mathbf{u}(\cdot,\kappa) &= \left\{u_1(x_i,t_j,\kappa),\ldots,u_{n_{\dim}}(x_i,t_j,\kappa)\ i=1,\ldots,n_{mp},\ j=1,\ldots,n_{tdat}\right\} \\ W_2\tilde{\rho}(\cdot,\kappa) &= \left\{\tilde{\rho}_1(x_i,t_j,\kappa),\ldots,\tilde{\rho}_{n_{\dim}}(x_i,t_j,\kappa)\ i=1,\ldots,n_{mp},\ j=1,\ldots,n_{tdat}\right\} \end{aligned} \tag{3}$$

Introduce the following definitions

$$\begin{aligned} \mathbf{u}_j(\kappa) &= \left\{u_1(x_i,t_j,\kappa),\ldots,u_{n_{\dim}}(x_i,t_j,\kappa)\ i=1,\ldots,n_{mp}\right\} \quad j=1,\ldots,n_{tdat} \\ \tilde{\rho}_j(\kappa) &= \left\{\tilde{\rho}_1(x_i,t_j,\kappa),\ldots,\tilde{\rho}_{n_{\dim}}(x_i,t_j,\kappa)\ i=1,\ldots,n_{mp}\right\} \quad j=1,\ldots,n_{tdat} \end{aligned} \tag{4}$$

for the simulated data and analogously for the experimental data $\bar{u}_j$ and $\bar{\tilde{\rho}}_j, j = 1,\ldots,n_{tdat}$, respectively. The least-squares problem (2) can be expressed as

$$\begin{aligned} J(\kappa) &= \frac{1}{2}\left\|\mathbf{u}(\kappa) - \bar{\mathbf{u}}\right\|_2^2 \to \min_{\kappa} \\ J_2(\kappa) &= \frac{1}{2}\left\|\tilde{\rho}(\kappa) - \bar{\tilde{\rho}}\right\|_2^2 \to \min_{\kappa} \end{aligned} \tag{5}$$

It should be noted that sometimes the problem (5) which is well posed, may lead to numerical instability in solutions because the small variations of $\bar{\mathbf{u}}$ may lead to large variations of the parameters κ. These difficulties are caused, e.g. if the material model has to many parameters. A mathematical tool, suitable to overcome the above-mentioned numerical instabilities is a regularization of the functional in (5), and this leads to the more general problem

$$\begin{aligned} J(\kappa) &= \frac{1}{2}\left\|\mathbf{B}_{\delta_1}\left(u(\kappa)-\bar{u}\right)\right\|_2^2 + \frac{\alpha_1}{2}\left\|\mathbf{B}_{\mu_1}\left(\kappa-\bar{\kappa}\right)\right\|_2^2 \to \min_{\kappa} \\ J_2(\kappa) &= \frac{1}{2}\left\|\mathbf{B}_{\delta_2}\left(\tilde{\rho}(\kappa)-\bar{\tilde{\rho}}\right)\right\|_2^2 + \frac{\alpha_2}{2}\left\|\mathbf{B}_{\mu_2}\left(\kappa-\bar{\kappa}\right)\right\|_2^2 \to \min_{\kappa} \end{aligned} \tag{6}$$

where the matrices $B_{\delta_{1,2}}$ and $B_{\mu_{1,2}}$, the scalars $\alpha_{1,2}$ and the parameters $\bar{\kappa}$ can be chosen based on information or statistical investigations.

The errors appearing in the identification problem can be divided into two kinds. The first type of errors is addressed throughout statistical investigations. In this respect, when considering the maximum method, on the basis of sufficient experimental results a normal distribution with known variances leads to the first part of the function (6). The second one is addressed throughout the complexity of the model, thus decreasing the model error. In doing so, it should be realized, that the additional material parameters may also result into the aforementioned numerical instability for the identification process, if appropriate steps are not performed when planning the experiment. Finally we can summarize that the requirements have to be carefully balanced to obtain numerical stability of calculations and reducing the model error.

