 ICCES, vol.6, no.3, pp.175-181

Computational Modeling of Cracked Plates Repaired with Adhesively Bonded Composite Patches Using the Boundary Element Method

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Summary

The computational fracture analysis of cracked thick plates repaired with adhesively bonded composite patches using a boundary element formulation is presented. The shear deformable cracked isotropic plate was modeled using the Reissner's plate theory. In order to model the repair, a three parameter boundary element formulation, based on Kirchhoff's theory for symmetric layered composite plates was established. Interaction forces and moments between the cracked plate and the composite repair were modeled as distributed loads. Coupling equations, based on kinematic compatibility and equilibrium considerations for the adhesive layer, were established. In-plane shear-deformable model with transversal stiffness was considered in order to modeling de mechanical response of the adhesive. Stress intensity factors are evaluated from crack opening displacements. A problem considering circular composite repair is presented and results compared with those presented in the literature.

Introduction

Advanced adhesively bonded composite repairs have been used in the aeronautical industry and they are accepted as efficient solutions for the repair of damaged structures. The main advantage, when compared to repairs fixed by bolts or rivets is that bonded repairs supply a load transfer relatively uniform among the structural components. The required holes for these fasteners act as stresses concentrators that reduces the useful life of the aeronautical panel.

The Boundary Element Method (BEM) is an atractive numerical alternative to treat fracture problems, mainly to its ability to model continuously high stress gradients without the need of domain discretization. The use of this method in structural analysis has strongly increased since 80s (see Aliabadi [1]). The analysis of cracked isotropic bending plates structures repaired with the application of adhesively bonded anisotropic patches using BEM hasn't been reported in the literature, to the authors knowledge.

Cracked plate formulation

The two dimensional boundary integral equation for displacements at the bound-

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ary point $\mathbf{x}' \in \Gamma$ that describes membrane effects can be written as Aliabadi [1]:

$$c_{ij}^{P}\left(\mathbf{x}'\right)u_{\beta}\left(\mathbf{x}'\right)=\int_{\Gamma}U_{\alpha\beta}^{P}\left(\mathbf{x}',\mathbf{x}\right)t_{\beta}d\Gamma-\int_{\Gamma}T_{\alpha\beta}^{P}\left(\mathbf{x}',\mathbf{x}\right)u_{\beta}d\Gamma+\frac{1}{h_{p}}\int_{A}U_{\alpha\beta}^{P}\left(\mathbf{x}',\mathbf{x}\right)f_{\beta}dA \tag{1}$$

where α, β = 1, 2 and $c_{ij}^{P}(\mathbf{x}')$ is a function of the geometry at the collocation points that can be determinated by considering rigid body movements. The boundary displacements and tractions for the sheet are denoted by u_{α} and $t_{\alpha}(=n_{\beta}\sigma_{\alpha\beta})$, respectively; displacement and traction fundamental solutions for the plane stress condition are $U_{\alpha\beta}^{P}(\mathbf{x}',\mathbf{x})$ and $T_{\alpha\beta}^{P}(\mathbf{x}',\mathbf{x})$ respectively, $f_{\beta}(\mathbf{x})$ denote two-dimensional body forces por unit area over a region A of patch and h_p is the thickness of the plate. The upper index refers to the isotropic plate.

In order to model cracked plates, the Dual Boundary Element Method (DBEM) will be used. In this method, the displacement integral formulation is written for source points on one crack surface and the traction integral equation on the other surface. The traction integral equation for two-dimensional problems in a smooth boundary can be derived as (see Dirgantara [3]):

