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Time Dependent Cyclic Constitutive Model and its Application to Some Geotechnical Problems

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Summary

The viscoplastic constitutive relations with kinematic strain hardening-softening model for geomaterials are developed. The constitutive models include strain-rate dependent properties based on the thoery by Duvaut-Lions. The dynamic relaxation method for static problems and the dynamic analysis for earthquake responses are applied to boundary value problems using finite element methods.

Introduction

The viscoplastic kinematic hardening model is developed. This model is based on the modified and extended soil model of isotropic strain hardening and softening elasto-plastic constitutive equation. The constitutive model is applied to the boundary value problems such as cyclic behavior of geotechnical problems. The explicit type dynamic relaxation method (Tanaka and Kawamoto, 1988) is used for the static cyclic retaining wall problem and the explicit dynamic response analysis is applied to the time integration of liquefaction problem of a buried pipe. The generalized return-mapping algorithm (Ortiz and Simo, 1986) is applied to the integration algorithms of viscoplastic constitutive relations. The return mapping algorithm is crucially important because the kinematic hardening model developed here has intersecting yield lines.

Elasto-Plastic Constitutive Model

The yield function (f) and the plastic potential function (Φ) are given by:

$$f = \alpha I_1 + \frac{\bar{\sigma}}{g(\theta_L)} = 0 \tag{1}$$

$$\Phi = \alpha' I_1 + \bar{\sigma} = 0 \tag{2}$$

where

$$\alpha = \frac{2\sin\phi}{\sqrt{3}(3-\sin\phi)} \quad \alpha' = \frac{2\sin\psi}{\sqrt{3}(3-\sin\psi)} \tag{3}$$

where I_1 is the first invariant (positive in tension) of deviatoric stresses and $\overline{\sigma}$ is the second invariant of deviatoric stress. With the Mohr-Coulomb model,

$g(\theta_L)$ takes the following form:

$$g(\theta_L) = \frac{3-\sin\phi}{2\sqrt{3}\cos\theta_L - 2\sin\theta_L\sin\phi} \tag{4}$$

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ϕ is the mobilized friction angle and θ_L is the Lode angle. The frictional hardening-softening functions expressed as follows were used:

$$\alpha(\kappa) = \{2\sqrt{\kappa\varepsilon_f}/(\kappa+\varepsilon_f)\}^m \alpha_p \ \text{ hardening-regime} \tag{5}$$

$$\alpha(\kappa) = \alpha_r + (\alpha_p - \alpha_r)\exp\{-(\kappa-\varepsilon_f)^2/\varepsilon_r^2\} \ \text{ softening-regime} \tag{6}$$

where κ is plastic parameter, m, ε_f and ε_r are the material constants (Yoshida et al. 1995) and α_p, α_r are the values of α at the peak and residual states.

The residual friction angle (ϕ_r) and Poisson's ratio (ν) were chosen based on the data from the test of air-dried Toyoura sand. The peak friction angle (ϕ_P) is estimated from the empirical relations based on the plane strain compression test and triaxial test on Toyoura sand. A dilatancy angle ψ is defined by Eq. (7) and Eq. (8).

$$\sin\psi = \frac{\sin\phi - \sin\phi_r'}{1-\sin\phi\sin\phi_r'} \tag{7}$$

$$\phi_r' = \phi_r[1-\beta\exp\{-(\frac{\kappa}{\varepsilon_d})^2\}] \tag{8}$$

where β and ε_d are material constants.

The kinematic hardening model considering the cumulative deformation by cyclic loading is developed. This is a modified and extended soil model of strain-hardening-softening property in order to take into account the cyclic behavior. Within bounding surface, plastic behavior is assumed and hardening modulus is much greater comparing the plastic behavior outside the bounding surface. The hardening function is given by Eq. (9). In this equation, κ' is plastic parameter and this parameter is cleared to zero at reversal point and a_f is material constant. The dilatancy angle ψ' is given by Eq. (10), and Eq. (11) and r_f in Eq. (11) is reduction factor for dilatancy.