Examples

Simple Compression of Rigid-Poroplastic Material

Assume the flow stress of the matrix material for a rigid-plastic material model in the form

$$\bar{\sigma} = Y_b\left[1+(\gamma\bar{\varepsilon})\right]^n \tag{7}$$

The above model is known as a material model with power hardening. In the case of linear hardening one puts in the above expression n = 1.0. The friction between the specimen and the flat dies is modeled by friction factor. Consider the simple compression, where a cylindrical sintered P/M perform is compressed between two flat dies. The following characteristics of forming process are assumed. The initial diameter-to-height ratio of the cylinder is 1.2 and the original height used is unity for calculation. The perform has uniform initial relative density of 0.800.The total reduction in height is 50%.

According to the symmetry of the workpiece, one quarter of the cylinder is employed for the analysis and is divided into 16 quadrilateral elements interconnected at 25 nodal points. Deformation is analyzed step-by-step with increments of 0.1% of the initial height of the cylinder. The simulations were carried out with friction

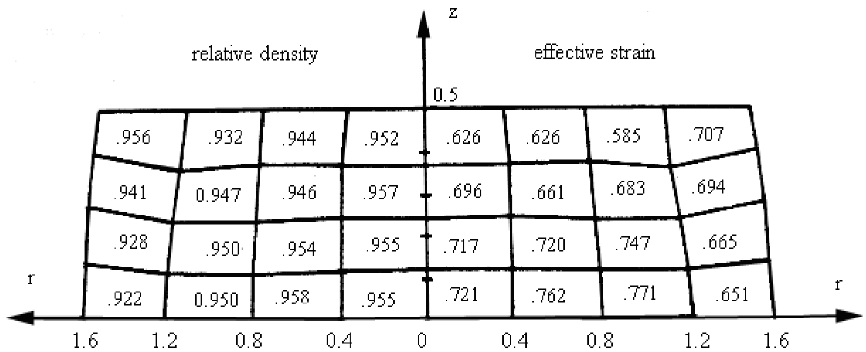

Figure 1: Distribution of relative density and effective strain for reduction in height 50%.

factor m = 0.1. The flow stress of the matrix material was assumed to be expressed by (7) with Y_b = 110.0, γ = 0.768 and n = 1.0. Fig. 3.1 shows the predicted relative density distributions and effective strain distributions at 50% reduction in height. It is seen from At 50% reduction in height, the density is lowest at the equator of the side surface, and at this stage, the side surface barrels considerably. The higher mean stress at the equator results in the low density at this point. It is also noted that at 50% reduction in height, densification near the center of the die contact surface has been accelerated. This can be explained by the fact that the pressure near the z axis increases as radius-to-thickness ratio increases. It is well known that higher pressure near the axis can be achieved with higher friction when a thin disk is compressed between two flat dies.

Re-identification of Flow Stress of Matrix Material

The examples intend to test the optimization algorithm in case of parameter re-identification of flow stress of matrix material. The solution procedure was as follows. First direct problem of metal forming of a rigid-poroplastic material is solved with assumed material data of the matrix material with Y_b = 110.0, γ = 0.768 and n = 1.0. The loading function was assumed by the expression (2). The material parameters A and μ are assumed in the forms (3) and (4), respectively. The following parameters of the process were assumed: initial relative density 0.8000, friction factor 0.5, total number of nodal points 25, total number of elements 16, the diameter of the specimen 2.4 cm and height 2.0 cm. Deformation is analyzed step-by-step with increments of 0.1% of the initial height of the cylinder. In the first example the parameter γ has been identified. Other parameters were assumed as constant. The displacement field $\mathbf{u} = \mathbf{u}(\gamma)$ at 30% and 40% reductions in height was calculated for various values of parameter γ with value of step 0.01. Next the sensitivity $\partial \mathbf{u} \partial \gamma$ was calculated for γ as the design parameter. Then the re-identification problem has been solved based on the displacements obtained from the solution of the direct problem. As an objective function the following function is examined

$$J(Y_b, \gamma, n) = \sum_k (u_k - \bar{u}_k)^2 \rightarrow \min_{(Y_b, \gamma, n)} \tag{8}$$

The results of re-identification tests are given in Table 3.1.

The second example concerns the identification of parameter n in a rigid-plastic material model given by expression (7) with parameters for the direct problem assumed as follows: Y_b = 110.0, γ = 0.768 and n = 1.0. The procedure was the same as described above for the identification of parameter γ. The results are given in Table 3.2. As an example displacements in r-direction of the middle-outer point of the specimen in function of parameters γ and n are given in Tables 3.3 and 3.4.

