

## Computation of Optimal Friction of Tuned Mass Damper for Controlling Base-Excited Structures

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### Summary

In this study, based on the results from the sinusoidal base excitation analyses of a single degree of freedom system with a tuned mass damper (TMD), the optimal friction is computed so that the rail friction improve the performance of the TMD. The magnitude of the optimal friction increases with increasing mass ratio of the TMD and decreases with increasing structural damping. Particularly, it is observed that the optimized friction force gives better control performance than the optimized viscous damping of the TMD. However, because the performance of the TMD considerably deteriorates when the friction force increases over the optimal value, it is required to keep the friction force from exceeding the optimal value. Based on the results from this study, it is possible to economically design the TMD by avoiding the unconditional minimization of the rail friction and in some cases by removing the additional damping devices of which function can be performed by the rail friction.

### Introduction

Mass type damping devices such as a tuned mass damper (TMD), an active mass damper (AMD), and a hybrid mass damper (HMD) have been applied utilized in order to suppress vibration induced by dynamic load. Most mass type damping devices consist of moving mass, spring, viscous damper, and a rail which secures the transversal movement of the mass. The most required condition for the rail is the minimization of the friction force, and the rail friction is generally not considered in the design of mass type damping devices under the assumption that the rail friction is minimized almost equal to zero.

The TMD provides good control performance when the motion of the structure is governed by the fundamental mode to which the TMD is tuned, but the TMD is ineffective in reducing the other modal responses and on the contrary it may excite those. In order to prevent this undesirable effect of TMD, linear viscous damper (LVD) is generally used to restrict the motion of the TMD and screen (or net) in a tuned liquid damper (TLD) play a similar role. However the LVD and the screen, respectively obstruct motions of the TMD and the TLD even when they operate very effectively in suppressing building vibration. Therefore the capacity of the LVD and the number of the screen should be restricted below a specific value. Extensive researches for finding optimal value of the damping ratio added by the LVD for the TMD or screen in the TLD have been conducted. Previous studies have

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shown that optimal value of the damping ratio is dependent of the mass ratio of the mass type damping devices to the structure, structural damping, loading characteristics, and the types of the structural responses which should be minimized. However, the results from the previous researches on optimal design of the TMD, are based on the assumption that friction in rail does not exist. This assumption does not reflect the practical condition, and furthermore the design process which unconditionally minimizes the rail friction force and then installs an additional damping device in the TMD is reciprocally contradictory because rail friction produces the effects identical to that of the LVD.

In this study, based on the results from the sinusoidal base excitation analyses of a single degree of freedom (SDOF) system with the TMD, the optimal friction is computed so that the rail friction improve the performance of a TMD. The rail friction is considered as Coulomb friction in the equation of SDOF system with the TMD and the transfer function of the displacement response of the SDOF system without/with the TMD to the base acceleration is obtained by conducting sine swept base excitation. The optimal friction is defined as one providing the minimum integration area of the transfer function, which equivalently means the minimization of the RMS displacement response. Also, the performance of the TMD with optimal friction is compared with one with optimal LVD. Finally, the variation of optimal friction with regard to the amplitude of the excitation force is investigated.

### Equation of motion

Equations of motion of a base-excited SDOF structure with a TMD having the rail friction are as follows

$$m_s \ddot{u} + c_s \dot{u} + k_s u = -m_s \ddot{x}_g + k_d u_d + c_d \dot{u}_d + f_r \text{sgn}(\dot{u}_d) \quad (1-a)$$

$$m_d (\ddot{u} + \ddot{u}_d) + k_d u_d + c_d \dot{u}_d + f_r \text{sgn}(\dot{u}_d) = -m_d \ddot{x}_g \quad (1-b)$$

where  $u$  denotes the structural displacement relative to the ground and  $u_d$  denotes the displacement of the TMD relative to the structure.  $m$ ,  $c$ , and  $k$  are, respectively, mass, damping, and stiffness of the structure or TMD, and subscript 's' and 'd' denote the structure and the TMD.  $f_r$  is the rail friction,  $\text{sgn}(\cdot)$  is the sign function, and  $\ddot{x}_g$  is the ground acceleration.

The first step in the design of the TMD is to determine the mass of the TMD, which is generally given by the ratio to the structural mass.

$$\mu = m_d / m_s \quad (2)$$

The higher mass ratio provides the better control performance, but, due to over-increase of gravity load, and construction/economic efficiency, mass ratio less than 2% is used in practical application of the general TMD, although Feng and Mita

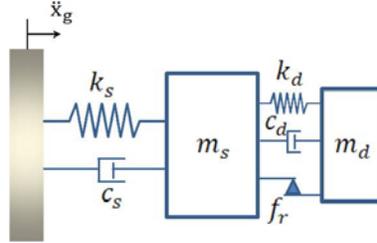


Figure 1: Model of SDOF structure and TMD subject to seismic load

(1995) proposed a system that the whole top floor acts like a TMD in a conception of Mega-substructure TMD with mass ratio more than 10%. With the given  $m_d$ , and  $k_d$  and  $c_d$  are determined by

$$k_d = \omega_d^2 m_d = f^2 \omega_s^2 m_d, \quad c_d = 2\xi_d \omega_d m_d \quad (3)$$

where  $f$  is tuning frequency ratio defined as the ratio of the TMD frequency  $\omega_d (= \sqrt{k_d/m_d})$  to the structural frequency  $\omega_s (= \sqrt{k_s/m_s})$  and  $\xi_d$  is the damping ratio of the TMD.