$$\begin{aligned}\frac{1}{2}t_{\alpha}\left(\mathbf{x}'\right) &= n_{\beta}\left(\mathbf{x}'\right)\int_{\Gamma}U_{\alpha\beta\gamma}^{P}\left(\mathbf{x}',\mathbf{x}\right)t_{\gamma}d\Gamma-n_{\beta}\left(\mathbf{x}'\right)\int_{\Gamma}T_{\alpha\beta\gamma}^{P}\left(\mathbf{x}',\mathbf{x}\right)u_{\gamma}d\Gamma\\ &+n_{\beta}\left(\mathbf{x}'\right)\frac{1}{h_{p}}\int_{A}U_{\alpha\beta\gamma}^{P}\left(\mathbf{x}',\mathbf{x}\right)f_{\beta}dA\end{aligned} \tag{2}$$

where $n_{\beta}(\mathbf{x}')$ is the normal to the boundary evaluated at collocation point. $U_{\alpha\beta\gamma}^{P}(\mathbf{x}',\mathbf{x})$ and $T_{\alpha\beta\gamma}^{P}(\mathbf{x}',\mathbf{x})$ are the displacement and traction fundamental solutions for two-dimensional problems.

If w_{α} are defined as rotations in the x_{α} direction, w_3 is the deflection of the plate along x_3, q_{α}^{P} and q_{3}^{P} are the distribution of moments and the out-of-plane body force per unit area, respectively, in the patch area A, and p_o is the pressure force applied in the domain of the plate Ω, the boundary integral formulation for the plate bending problem can be written as:

$$\begin{aligned}c_{ik}^{P}\left(\mathbf{x}'\right)w_{k}\left(\mathbf{x}'\right) &= \int_{\Gamma}W_{ik}^{P}\left(\mathbf{x}',\mathbf{x}\right)p_{k}d\Gamma-\int_{\Gamma}P_{ik}^{P}\left(\mathbf{x}',\mathbf{x}\right)w_{k}d\Gamma+p_{o}\int_{\Omega}W_{i3}^{P}\left(\mathbf{x}',\mathbf{x}\right)d\Omega\\ &+ \int_{A}W_{ik}^{P}\left(\mathbf{x}',\mathbf{x}\right)q_{k}^{P}dA\end{aligned} \tag{3}$$

where $k=1\ldots3$. $W_{\alpha\beta}^{P}(\mathbf{x}',\mathbf{x})$ and $P_{\alpha\beta}^{P}(\mathbf{x}',\mathbf{x})$ are the fundamental solutions for Reissner's plate model and $p_{\alpha}=M_{\alpha\beta}n_{\beta}$, $p_3=Q_{\beta}n_{\beta}$. Constant c_{ik}^{P} is similar with those at in-plane displacement problem.

In a similar way, fracture mechanics problems involving plate bending can be modeled using DBEM. In this case, the traction equation can be written as:

$$\begin{aligned}\frac{1}{2}p_i(\mathbf{x}') &= n_\beta(\mathbf{x}')\int_\Gamma W^P_{i\beta k}(\mathbf{x}',\mathbf{x})\,p_k d\Gamma - n_\beta(\mathbf{x}')\int_\Gamma P^P_{i\beta k}(\mathbf{x}',\mathbf{x})\,w_k d\Gamma \\ &+ n_\beta(\mathbf{x}')\,p_o\int_\Omega W^P_{i\beta 3}(\mathbf{x}',\mathbf{x})\,d\Omega + n_\beta(\mathbf{x}')\int_A W^P_{i\beta k}(\mathbf{x}',\mathbf{x})\,q^P_k dA \qquad (4)\end{aligned}$$

where $W^P_{i\beta\gamma}(\mathbf{x}',\mathbf{x})$ and $P^P_{i\beta\gamma}(\mathbf{x}',\mathbf{x})$ are the displacement and traction fundamental solutions for isotropic Reissner's plate (see Dirgantara [3]).

