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Generalized Fatigue Model For Ploymer Matrix Composites

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Summary

A new fatigue model suitable for polymer matrix composites is proposed, based on dimensional analysis of testing and material variables involved in a fatigue test. The new model has a good physical and theoretical foundation based on progressive damage, and unifies all the material and testing parameters into one general equation. Especially, the model can be simplified to the Goodman relation or Gerber relation when capturing the effect of mean stresses. The model suggested can be used to substitute the classical S-N curves and reduce the testing number by 50% required by ASTM standards. A series of experiments have also been conducted to validate the model using E-Glass polyurethane materials along with literature results.

Introduction

It is well known that the characterization of fatigue behavior is a relatively complicated process, particularly for composite materials, which is actually involving the degradation of strength or modulus with the growth and coalescence of microvoids. Several studies in the literature addressed the modeling of fatigue process. Manson[1] and Coffin[2] studied the fatigue behavior of metallic materials, and found that the strain range in the low cycle region controlled the fatigue life. James et al.[3] investigated the dependence of stress and strain range on fatigue life of composite materials. Agarwal and James[4] studied the effect of stress ratio R on the fatigue of composites, and shown that the stress ratio had a strong influence on the fatigue life of composites and the micro matrix cracks were observed prior to gross failure of composites under both static and cyclic loading. Abd-Allah et al.[5] studied the effect of fiber volume fraction on the fatigue life of composites, and they concluded that the fatigue strength was proportional to the fiber volume fraction, which gives us a hint that the fatigue life could be related to the static stiffness or strength of the material. Mandell et al.[6]conducted the study on effect of matrix and fiber on the fatigue life of composite, and they concluded that more ductile resin systems provided better structural integrity in term of fatigue process and strength. Sun and Chan[7] discussed the influence of frequency on fatigue behavior of laminated composites, and their study showed that the fatigue life of composites was extended by the increased load frequency, provided that the temperature of the specimens

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remained unchanged. Piggott[8] also reported that the fatigue life of FRP composites was insensitive to the temperature increase at frequencies below 30 Hz. Ferreira et al.[9] investigated the influence of fiber orientation and loading mode on fatigue behavior of composites, and their study showed that the lay-up had a strong effect on fatigue strength while the displacement control mode slightly increased the fatigue strength of composites (less than 5%). Hwang and Han[10,11] used the residual stiffness as a parameter to describe the degradation process during the fatigue and to predict the fatigue life in graphite/epoxy composite laminates. Ellyin and El-Kadi[12] demonstrated that the strain energy density could be used as a fatigue failure criterion for FRP composites, and the fatigue life was related to the total strain energy of the samples using a power law fitting.

In this study, a combined analytical and experimental approach to characterize the fatigue of fiber-reinforced plastic composites is developed, and the factors considered include stress ratio, stress level, and frequency. First, a non-dimensional analysis is proposed to establish a prediction equation for fatigue life of composites. Then, a series of experimental tests of E-glass/polyurethane composites under cyclic loading are conducted, and based on the testing data and proposed prediction equation, the modified S-N curve and modified Goodman diagram are constructed to show the fatigue life of pultruded E-glass/polyurethane composites and better represented the experimental data compared to the classical S-N curves.

The New Fatigue Model

Non-Dimensional Analysis of variables

To develop a general model for fatigue life predictions of composites, two types of parameters, i.e., the testing and material related ones, should be considered. For an ideal coupon sample subjected to constant amplitude and constant frequency cyclic loading, the testing parameters, i.e., the stress amplitude (or stress ratio), maximum cyclic stress (or mean stress), and frequency, are included in the analysis; while the material parameters, such as the elastic Young's modulus, mass, and ultimate strength, are taken into account.