Many researchers have shown that optimal  $f$  is given as a function of mass ratio and the structural damping ratio and the function varies according to the type of excitation and optimization criteria. Sadek et al.(1997) proposed an optimal  $f$  for a base-excited structure with non-zero damping and a TMD so that the first two modes of the structure with the TMD has equal damping ratio. The stiffness of the TMD in this study is determined by using the following optimal  $f$  proposed by Sadek et al. and then the effect of rail friction is considered.

$$f = \frac{1}{1 + \mu} \left[ 1 - \xi \sqrt{\frac{\mu}{1 + \mu}} \right] \quad (4)$$

where  $\xi (= c_s/2\omega_s m_s)$  is the damping ratio of the structure

### Numerical Analysis

In this section, the effects of LVD and the rail friction on the control performance of the TMD are investigated through sine-swept excitation analyses for a damped SDOF structure. The ground acceleration is given by  $\ddot{x}_g = P_o \sin \omega t$  and the excitation frequency  $\omega$  varies in the range of  $0.1 \leq \beta (= \omega/\omega_s) \leq 2.0$ .

Figure 2 shows the peak displacement of a mass-normalized SDOF system, which has 2% structural damping ratio and 0.4 second natural period.  $P_o=1N$  is used for all excitation frequency so that the resulting structural response should have form identical to the transfer function and  $\mu=2\%$  are used. The results shown in figure 2(a) are obtained from the case that friction force is zero and  $\xi_d$  varies, and the results shown in figure 2(b) are obtained from the case that friction force varies

and  $\xi_d$  is zero. It is observed from figure 2 that the LVD and the rail friction affect frequency response of the structure-TMD in a similar way. When the magnitude of LVD damping or rail friction is little, the single peak of a structure without the TMD divides into two peaks and the peak displacement corresponding to the structural frequency to which the frequency of the TMD is tuned approaches near to zero. Admitting that the TMD without damping or friction is efficient in reducing the resonant response, two peak phenomenon indicates that the effect of the TMD becomes negligible in controlling the other frequency responses and on the contrary TMD amplifies those. As  $\xi_d$  and  $f_r$  increase, the two peak values divided around the structural frequency reduce gradually while the resonant response increases. And then the two peak system is changed to one peak system. This fact means that LVD damping and rail friction play a role of eliminating undesirable effect of the TMD in amplifying the structural response other than resonant one. However, because the resonant response, which generally dominates the structural behaviors, increases with regard to the LVD damping and rail friction, they should be limited below a specific value and optimally determined.

Various criteria such as maximization of the damping ratio of the structure-TMD system and minimization of displacement/acceleration of the structure can be used in determining optimal value of the damping ratio or rail friction. In this study, a following index, the area of the peak displacement curve  $D(\omega)$  in figure 2, which is known to be equivalent to the RMS response induced by white noise having wide bandwidth frequency contents, is chosen.

