 ICCES, vol.7, no.2, pp.83-87

On the Use of the Tangential Differential Operator in the Traction Boundary Integral Equation of the Dual Boundary Element Method for Three Dimensional Problems

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Summary

The differentiation of the kernels of integrals in the displacement BIE to obtain one for stresses increases the order of the kernel singularity and additional care are necessary to treat the improper integrals. The application of the tangential differential operator (TDO) can reduce the order of the kernel singularity when the stress BIE employs Kelvin type fundamental solutions. This paper presents the numerical formulation for the TDO to three-dimensional problems. The TDO uses the derivatives of the shape function for displacements instead of introducing another interpolation function. Furthermore, the paper shows the additional integrals for the TDO to be applied to non-conformal interpolations or when the boundary surface is piecewise smooth.

Introduction

The dual boundary element method (DBEM) [1, 2, 3] is one of the most efficient methods to analyze crack problems. The collocation point position to perform the traction BIE and the strategy used to treat the improper integrals are the essential features of the formulation. On the other hand, the differentiation of the kernels of integrals in the displacement BIE to obtain one for stresses increases the order of the kernel singularity and additional care are necessary to treat the improper integrals. The order of the kernel singularity can be reduced with the application of the tangential differential operator (TDO) when the stress BIE employs Kelvin type fundamental solutions [4, 5, 6]. The analyses of plane problems containing an internal or an edge crack employed the DBEM with the TDO in the traction BIE in [7]: conformal and non-conformal interpolations used the same shape function with the collocation points shifted to the interior of elements, which had the nodal parameters positioned at their ends. The adopted mesh on the crack surface employed conformal interpolations and the numerical implementation of the TDO used the derivatives of the shape function for displacements instead of introducing another interpolation function. The results obtained were close to values of the literature even considering that the traction BIE employed the TDO with low order elements. No significant changes appeared in the analysis presented in [8], which had only the collocation points of the displacement BIE positioned at the ends of the elements in the conformal interpolations.

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The TDO in the Traction Boundary Integral Equation

A simple algebraic manipulation is necessary to introduce the TDO in the kernel containing the higher singularity of the stress BIE. Starting from the BIE for the gradient of the displacements at an internal point x with the differentiation written in terms of field variables:

$$u_{i,m}(x) = \int_{\Gamma} T_{ij,m}(x,y)u_j(y)d\Gamma(y) - \int_{\Gamma} U_{ij,m}(x,y)t_j(y)d\Gamma(y) \quad (1)$$

$U_{ij}(x,y)$ and $\mathrm{T}_{ij}(x,y)$ are the displacement and the traction in the direction j at the boundary point y due to a singular load in the direction i at the collocation point x according to the Kelvin solution, respectively; $u_j(y)$ and $t_j(y)$ are the displacement and the traction at the field point, respectively. The first and the second integrals of equation (1) are regular for internal points and exhibit singularities of order $1/r^3$ and $1/r^2$ in three-dimensional problems, respectively, when the field point approaches the collocation point.

The introduction of the TDO in the first integral of the right member of equation (1) yields:

$$\int_{\Gamma} T_{ij,m}(x,y)u_j(y)d\Gamma(y) = \int_{\Gamma} \left\{D_{bm}\left[\sigma_{ibj}(x,y)\right] + n_m(y)\sigma_{ibj,b}(x,y)\right\}u_j(y)d\Gamma(y) \quad (2)$$

$D_{bm}(\ )$ is the tangential differential operator that has the following definition:

$$D_{bm}[f(y)] = n_b(y)f_{,m}(y) - n_m(y)f_{,b}(y) \quad (3)$$

The second term in the integral of the right member of equation (2) is turned null at points y not coincident with x ($y \neq x$) when the Kelvin solution is used. The application of the integration by parts on the first term of the integral of the right member of equation (2) reduces the order of the kernel singularity, [4], i.e.:

$$\int_{\Gamma} D_{bm}\left[\sigma_{ibj}(x,y)\right]u_j(y)d\Gamma(y) = \int_{\Gamma} \sigma_{ibj}(x,y)D_{mb}\left[u_j(y)\right]d\Gamma(y) \quad (4)$$

The BIE for the gradient of displacements using the TDO is given by:

$$u_{i,m}(x) = \int_{\Gamma} \sigma_{ibj}(x,y)D_{mb}\left[u_j(y)\right]d\Gamma(y) - \int_{\Gamma} U_{ij,m}(x,y)t_j(y)d\Gamma(y) \quad (5)$$

The BIE for stresses using the TDO is obtained from equation (5) using the Hooke tensor (C_{akim}) and the symmetry property of $U_{ij,m}(x,y)$ [7]:

$$\sigma_{ak}(x) = C_{\mathrm{aki\ m}} \int_{\Gamma} \sigma_{ibj}(x,y)D_{mb}\left[u_j(y)\right]d\Gamma(y) - \int_{\Gamma} \sigma_{jak}(x,y)t_j(y)d\Gamma(y) \quad (6)$$