Based on the above parameters in fatigue modeling and the nature of dynamic effect on fatigue due to cyclic loading, a general model for the fatigue life (N) can be given as

$$N = F(\Delta\sigma, \sigma_{\max}, f, E, m, \sigma_u) \tag{1}$$

where $\Delta\sigma$ is the stress amplitude, and $\sigma_{\max}$ is the maximal cyclic stress; f is the frequency of applied cyclic loading; E is the elastic Young's modulus; m is the mass of the composite material; and σ_u is the uniaxial tensile strength of the composite material. In this study, a global characterization approach was used, in which the laminates were tested under the tension-tension uniaxial fatigue loading, and their mechanical responses (e.g., stress and strain) along the loading direction under cyclic loading were measured. The effects of fiber volume fraction, composite lay-up, and micro-level failure mechanism were not studied. Thus, the initial Young's modules E_0 along the tensile loading direction is chosen as the Young's modules (E), and the static uniaxial tensile strength σ_{u0} is taken as the uniaxial tensile strength (σ_u) of the composite material, assuming that the dynamic strength under low frequency is very close to the static strength.

Normalizing Eq. (1) with respect to the material parameters, the fatigue life is expressed as a function of non-dimensional factors as

$$N = F(\frac{\Delta\sigma}{\sigma_u}, \frac{\sigma_{\max}}{\sigma_u}, \frac{\rho A f^2}{E}) \tag{2}$$

where ρ is the density of composite material, and A is the cross-section area of the specimen.

Sendeckyj[13] suggested that the residual strength after the constant amplitude cycles was related to the initial static strength (σ_{u0}) by a deterministic equation. Similar to the one by Caprino and D'Amore[14], the proposed fatigue model in this study begins with a deterministic equation for the rate of strength degradation with inclusion of non-dimensional factor as,

$$\frac{d(\sigma_r/\sigma_u)}{dn} = -C_1 n^{-m_1} \tag{3}$$

where σ_r is the residual strength after n cycles; C_1 and m_1 are the material constants; and n is the number of fatigue cycles.

Based on the results postulated by Sendeckyj[13] and Hetrzberg and Manson[15],C_1 can be formulated in term of non-dimensional factors as

$$C_1 = a(\frac{\Delta\sigma}{\sigma_u})^r(\frac{\rho A f^2}{E})^{m_2} \tag{4}$$

where a, r and m_2 are the non-unit constants.

Integrating Eq. (3) and considering the corresponding fatigue boundary conditions (i.e., $n = 1$, $\sigma_r = \sigma_u$, and $n = N$, $\sigma_r = \sigma_{\max}$), the fatigue life are obtained in terms of non-dimensional factors as

$$N^\beta - 1 = \frac{1}{\alpha}(\frac{\Delta\sigma}{\sigma_u})^{-r}(\frac{\rho A f^2}{E})^{-m_2}(1 - \frac{\sigma_{\max}}{\sigma_u}) \tag{5}$$

where γ and m_2 are the material constants; and the non-unit constants α and β are

$$\alpha = \frac{a}{1 - m_1}, \quad \beta = 1 - m_1$$

Note that the effects of the testing parameters, particularly, the frequency, and material properties are included in Eq. (5).

Now considering the general fatigue model proposed in Eq. (5) and applying the log transformation over both sides of the equation, we can obtain

$$Log(N^{\beta}-1)+Log\alpha+\gamma Log(\frac{\Delta\sigma}{\sigma_u})+m_2 Log(\frac{\rho A f^2}{E})=Log(1-\frac{\sigma_{\max}}{\sigma_u}) \tag{6}$$

The constants for the given influential parameters (i.e., stress amplitude, frequency, and maximum stress) can be respectively obtained, and their corresponding effects can be analyzed.

Implication of Fatigue Model

The so-called "Goodman Line"[15] is commonly used to describe the effect of mean stress on fatigue strength. The fatigue strength for a given cyclic life time depends on the cyclic load amplitude (σ_a) and the mean stress (σ_m) (Figure 1). Thus, the classical Goodman relationship is expressed as

$$\frac{\sigma_a}{\sigma_N}=1-\frac{\sigma_m}{\sigma_u} \tag{7}$$

where σ_N is the fatigue strength under the cyclic loading with the stress ratio of R = -1.0.