$$\alpha_{iy}(\kappa') = a_f\left(\frac{2\sqrt{\kappa'\varepsilon_f}}{\kappa'+\varepsilon_f}\right)^m \alpha_p \tag{9}$$

$$\alpha_{id}'(\kappa') = (\alpha_{iy}-\alpha_p)r_f \tag{10}$$

$$\sin\psi' = \frac{3\sqrt{3}\alpha_{id}}{2+\sqrt{3}\alpha_{id}} \tag{11}$$

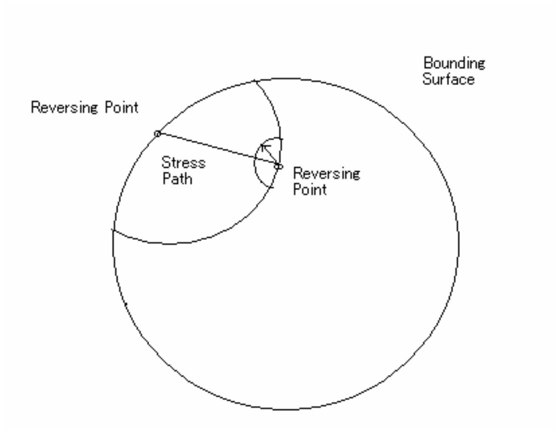

Figure 1: Kinematic hardening model on π plane (Mohr-Coulomb model takes pyramid shape)

Viscoplastic Constitutive Model

The constitutive equations of rate-dependent plasticity (Simo et al., 1988) originally proposed by Duvaut-Lions are as follows.

$$\sigma^{v} = \eta C \dot{\varepsilon}^{vp} = \sigma - \bar{\sigma} \tag{12}$$

$$\varepsilon_{B}^{p} = \varepsilon_{A}^{p} + \lambda(\partial\Phi/\partial\sigma \tag{13}$$

where η fluidity parameter, q is internal variables, $\overline{\sigma}$ and $\overline{q}$ are rate independent solution, C is elastic modulus. Eq. (12) can be rewritten in incremental form as follows.

$$\Delta\varepsilon^{ir} = \frac{\Delta t_{n+1}}{\eta}(\sigma_{n+1} - \bar{\sigma}_{n+1})C^{-1} \tag{14}$$

We can obtain the following equation.

$$\begin{aligned}\sigma_{n+1} &= \frac{\eta C \Delta\varepsilon_{n+1} + \eta\sigma_{n} + \Delta t_{n+1}\bar{\sigma}_{n+1}}{\Delta t_{n+1} + \eta} \\ &= \frac{\eta\sigma_{n+1}^{trial} + \Delta t_{n+1}\bar{\sigma}_{n+1}}{\Delta t_{n+1} + \eta}\end{aligned} \tag{15}$$

A great deal of experimental results indicates that the stress is a unique function of irreversible strain and its rate (Tatsuoka et al. 2002) and following the framework of the three component model, Tatsuoka et al. proposed the TESRA (temporary effect of strain rate and acceleration) model. We employed the simplified TESRA model for viscoplsticity.

Fig.2 shows a model experiment of retaining wall using air-dried dense Toyoura sand. In the Fig.3. H=19cm, H_1=4cm, H_2=9cm, H_3=14cm.The retaining wall

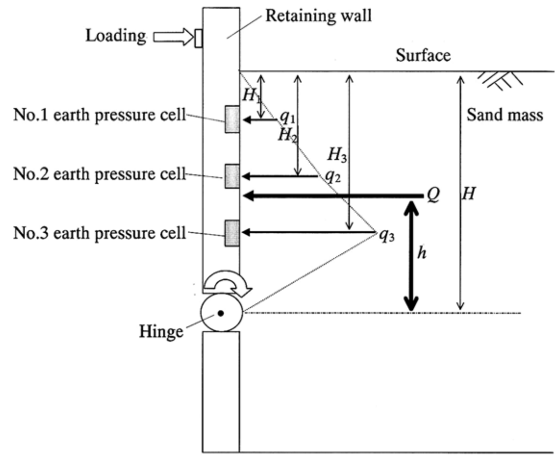

Figure 2: Retaining wall model using air dried sand

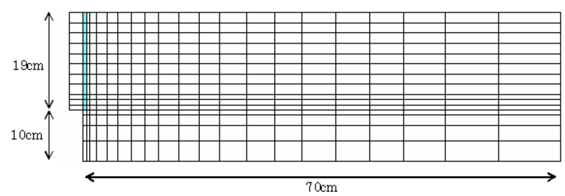

Figure 3: Finite element mesh of retaining wall

with a rough wall face was analyzed and the result of analysis was compared with the model experiment. The experiment was performed with the wall rotated about its bottom into the sand mass cyclically. The finite element mesh of retaining wall is shown in Fig.3. The wall friction angle (δ) employed in these analyses was assumed to be equal to the mobilized internal friction angle (ϕ) of the sand. Fig.4 shows observed relationship between the earth pressure of no.2 pressure cell and angle (θ) of wall rotation. Fig.5 shows the earth pressure and wall rotation relationships calculated by the simplified TESRA model.