Composite patch formulation

Similary to the isotropic case, the in-plane displacements of a point $\mathbf{x}'$ in the anisotropic patch are given by:

$$c^R_{\alpha\beta}(\mathbf{x}')\,u^R_\beta + \int_{\Gamma_R} T^R_{\alpha\beta}(\mathbf{x}',\mathbf{x})\,u^R_\beta d\Gamma = \frac{1}{h_R}\int_A U^R_{\alpha\beta}(\mathbf{x}',\mathbf{x})\,f^R_\beta dA \qquad (5)$$

where $T^R_{\alpha\beta}(\mathbf{x}',\mathbf{x})$ and $T^R_{\alpha\beta}(\mathbf{x}',\mathbf{x})$ are the traction and displacements fundamental solutions for anisotropic plane elasticity ploblems and h_R is the repair thickness. Other variables have similar meaning to the isotropic case.

To model the bending response of the repair, a boundary integral formulation for Kichhoff's plate model with three unknows at every point is used in this work as presented by Palermo [4]:

$$\begin{aligned} w(\mathbf{x}') &+ \int_\Gamma \left[V_n(\mathbf{x}',\mathbf{x})\,w^R(\mathbf{x}) - M_n(\mathbf{x}',\mathbf{x})\,w^R_{,n}(\mathbf{x}) - T_s w^R_{,s}(\mathbf{x})\right] d\Gamma \\ &= \int_\Gamma \left[W(\mathbf{x}',\mathbf{x})\,v_n - W_{,n}(\mathbf{x}',\mathbf{x}) m_n(\mathbf{x})\right] d\Gamma + \int_A W_{,\alpha}(\mathbf{x}',\mathbf{x})\,q^R_\alpha dA \\ &+ \int_A W(\mathbf{x}',\mathbf{x})\,q^R_3 dA \qquad (6)\end{aligned}$$

where, $w(\mathbf{x})$ and $w_{,n}(\mathbf{x})$ are the bending deflexion and the normal rotation, respectively, $v_n(\mathbf{x})$ and $m_n(\mathbf{x})$ are the shear force and the normal moment, respectively, and $t_s(\mathbf{x})$ is the tangent moment. ${q_\alpha}^R$ and q^R_3 are distributed body moments and out-of-plane body force by unit area, respectively, generated by interaction with the adhesive layer. $W(\mathbf{x}',\mathbf{x})$, $V_n(\mathbf{x}',\mathbf{x})$, $M_n(\mathbf{x}',\mathbf{x})$, $T_s(\mathbf{x}',\mathbf{x})$ are the fundamental solutions for Kirchhoff's anisotropic plates. The upper index R refers to the composite repair.

A second boundary integral equation is obtained by differentiating Equation (6) with respect to point $\mathbf{x}'$ in the tangent direction:

$$w_{,s}+\int_{\Gamma}\left[V_{n,s}\left(\mathbf{x}',\mathbf{x}\right)w^{R}\left(\mathbf{x}\right)-M_{n,s}\left(\mathbf{x}',\mathbf{x}\right)w^{R}_{,n}\left(\mathbf{x}\right)-T_{s,s}w^{R}_{,s}\left(\mathbf{x}\right)\right]d\Gamma$$
$$=\int_{\Gamma}\left[W_{,s}\left(\mathbf{x}',\mathbf{x}\right)v_{n}-W_{,ns}\left(\mathbf{x}',\mathbf{x}\right)m_{n}\left(\mathbf{x}\right)\right]d\Gamma+\int_{A}W_{,\alpha s}\left(\mathbf{x}',\mathbf{x}\right)q^{R}_{\alpha}dA$$
$$+\int_{A}W_{,s}\left(\mathbf{x}',\mathbf{x}\right)q^{R}_{3}dA \tag{7}$$