By substituting $\sigma_a=\Delta\sigma/2$ and $\sigma_m=\sigma_{\max}-\Delta\sigma/2$ (see Figure 1) into Eq. (7), the Goodman relationship thus becomes

$$\frac{\Delta\sigma}{\sigma_u}(\frac{\sigma_u}{2\sigma_N}-\frac{1}{2})=1-\frac{\sigma_{\max}}{\sigma_u} \tag{8}$$

To compare the proposed model in Eq. (5) with the classical Goodman Line equation (i.e., Eq. (7) or (8)) and further simplify it to approximate the classical S-N curves, a constant frequency is applied (i.e., f is a constant) and $\gamma = 1$ is taken. For each given fatigue life N (e.g., 10^3, 10^5 ...), Eq. (5) is then reduced to

$$B(\frac{\Delta\sigma}{\sigma_u})=1-\frac{\sigma_{\max}}{\sigma_u} \tag{9}$$

where $B=\alpha\left(\frac{\rho A f^2}{E}\right)^{m_2}(N^{\beta}-1)$. Eq. (9) indicates that the proposed model of Eq. (5) has a similar form as the classical Goodman relationship; however, the present model incorporates the three testing parameter effects (i.e., stress amplitude, frequency, and maximum stress) into one equation (Eq. (5) or Eq. (9) for the modified Goodman Line). Thus, the proposed model represents a more generic and improved one compared to the classical S-N curve in the literature, and it has a clear physical meaning on fatigue life prediction.

Eq. (9) is further written as

$$B'(\frac{\sigma_a}{\sigma_N}) = 1 - \frac{\sigma_m}{\sigma_u} \tag{10}$$

where $B' = (2B+1)\frac{\sigma_N}{\sigma_u}$. Thus, for a given fatigue life (N), $\frac{\sigma_a}{\sigma_N}$ is linearly related to $\frac{\sigma_m}{\sigma_u}$.

Now including the term of fatigue life (N) in the model, Eq. (9) is rewritten as

$$(\frac{\Delta\sigma}{\sigma_u})C(N^\beta - 1) = 1 - \frac{\sigma_{\max}}{\sigma_u} \tag{11}$$

where $C = \alpha\left(\frac{\rho A f^2}{E}\right)^{m_2}$. Considering the stress ratio $R = \sigma_{\min}/\sigma_{\max}$ (Figure 1) and the following relationship

$$\frac{\Delta\sigma}{\sigma_u} = (1-R)(\frac{\sigma_{\max}}{\sigma_u}) \tag{12}$$

Eq. (11) thus becomes

$C(N^\beta - 1)(1-R)(\frac{\sigma_{\max}}{\sigma_u}) = (1 - \frac{\sigma_{\max}}{\sigma_u})$ or $C(N^\beta - 1)(1-R) = (\frac{\sigma_u}{\sigma_{\max}} - 1)$ (13)

Eq. (13) has a clear physical meaning. For R = -1, the fatigue strength $\sigma_N = \sigma_{\max}$. By substituting $\sigma_{max} = \sigma_N$ and R= -1 into Eq. (13), we could find

$$C(N^\beta - 1) = \frac{\sigma_u}{2\sigma_N} - \frac{1}{2} \tag{13}$$

Considering the relationship given in Eq. (13) which is derived from Eq. (5) and comparing it with Eqs. (7) and (8), we can conclude that the proposed model (Eq. (5)) is coincident with the classical Goodman Line equation (Eq. (7)) at R = -1 (i.e., along the vertical axis of the Goodman Line diagram as illustrated later).

Similarly, the following relationships exist for R = 1,

$$\sigma_m = \sigma_u; \quad \Delta\sigma = 0 \tag{14}$$

where σ_N and σ_u are the constants for each Goodman line of a given material. The conditions in Eq. (15) represent the horizontal axis of the Goodman Line diagram as indicated later.