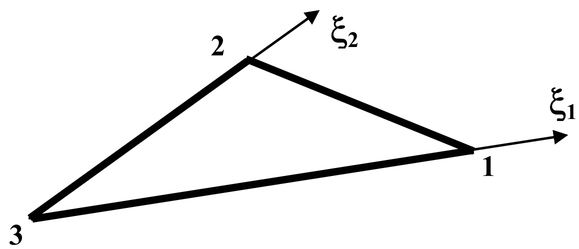

Figure 1: Three-node isoparametric triangular element

The limiting form of the stress BIE at an internal point when it is led to a point on the boundary defines one for points at the boundary and that for tractions by using the direction cosines of the outward normal at the collocation point (n'). The traction BIE for a point x' on a smooth boundary is given by:

$$\frac{1}{2}t_k\left(x'\right) = n'_a\left(x'\right)C_{\text{aki m}}\int_{\Gamma}\sigma_{ibj}\left(x',y\right)D_{mb}\left[u_j(y)\right]d\Gamma(y) + \dots$$
$$- n'_a\left(x'\right)\int_{\Gamma}\sigma_{jak}\left(x',y\right)t_j(y)d\Gamma(y) \quad (7)$$

The traction BIE using the TDO exhibits singularities of order $1/\mathrm{r}^2$ when the field point approaches the collocation point and only the Cauchy principal value sense is necessary to treat the improper integrals. Nevertheless, it is necessary to verify the continuity requirement for the derivative of the displacement function at the collocation point x' even for BIEs using the TDO.

Numerical Implementation of the TDO for three dimensional problems

Linear shape functions approximated displacements and efforts in the adopted triangular element, which had the same function for conformal and non-conformal interpolations. This element had the nodal parameters on its boundary line with the position of the collocation point shifted to its interior as it was done in [7] for elements used in plane problems. The use of derivatives of the shape function for displacements in the TDO carries to constant values as the tangent derivatives and the three-node isoparametric triangular element becomes the lowest order element for the TDO. The numerical implementation for the TDO on a surface element uses the partial derivatives written in terms of local coordinates and directions [4], which has the following expression for an arbitrary scalar function $f(y)$ in terms of field variables:

$$D_{jk}[f(y)] = e_{rjk}\frac{1}{|J|}\left[\frac{\partial f(y)}{\partial \xi_1}\left(\vec{\xi}_2\right)_r - \frac{\partial f(y)}{\partial \xi_2}\left(\vec{\xi}_1\right)_r\right] \quad (8)$$

e_{rjk} is the permutation symbol. $|J|$ is the Jacobian of the element mapping or the double of the area of the used triangular element. $\left(\vec{\xi}_i\right)_r$ is the component in the direction r of the vector $\vec{\xi}_i$. The introduction of the result of equation (8) in TDO of the equation (7) and considering the element shown in Figure 1 yields:

$$D_{mb}\left[u_j(y)\right] = e_{rmb}\frac{1}{|J|}\left[u_j^1\left(y_r^2-y_r^3\right)+u_j^2\left(y_r^3-y_r^1\right)+u_j^3\left(y_r^1-y_r^2\right)\right] \tag{9}$$

The upper indices in equation (9) are numbers of the nodes of the element shown in Figure 1, u_r^q and y_r^q are the displacement and the coordinate, respectively, in the direction r at the node q. An additional integral for the TDO is required to compute the effect of interfaces between adjacent elements when the boundary surface is piecewise smooth or in case of non-conformal interpolations. The traction BIE using the effect of one interface is given by:

$$\begin{aligned}\frac{1}{2}t_k\left(x'\right) = n'_a\left(x'\right)C_{\text{aki m}}\int_{\Gamma}\sigma_{ibj}\left(x',y\right)D_{mb}\left[u_j(y)\right]d\Gamma(y)+\ldots \\ -n'_a\left(x'\right)\int_{\Gamma}\sigma_{jak}\left(x',y\right)t_j(y)d\Gamma(y)+\ldots \\ +n'_a\left(x'\right)C_{\text{aki m}}\int_{\ell}e_{rbm}\sigma_{ibj}(x,y)u_j(y)s_r(y)d\ell(y)\end{aligned} \tag{10}$$

The effect of the interface is the result from the integration by parts presented in equation (4) with the use of the Stokes identity [4]. The third integral of the right member of equation (10) should be performed along boundary lines of surface elements where their interfaces are not continuous and s_r is the direction cosine in the direction r of the tangent vector to the boundary line at the integration point. The integral along the boundary line of the surface was reduced to the effect of the ends of the linear element in the formulation presented for plane problems [7].

Conclusion

This study presented a general formulation to employ the TDO in the traction BIE for three-dimensional problems and generalized that presented in [7] for plane problems. The effect of the interface in equation (10) should be used to non-conformal interpolations in case of surface boundary element having nodal parameters positioned on its boundary line and collocation points shifted to its interior or for nodes and collocation points shifted to the interior of the element. The first strategy, which uses nodal parameters positioned on the boundary line and collocation points shifted to the interior of the element, allows employing conformal interpolations along the crack surface as shown in [7] for plane problems. The equation (8) shows the general expression to apply to high order boundary elements.

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