Finally, a third integral equation can be obtained differentiating Equation (6) with respect to the normal direction:

$$w_{,n}+\int_{\Gamma}\left[V_{n,n}\left(\mathbf{x}',\mathbf{x}\right)w^{R}\left(\mathbf{x}\right)-M_{n,n}\left(\mathbf{x}',\mathbf{x}\right)w^{R}_{,n}\left(\mathbf{x}\right)-T_{s,n}\left(\mathbf{x}',\mathbf{x}\right)w^{R}_{,s}\left(\mathbf{x}\right)\right]d\Gamma$$
$$=\int_{\Gamma}\left[W_{,n}\left(\mathbf{x}',\mathbf{x}\right)v_{n}-W_{,nn}\left(\mathbf{x}',\mathbf{x}\right)m_{n}\right]d\Gamma+\int_{A}W_{,\alpha n}\left(\mathbf{x}',\mathbf{x}\right)q^{R}_{\alpha}dA$$
$$+\int_{A}W_{,n}\left(\mathbf{x}',\mathbf{x}\right)q^{R}_{3}dA \tag{8}$$

Coupling equations

Aditional equations can be written if displacement compatibility between plate and repair and the equilibrium conditions at the adhesive layer, are considered. The equilibrium of forces acting in the adhesive layer can be written as:

$$f^{P}_{\alpha}+f^{R}_{\alpha}=0$$

$$q^{P}_{3}+q^{R}_{3}=0$$

$$q^{P}_{\alpha}+q^{R}_{\alpha}+f^{R}_{\alpha}\left(h_{A}+\frac{h_{P}+h_{R}}{2}\right)=0 \tag{9}$$

Where h_A is the thickness of the adhesive. The shear force $\tau^{A}_{3\alpha}$, acting at the adhesive layer can be written as:

$$\tau^{A}_{3\alpha}=f^{R}_{\alpha}=\frac{\mu_{A}}{h_{A}}\left(u^{R}_{\alpha}-\frac{h_{R}}{2}w^{R}_{\alpha}\right)-\left(u^{P}_{\alpha}+\frac{h_{P}}{2}w^{P}_{\alpha}\right) \tag{10}$$

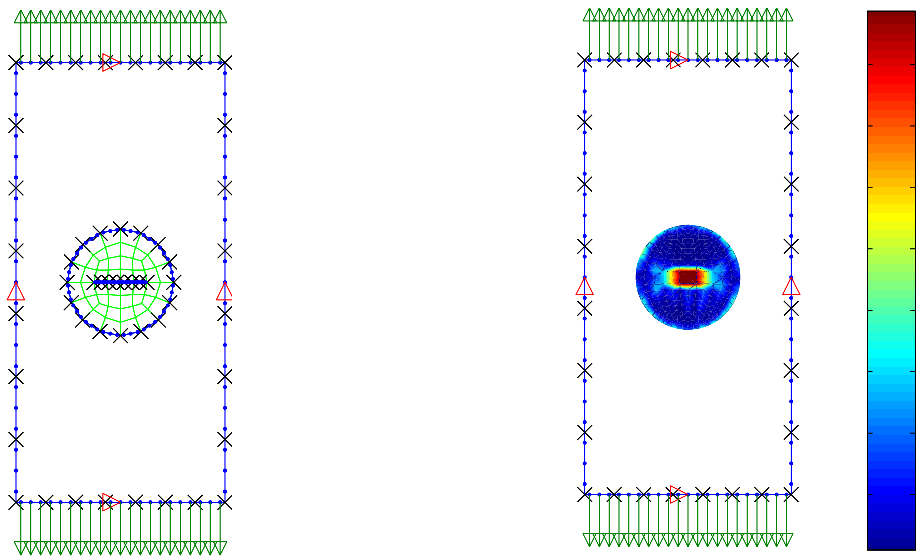

Figure 1: Left: Boundary element model for cracked isotropic plate repaires with composite circular patch. Right: shear stress distribution in the adhesive

where μ_A is the shear modulus of the adhesive. Finally, we can consider that deflexion and rotation angles at coincident points at plate and repair are related as:

$$q_3^P = E(w_3^P - w_3^R)$$

$$q_\alpha^P = C\left(w_\alpha^R + w_\alpha^P\right) \tag{11}$$

where, $E = 2\mu_A(1+\nu_A)/h_A$ is the transversal stiffness of the adhesive and $C = D(1-\nu_P)\lambda^2/2$ is the flexural bending stiffness of the isotropic plate. D is the bending stiffness of the cracked plate, ν_A and ν_P are the Poisson ratio for the adhesive and the isotropic plate, respectively, and $\lambda = \sqrt{10}/h_P$.