In summary, the present model (Eq. (5)) can be simplified to the classical Goodman Line model based on the above derivations. However, the proposed model has a strict physical and mathematical basis compared to the classical Goodman line model. Also it is noticed that if $r = 1/2$, we could obtain the Gerber formula[15]. Thus, the present fatigue model represents a more generic and improved fatigue model, of which the effects of frequency (f), load level (i.e., the maximum cyclic stress or mean stress), and stress ratio (R) (or stress amplitude) are included. The proposed model is used next to study the effects of stress ratio, frequency, and maximum cyclic stress to the fatigue life prediction of newly developed E-glass/polyurethane composites and other common polymer matrix composites. Further, it is interesting to observe that the proposed model can dramatically reduce the fatigue test samples required by the ASTM standard[17], as illustrated later in the experimental section.

Experimental Validation

In this section, the experimental tests of pultruded E-glass/polyurethane composites were conducted. The experiments were designed to provide sufficient data to establish the dependence of low cyclic fatigue life on the experimental parameters and validate the model proposed in Eq. (5). To avoid the effect of hysteretic heating of the specimen, the fatigue tests were performed at the low frequency range (f = 1, 3, and 5 Hz) using a servo-hydraulic MTS machine. Both the stress range and the number of cycles to failure were recorded for all the tests.

Materials

The material used for this study was the E-glass/polyurethane composites manufactured by the pultrusion process. The lay-up of the material is [CSM/0/90/0/-90/0/CSM] including (1) two plies of Nexus veil stitched to 1 oz/ft^2 continuous strand mats (CSM), (2) two plies of [0/90] stitched fabrics (18 oz/yd^2), and (3) one unidirectional layer consisting of continuous glass roving of 113 yield (73 tows). The plate panels were cut with a saw-shear machine, and the coupon samples with dimensions of 25.4 x 254 x 3.175 mm ($1.0\times10.0\times0.125$ in) were obtained. These coupon samples were then polished, and the plastic tabs were bonded to the ends of each coupon using the epoxy adhesive.

The static behavior of the material was first established by testing five specimens using the ASTM standard and under the displacement control of loading (loading rate = 2.0 mm/min), and the stress-strain curves of the corresponding specimens were recorded and analyzed. The static experimental tensile Young's modulus (E_0) and strength (σ_{u0}) of the composite is given in Table 1. The common failure modes exhibited during the static tests included the fiber breaking and interlamina delamination.

Fatigue Testing Procedures

In this study, the effects of stress ratio, frequency, and maximum stress on fatigue life prediction of E-glass/polyurethane composites were investigated. To obtain the data necessary for establishing the relationship between the low cycle fatigue life and respective testing parameters, a range of test combinations with R = 0.05, 0.1, 0.5 and 0.9; f= 1, 3, and 5 Hz; and $\sigma_{\max}$ = 0.20, 0.50, and $0.80\sigma_{u0}$ was conducted. A load control mode was adopted, and the fatigue failure is defined as

Table 1: Experimental tensile stiffness and strength of E-glass/polyurethane composites

Specimen No.	Young's Modulus (GPa)	Tensile Strength (MPa)
1	23.17	689.48
2	33.16	829.00
3	29.38	758.42
4	30.13	749.34
5	28.29	715.23
Mean Values	28.83 (COV = 12.64%)	748.29 (COV = 7.06%)

the complete breaking/separation of the samples. For the classical S-N model to the proposed test combination, a total of 12 S-N curves is needed corresponding to the combined frequency (f) and load ratio (R), resulting in a total number of 72 tests if a minimum number of six samples is used for each curve. Per ASTM D 3479 requirements (see Table 1 in Ref. 20), the minimum number of specimens required for each S-N curve is six for preliminary and exploratory study, 12 for research and development testing, and 24 for design allowable and reliability data. Based on the present model, of which the testing parameters (e.g., the frequency and the load ratio) are interrelated as shown in Eq. (5), all the tests under different frequencies and load ratios can be used to obtain a particular S-N curve, which reduces the total number of tests to 36. Thus, a significant of time and effort could be saved compared to the classical S-N model using more test samples.