$$J_x = \int_{\omega_1}^{\omega_2} D(\omega) d\omega \quad (5)$$

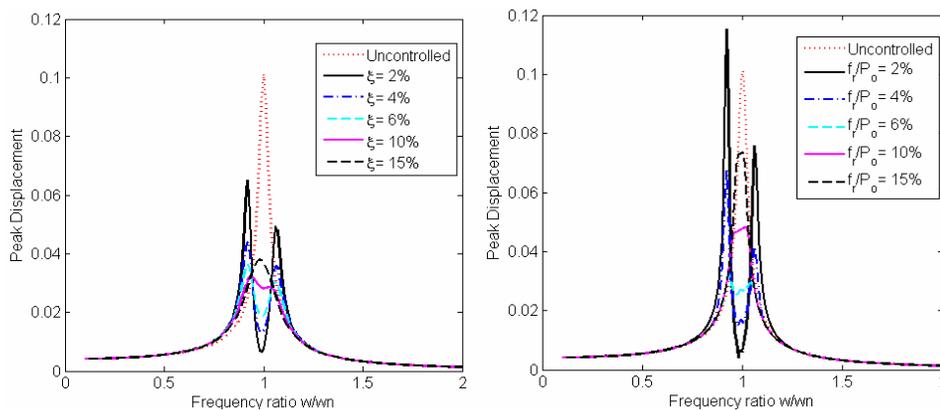


Figure 2: Peak displacement induced by unit sinusoidal base excitation

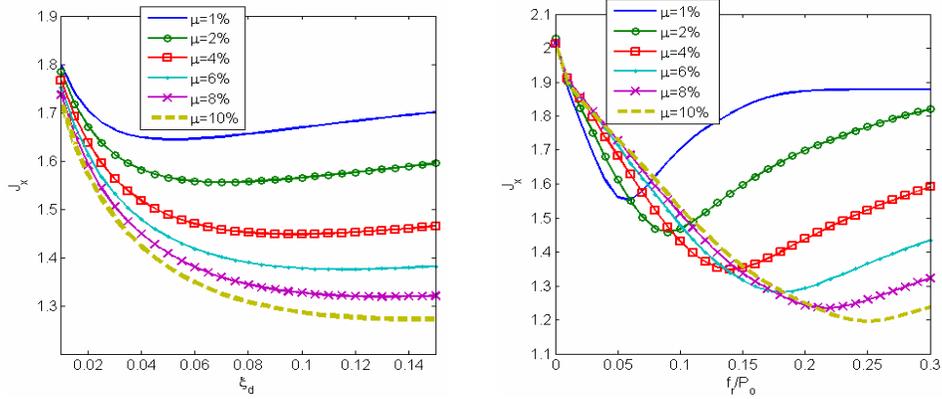


Figure 3: The variation of  $J_x$

Figure 3 shows variation of  $J_x$  with regard to  $\xi_d$  and  $f_r$  for various mass ratio. It is clearly observed that there exist optimal  $\xi_d$  and  $f_r$  providing the minimum value of  $J_x$  and this tendency is more explicit for the case of  $f_r$ . As  $\xi_d$  and  $f_r$  increase from zero,  $J_x$  starts to decrease and then increase when  $\xi_d$  and  $f_r$  increase over a specific value. Especially, increasing  $f_r$  over the specific value brings about the significant increase of  $J_x$  while the variation of  $J_x$  with regard to  $\xi_d$  is inconsiderable when  $\xi_d$  increases over the specific value. Accordingly, the rail friction should be cautiously designed not to exceed the optimal value unlike the LVD which comparatively secures the consistent performance for the large damping. It is also noted that the optimal  $\xi_d$  and  $f_r$  increase as  $\mu$  increases.

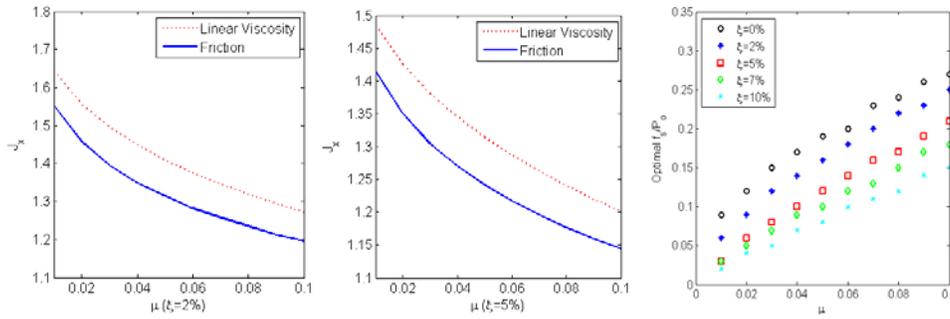


Figure 4:  $J_x$  by optimal linear viscosity and friction      Figure 5: Optimal friction

Figure 4 compares  $J_x$  obtained by assuming that the TMD is designed to have optimal linear viscous damping or optimal rail friction. The 2% and 5% structural damping ratios are considered. It is observed that the TMD with optimized rail friction provides the less  $J_x$  than one with optimized linear viscous damping. This implies that if optimizing the rail friction is possible, greater control efficiency can

be ensured in spite of not using supplemental LVD of which installation increases additional construction cost. Figure 5 shows optimal friction normalized by excitation load magnitude with regard to  $\mu$  and  $\xi$ . Optimal friction increases with increasing  $\mu$  and with decreasing  $\xi$ .

### **Conclusion**

This study showed that the rail friction, which was unconditionally minimized in practical application or ignored in the optimization process for the design of the TMD, could be used to improve the performance of the TMD. Through numerical analyses of a base-excited SDOF system controlled by the TMD with the rail friction, optimal rail friction was obtained. Optimal value of rail friction increases as mass ratio of the TMD increases and decreases as structural damping ratio increases. Particularly, optimized rail friction provides even better control performance than the optimized viscous damper, but it is essential to keep the magnitude of friction force below the optimal value because increasing friction force over the optimal value significantly deteriorates the performance of the TMD. Based on the results from this study, it is possible to economically design the TMD by avoiding the unconditional minimization of the rail friction and in some cases by removing the additional damping devices of which function can be performed by the rail friction.

### **Acknowledgement**

The authors would like to acknowledge the financial support from Korea Science and Engineering Foundation (KOSEF, Project No. R01-2007-000-10872-0), Korea Institute of Construction & Transportation Technology Evaluation and Planning (KICTTEP, Project No. 05 CTRM D06: High-tech Urban Development Program), and Seoul Regional Small and Medium Business Administration (Seoul SMBA, Project No.00023749).

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