Cyclic viscoplastic constitutive model was applied to dynamic analysis of two-dimensional buried pipe problem. Fig.6 shows the finite element mesh used for the analysis. The pipe was buried within a saturated sand layer with relative density 85%. Fig.7 shows the horizontal acceleration applied and Fig.8 shows observed and calculated displacements at the top of sand layer (center).

References

1. Tanaka, T. and Kawamoto, O., (1988), Three Dimensional Finite Element Collapse Analysis for Foundations and Slopes Using Dynamic Relaxation, in

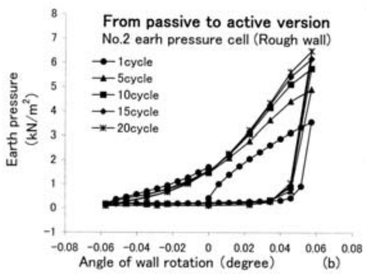

Figure 4: Observed relationships between earth pressure and wall rotation

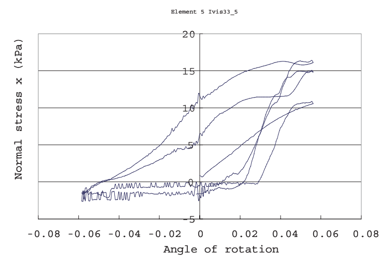

Figure 5: Calculated relationships between earth pressure and wall rotation

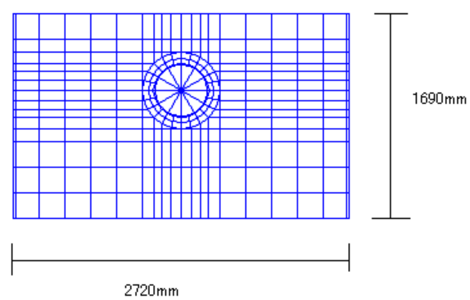

Figure 6: Finite element mesh of buried pipe

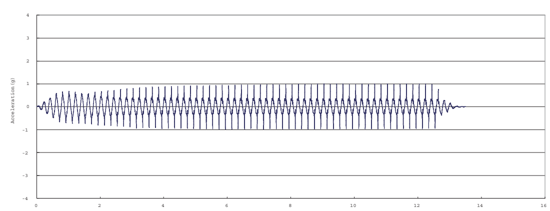

Figure 7: Input acceleration for buried pipe experiment

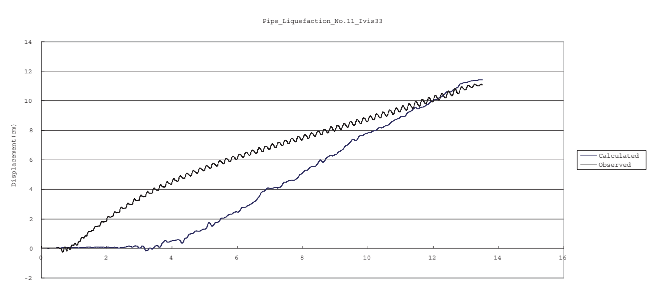

Figure 8: Calculated displacement at the top of sand layer

Proceedings of Numerical Methods in Geomechanics, Insbruch, 1213-1218.

2. Ortiz, M., Simo, J. C. 1986. An analysis of a new class of integration algorithms for elasto-plastic constitutive relations, Int. J. Numer. Methods Eng., 23, 353-366.
3. Yoshida, T., Tatsuoka, F., Siddiquee, M. S. A. and Kamegai, Y. (1995), Shear Banding in Sands Observed in Plane Strain Compression, Localization and Bifurcation Theory for Soils and Rocks, (Chambon et al., eds.), Balkema, 165-179.
4. Tatsuoka, F., Ishihara, M., Di Benedetto, H. and Kuwano, R. (2002): Time-dependent Shear Deformation Characteristics of Geomaterials and Simulation, Soils and Foundations, Vol.42, No.2, pp.103-129.