The experimental process involved: (1) let the loading ramped to the mean stress (the stress set point); (2) let the load ran cyclically to the full amplitude, and count the transition cycle numbers; and (3) monitor the peak and valley values for the test, and record the instance of 2% deviations. A total of 36 samples were tested, and their fatigue life (N) under a given testing parameter combination was recorded. No significant temperature increase was observed during the fatigue experiments, indicating that the temperature effect on fatigue life could be excluded.

Results and Discussions

In this section, the experimental fatigue data of E-glass/polyurethane composites were analyzed using the proposed model, and the effects of stress ratio, frequency, and mean stress on the fatigue life were discussed and compared with the classical S-N curves. The master fatigue design diagram based on the proposed model was established for E-glass/polyurethane composites and compared with the Goodman Line model. Degradation of tensile Young's modulus and fatigue failure modes were analyzed. Finally, the bounds for the S-N curves based on the statistical analysis were provided.

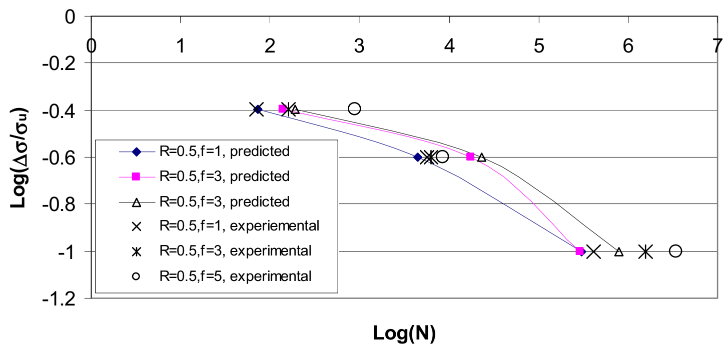

Figure 1: Comparisons of predicted S-N curves with testing data for E-glass/polyurethane composites

The modified S-N Curves

As expected and also indicated in Eq. (6) and Figure 1, the larger the stress ratio, the longer the fatigue life proceeds[16]. Also from Eq. (6) and Figure 1, the increased frequency extends the fatigue life of the composites as expected.

Modified Goodman Line

Besides the effects of stress ratio and frequency, the fatigue life is also dependent on the mean stress at any given load amplitude. The effect of mean stress can be described by the modified Goodman Line concept as suggested by the present model. Based on the experimental data, the modified Goodman master diagram is developed and given in Figure 2 for $f = 1$ Hz. As shown in Figure 2, the fatigue life of E-glass/polyurethane composites decreases as the mean stress increases. Similarly, as the stress amplitude increases, the fatigue life is correspondingly reduced.

Comparison the present model with the classical S-N curves using the existed database

As an extended application, the proposed fatigue model (Eq. (5)) is applied to analyze the existing available data in the literature. The proposed model is also used to fit the experimental data in the S-N curves, and the experimental fatigue life and the material constants in Eq. (5) are given in Table 2 for four different E-glass fiber-reinforced plastic composites. In order to compare with the classical S-N curve, the 4-parameter non-linear model (Eq. (5)) developed in this study is used to predict the fatigue life N and the corresponding stress amplitude, of which the predicted S-N curve is obtained. Compared to the classical S-N curve (the linear experimental data) fitting, the proposed model (the predicted results) captures the trend of S-N data from the experiments and provides a consistent prediction of fatigue life (see Figures 2 to 5). Besides capturing the effects of stress ratio (or stress amplitude) and maximum stress (or mean stress), the proposed model is