Numerical results

A rectangular cracked plate repaired with a bonded composite patch is analized. The plate is 248 mm × 118 mm, thickness h_P =2.0 mm and it is subject to a in-plane load σ_0 =79.4 MPa. The material constants of the cracked plate are chosen as E = 72.39 GPa, ν =0.33. A circular anisotropic patch of radius R = 25 mm and thickness h_R = 3.2 mm is bonded to the plate (see Figure 1). The mechanical properties are of patch are: E_1 = 11.38 GPa, E_2 = 37.35 GPa, G_{12} = 5.97 GPa and ν = 0.38. The adhesive layer has thickness h_a = 0.1 mm and shear modulus μ_a = 0.44 GPa. The same problem was analized by Sekine, Yan and Yasuho [5] where the cracked plate is modeled using a 3D BEM model and the repair using a finite element plate model.

Table 1: Stress intensity factors for cracked plate repaired with composite patch

z(mm)	$K_{Imax}(MPa.m^{1/2})$ **BEM**	$K_I(MPa.m^{1/2})$**-Ref.[5]**	Difference
0.40	12.92	12.60	2.54%
0.80	11.35	11.09	2.34%
1.20	9.33	9.52	1.99%
1.60	7.95	7.84	1.40%

A total of 28 quadratic discontinuous boundary elements were used to discretize the boundary of the isotropic cracked plate. Meshes from 4 to 16 quadratic discontinuous boundary elements were used to discretize the crack faces. Patch domain was discretized using 128 cells and 24 quadratic discontinuous boundary elements, see figure 1(left). Simply supported conditions were applyed to all sides. The resultant shear stress distribution in the adhesive layer is showed in figure 1(right).

Table 1 compares values for the maximum stress intensity factor: $K_{Imax} = K_{Im} + 6/h_P^2 K_I^b$ evaluated along plate thickness with those K_I reported by Sekine, Yan and Yasuho [5]. In this equation, K_{Im} represents the stress intensity factor in mode I for membrane response and K_I^b represents the stress intensity factor in mode I for bending response.

Conclusions

The analysis of cracked isotropic thick plates repaired with symmetrical laminate composite plates using the boundary element method, was presented. The equations for the repair is based on boundary integral formulation considering three parameters, based on the theory of Kirchhoff's plates as a generalization of the integral formulation of thin plates. The linear isotropic model proposed for the adhesive, considers shear forces and bending moments acting on it. This way, equations for kinematic coupling for displacements and rotations, as well as, a system of equations that describe the equilibrium of forces and moments that act on the adhesive, were stablished. Reasonable accuracy results were obtained when compared with those reported in the literature.

References

1. Aliabadi, M. H., (2003): *The Boundary Element Method: Applications in Solids and Structures, V.2*, John Wiley and Sons.

2. Dirgantara, T. and Aliabadi, M. H., (2001): "Dual boundary element formulation for fracture mechanics analysis of shear deformable shells", *International Journal of Solid and Structures*, Vol. 38, pp. 7769-7800.

3. Dirgantara, T., (2002): "Boundary Element Analysis of Cracks in Shear Deformable Plates and Shells", *Topics in Engineering, V. 43*, WIT Press.

4. Palermo Jr, L., (2003): "Plate bending analysis using the classical or the Reissner-Mindlin models", *Engineering Analysis with Boundary Elements*, Vol. 27, pp. 603-609.
5. Sekine, H., Yan, B. and Yasuho, T., (2005): "Numerical simulation study of fatigue crack growth behaviour of cracked aluminium panels repaired with a FRP composite patch using combined BEM/FEM", *Engineering Fracture Mechanics*, Vol. 72, pp. 2549-2563