It should be noted that one can identify simultaneously two material parameters in the power law of material model.

Table 1: Starting and obtained values in the re-identification process for the material parameter γ in a rigid poroplastic material law of matrix material

parameter	starting	obtained
Y_b	110.0	110.0
n	1.0	1.0
γ	1.0	0.768

Table 2: Starting and obtained values in the re-identification process for the material parameter n in a rigid poroplastic material law of matrix material

parameter	starting	obtained
Y_b	110.0	110.0
n	1.5	1.0
γ	0.768	0.768

Table 3: Examples of displacements in r-direction of the middle-outer point of the specimen in function of parameter γ (reduction 30% in height)

parameter γ	0.6	0.7	0.8	0.9	1.0	1.1
displacement [cm]	1.3921	1.3917	1.3914	1.3911	1.3908	1.3905

Table 4: Examples of displacements in r-direction of the middle-outer point of the specimen in function of parameter n (reduction 30% in height)

parameter n	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6
displacement [cm]	1.3918	1.3915	1.3912	1.3910	1.3907	1.3904	1.3902	1.3899

Concluding Remarks

Powder metallurgy has evolved into a manufacturing technique for producing high performance components economically in metalworking industry because of its low manufacturing cost compared to conventional metal forming processes. The development and solution of metallurgical and mechanical problems of powder forming processes seems recently to be very important. This work has been focused on a unified approach for parameters identification of P/M specimens in powder forming processes. The first step was to describe the numerical solution of the direct problem. In order to solve the associated inverse problem a chosen functional is minimized by the use of gradient based methods and a sensitivity analysis. From the computational standpoint it follows that the determination of the gradient can be performed to the step-by-step solution of the direct problem. Several numerical and experimental results were presented. These are: the direct problem of simple compression of rigid-poroplastic material, re-identification of flow stress of matrix material, determination of apparent yield stress of poroplastic as well as the identification of loading functions. The experimental studies which in particular were the basis for the subsequent identification were conducted on unique testing machines.

The concept proposed in this work is a flexible approach for identification of mentioned above poroplastic material models and can be useful to apply the results to metalworking industry. The information derived can be used for the subsequent quantitative design as well as optimization of the powder metallurgy processes.

References

1. Kysiak A., Pokorska I., RIGID-PLAST – Rigid-plastic materials analysis – computer code, Civil Engineering Department, Technical University of Czestochowa, 1994.

2. Lee T. H., Arora J. S., A computational method for design sensitivity analysis of elasto-plastic structures, Comput. Methods Appl. Mech. Eng., 122, 27-50, 1995.

3. Lee T. H., Arora J. S., Kumar V., Shape design sensitivity analysis of viscoplastic structures, Comput. Methods Appl. Mech. Eng., 108, 237-259, 1993.

4. Pokorska I., An analysis of sensitivity of metal forming process for rigid-poroplastic materials, PhD Dissertation, Technical University of Czestochowa, 2002.

5. Pokorska I., Fundamentals of sensivity analysis of porous materials, Bull. Technical University of Czestochowa, 11, 80-86, 2004.

6. Pokorska I., Koniecko M., Kowalik M., An analysis of density changes of porous material, Bull. Technical University of Czestochowa, 7, 49-57, 1997.
7. Pokorska I., Kubicki K., Kysiak A., Design Sensitivity of Structures, Bull. Technical University of Czestochowa, 12, 59-65, 2004.
8. Pokorska I., Material derivative in problems of optimization and sensitivity, Bull. Technical University of Czestochowa, 11, 172-179, 2004.
9. Pokorska I., Material hardening in porous lightweight structure undergoing thermomechanical loading, Lightweight Structures in Civil Engineering, International Seminar of IASS Polish Chapter, Warsaw , 175-179, 2004.
10. Pokorska I., Modelling of isotropic and kinematic hardening in porous materials, Bull. Technical University of Czestochowa, 10, 143-156, 2004.
11. Pokorska I., Models of rigid-plastic porous building materials, Bull. Technical University of Czestochowa, 10, 137-142, 2004.
12. Pokorska I., Paluszyński J., Thermomechanics of porous materials undergoing finite deformations, Bull. Technical University of Czestochowa, 7, 29-40, 1997.