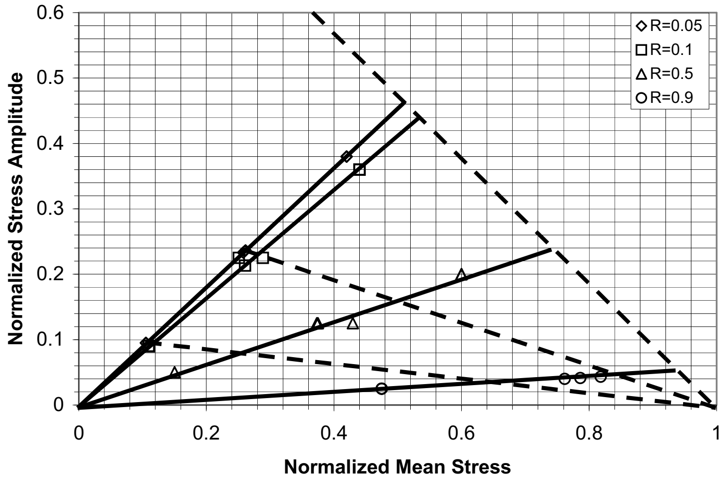

Figure 2: Modified Goodman Line diagram of E-glass/polyurethane composites for $f = 1$ Hz (both the mean stress and the stress amplitude are normalized by the ultimate static strength)

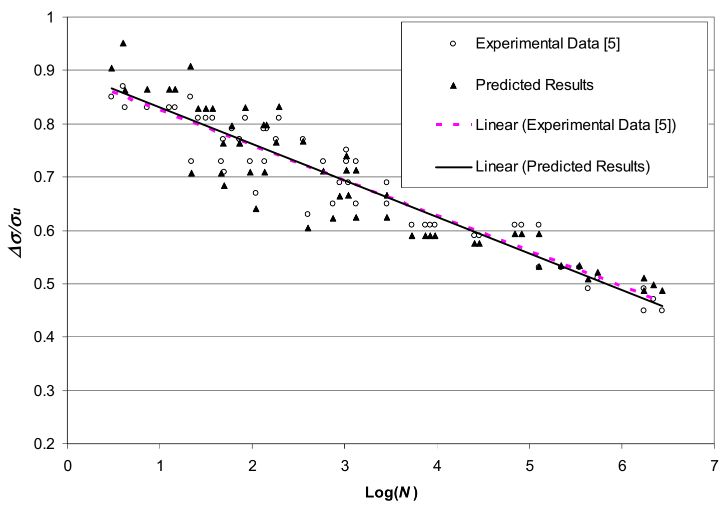

Figure 3: Comparison of S-N curves for E-glass/epoxy composites [5]

capable of studying the influence of frequency, as illustrated before for pultruded E-glass/polyurethane composites in this study. Compared to the model proposed by Epaarachchi and Clausen[16], the effect of frequency was included, but the reason for changing the integration from the frequency domain to the time domain was not given in their study. While compared to the model provided by Caprino and D'Amore[14], the proposed model can be applied to analyze the fatigue of general composites with different material architecture and under different loading condi-

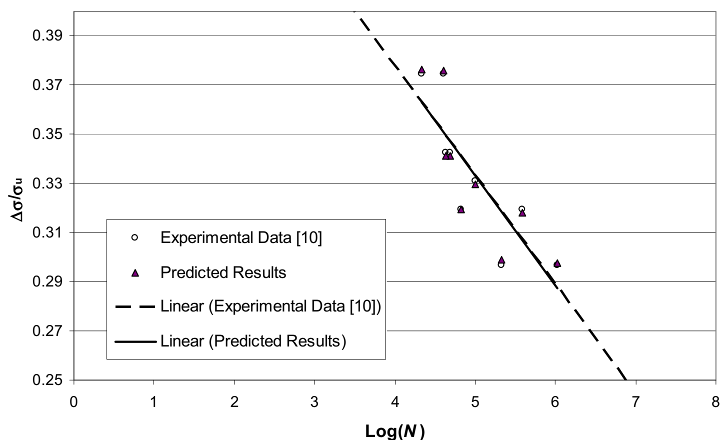

Figure 4: Comparison of S-N curves for E-glass/polypropylene composites [10]

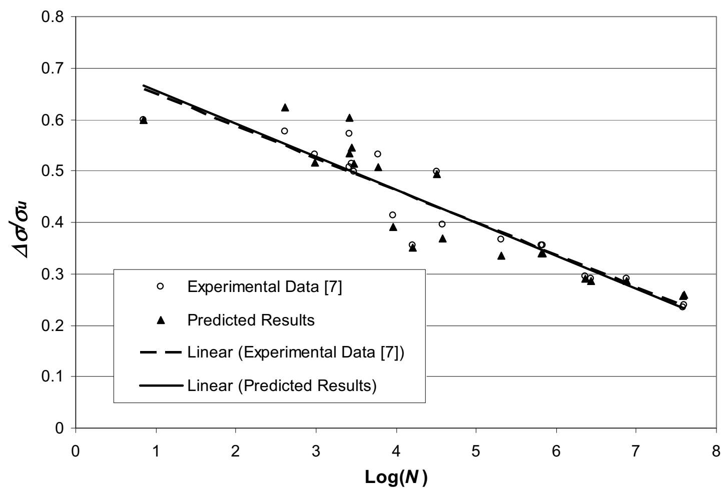

Figure 5: Comparison of S-N curves for E-glass/vinyl ester composites [7]

tions. Hence, the fatigue model proposed is an improved and generic model with a better predictability and clear physical meaning.

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References

1. Manson, S.S. (1965). Fatigue: A Complex Subject - Some Simple Approximations. *Experimental Mechanics*, 5(7): 193-226.

2. Coffin, L.F. (1954). A Study of the Effects of Cyclic Thermal Stresses on a Ductile Metal. *Transactions ASME*, 76(6): 931-950.

Table 2: Values of model parameters and fatigue life model

σ_u, *E*, ρ, *A*	Experimental parameters	Classical S-N Curve	Parameters in Eq. (5)	Loading	Reference
E-glass/Epoxy: Lay-up: Cross-plies $\sigma_u = 66000$ psi $E = 3.6 \times 10^6$ psi For $\sigma < 36000$ psi; $E = 2.7 \times 10^6$ psi for $\sigma > 36000$ psi $A = ¾ \times 1/8$ (in^2) $\rho = 2.05$g/cc*	$\sigma_{max} = 0.4 \sim 0.9\sigma_u$; $R = 0.05$; $f = 0.01 \sim 2.0$ Hz	$\Delta\sigma / \sigma_u = 0.934$ - 0.0815Log(N)	α = 0.32619 γ =-1.17606 $m_2 = 0$ β = 0.0026454 COC** = 0.908	Tension-Tension	$E = 3.6 \times 10^6$ psi is chosen [4]
E-Glass/polypropylene: Lay-up: Unidirectional $\sigma_u = 438$ MPa $E = 15{,}916$ MPa $A = 3.0 \times 30.0$ (mm^2) $\rho = 1.75$g/cc* $V_f = 0.338$	$R = 0.025$; $f = 10$ Hz; $\Delta\sigma = 130 \sim 164$ MPa	$\Delta\sigma / \sigma_u =$ 0.5552 - 0.0444Log(N)	α = 0.36821 γ =-0.53896 $m_2 = 0$ β =-0.0012194 COC** = 0.838	Tension-Tension	[9]
E-Glass/Vinyl ester: Lay-up: $[0]_5$ $\sigma_u = 581$ MPa $E = 21{,}000$ MPa $A = 25.0 \times 3.68$ (mm^2) ρ = *1.95 g/cc** $V_f = 0.27$	$R = 0.1$; $f = 0.1 \sim 15$ Hz; $\Delta\sigma = 270 \sim 430$ MPa	$\Delta\sigma / \sigma_u =$ 0.7905 - 0.0694Log(N)	α = 0.55027 γ = -0.72798 $m_2 = 0.028152$ β =-0.0067362 COC** = 0.939	Tension-Tension	[6]
E-Glass/Polyurethane: [CSM/0/90/0/90/0/CSM] $\sigma_u = 748.294$ MPa $E = 28{,}826$ MPa $A = 25.4 \times 3.175$ (mm^2) ρ = *1.85 g/cc*	$\sigma_{max} = 0.2 \sim 0.8\sigma_u$; $R = 0.05 \sim 0.9$; $f = 1 \sim 5$ Hz	$\Delta\sigma / \sigma_u =$ 0.8751 - 0.1428Log(N) or Log($\Delta\sigma / \sigma_u$) = 0.5196 - 0.3264Log(N)	α = 0.0269 γ = -0.367 $m_2 = -0.068$ β = 0.228 COC** = 0.773	Tension-Tension	Test data from the present study

* An estimated density is used, due to unavailability of the data in the reference.
** COC = the Coefficient of Correlation

3. James, T.K., Appl, F.J., and Bert, C.W. (1968). Low-cycle Fatigue of a Glass Fiber-reinforced Plastic Laminate. *Experimental Mechanics*, 8(10): 327-330.

4. Agarwal, B.D. and James, W.D. (1975). Prediction of Low-cycle Fatigue Behavior of GFRP: An Experimental Approach. *Journal of Materials Science*, 10(2): 193-199.

5. Abd-Allah, M.H., Abdin, E.M., Selmy, A.I. and Khashaba, U.A. (1996). Effect of Fiber Volume Fraction on the Fatigue Behavior of GRP Pultruded Rod Composites. *Composites Science and Technology*, 56(1): 23-29.

6. Mandell, J.F., Samborsky, D.D. and Scott, M.E. (2003). *DOE-MSU Wind Turbine Blade Composite Material Fatigue Database*. Department of Chemical Engineering, Montana State University at Bozman, Montana, USA.

7. Sun, C.T. and Chan, W.S. (1979). Frequency Effect on the Fatigue Life of a Laminated Composite. Composite Materials: Testing and Design (Fifth Conference), *ASTM STP 674*, 418-430.

8. Piggott, M.R. (1980). *Load-Bearing Fiber Composites*. Pergamon Press, New York, NY.

9. Ferreira, J.A.M, Costa, J.D.M, Reis, P.N.B., and Richardson, M.O.W. (1999). Analysis of Fatigue and Damage in Glass Fiber-reinforced Polypropylene Composite Materials. *Composites Science and Technology*, 59(10): 1461-1467.

10. Hwang, W. and Han, K.S. (1986a). Cumulative Damage Models and Multi-Stress Fatigue Life Prediction. *Journal of Composite Materials*, 20(2): 125-153.

11. Hwang, W. and Han, K.S. (1986b). Fatigue of Composites-Fatigue Modulus Concept and Life Prediction. *Journal of Composite Materials*, 20(2): 154-165.

12. Ellyin, F. and El-Kadi, H.A. (1990). A Fatigue Failure Criterion of Fiber Reinforced Laminates. *Composite Structures*, 15(1): 61-74.

13. Sendeckyj, G. P. (1990). Life Prediction for Resin Matrix Composite Materials. In: Reifsnider KL, editor, *Fatigue of Composite Materials*, Elsevier, Amsterdam.

14. Caprino, G. and D'Amore, A. (1998). Flexural Fatigue Behavior of Random Continuous-Fiber-Reinforced Thermoplastic Composites. *Composites Science and Technology*, 58(6): 957-965.

15. Hertzberg, R.W. and Manson, J.A. (1980). *Fatigue of Engineering Plastics*. Academic Press, New York, NY.

16. Epaarachchi, J.A. and Clausen, P.D. (2003). An Empirical Model for Fatigue Behavior Prediction of Glass Fiber-reinforced Plastic Composites for Various Stress Ratios and Test Frequencies. *Composites, Part A: Applied Science and Manufacturing*, 34(4): 313-320.

17. ASTM D3479. (1996). *Standard Test Method for Tension-Tension Fatigue of Polymer Matrix Composite Materials*. ASTM, PA.