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Mechanics of Fragmentable Geomaterials

A.P.S. Selvadurai[1]

Summary

This paper examines the mechanics of brittle fragmentation that can occur in geomaterials subjected to critical stress states that promote fragmentation as opposed to plastic flow. The discrete element technique is used to examine the dynamic interaction between a moving ice floe and a stationary structure. The results illustrate the influence of fragmentation on the reduction of the dynamic interactive stresses.

Introduction

The classical experimental studies by von Karman and Prandtl and Rinne (see e.g. Nadai[1])were among the first to illustrate the phenomenon that geomaterials can exhibit traits of either a plastic yield-type response or a brittle fragmentation by suitably altering the confining stress states acting on the geomaterial. They showed that at relatively low confining stresses in comparison to their uniaxial failure stress, sandstone can fragment in a brittle fashion and, as the confining stresses increase, marble can exhibit plastic flow. Therefore as a geomaterial is subjected to loads that have a low stress state with respect to confinement, the brittle mode of fragmentation can dominate. The propensity for brittle fragmentation is not restricted to the presence of low confining stresses; materials such as asphalt can exhibit a transition from viscoplastic flow to brittle behaviour even under one-dimensional loading by a reduction in the temperature at which the tests are conducted. In this paper we examine the modelling of the development of fragmentation in materials whose mechanical behaviour is predominantly brittle. A typical example of such a material is ice that can fragment when it is subjected to loadings applied rapidly and under conditions where there is little or no confinement. The paper describes the constitutive relationships that are employed in the modelling of the mechanical behaviour of the geomaterial, and the fragmentation criteria, and applies the developments to the study of the impact of an ice floe with a stationary object. The results shown in the paper illustrate the influence of the fragmentation process on the time history of the interactive dynamic force between the impacting ice floe and a stationary structure.

Constitutive models

We consider the dynamic interaction between a moving ice sheet and a stationary object. The constitutive properties of sea ice in particular are complex and highly variable, with anisotropy and inhomogeneity featuring dominantly in the description of the material. The identification of this variability is a restriction on the

[1]William Scott Professor and James McGill Professor, Department of Civil Engineering and Applied Mechanics, McGill University, Montreal, QC, Canada

modelling process and for this reason attention is restricted to relatively simplified models of ice behaviour that are observed in both laboratory and in situ testing. We assume that the rate form of the elastic response of the ice is given by an isotropic model of the form

$$\dot{\varepsilon}_{ij} = \frac{\dot{\sigma}'_{ij}}{2G} + \frac{\dot{\sigma}_{kk}}{9K}\delta_{ij} \tag{1}$$

where the dot denotes the time derivative, $\dot{\sigma'}_{ij}$ is the stress deviator rate and $K$and G are, respectively, the linear elastic bulk modulus and shear modulus. The rate-dependent failure behaviour of ice is defined by a viscoplastic model of the form proposed by Perzyna [2]. This model has been successfully applied by a number of investigators, including Zienkiewicz and Cormeau [3] to geomaterials and by Selvadurai and Sepehr [4] to describe the failure behaviour of ice in particular. The viscoplastic strain rate is given by

$$\dot{\varepsilon}_{ij}^{vp} = \gamma \langle \Phi(F) \rangle \frac{\partial F}{\partial \sigma_{ij}} \tag{2}$$

where $\gamma$is a flow parameter,F is the current yield function and $\Phi(F)$is a flow function defined by

$$\langle \Phi(F) \rangle = \Phi(F) \quad \text{if} \quad F \geq 0; \quad \langle \Phi(F) \rangle = 0 \quad \text{if} \quad F < 0 \tag{3}$$

A number of flow functions have been proposed in the literature and in the present computations we have assumed that $\Phi = F$. The failure criterion for ice again depends on the type of ice and the microstructures and inhomogeneities that can develop during its formation. The failure criteria can be anisotropic due to the columnar or polycrystalline nature of the ice. In this study we assume that the failure criterion for ice can be represented by a Mohr-Coulomb failure criterion of the form

$$F = \frac{I_1}{3}\sin\phi + \sqrt{J_2}\left(\cos\Theta - \frac{1}{\sqrt{3}}\sin\Theta\,\sin\phi\right) - c\cos\phi \tag{4}$$

where c and ϕ are, respectively, the cohesion and angle of friction of the ice at failure and

$$\Theta = \frac{1}{3}\sin^{-1}\left(-\frac{3\sqrt{3}J_3}{2J_2^{3/2}}\right); \quad I_1 = tr\sigma_{ij}; \quad J_2 = \frac{1}{2}\left\{(tr\sigma'_{ij})^2 - tr(\sigma'_{ij})^2\right\}; \quad J_3 = \left|\sigma'_{ij}\right| \tag{5}$$

The yield behaviour in the tension range is truncated by a tension cut-off that introduces an additional parameter σ_T. The fragmentation process, as opposed to yield, largely depends on the stress state within the ice mass. Stress fields that introduce tensile stress states are more susceptible to fragment development. In an intact ice mass, processes that initiate fragment development are directly related to the strength parameters associated with the attainment of peak strength. The principal stresses and their orientations can be used to determine the stress levels and the orientations of the fragmentation planes. For example, the fragment development during compressive-shear failure can be determined by considering the Mohr-Coulomb criterion

$$\sigma_1 \geq \sigma_c + \sigma_3 \tan^2(45 + \phi/2) \tag{6}$$

where σ_c is the unconfined strength in compression and σ_1 and σ_3 are, respectively, the maximum and minimum principal stresses; the unconfined compressive strength is related to shear strength parameters c and ϕ according to

$$\sigma_c = 2c\left\{(\tan^2\phi + 1)^{1/2} + \tan\phi\right\} \tag{7}$$

The orientation of the fragmentation plane in the compressive mode has two possible conjugate planes inclined at equal angles $(\pi/2 - \phi/2)$ to the direction of the stress on either side of it. In two dimensions, these are defined by

$$\theta = \tan^{-1}\left(\frac{(\sigma_1 - \sigma_{xx})}{\sigma_{xy}}\right) \pm \left(\frac{\pi}{4} - \frac{\phi}{2}\right) \tag{8}$$

where θ is the angle between the fragmentation plane and the positive global axis, σ_{xx} is the x-component of the stress tensor and σ_{xy} is the shear stress component. The second term on the right hand side of (8) is determined using a random number generator. For fragment initiation in tension, it is assumed that the material will fragment if the minimum principal stress reaches the tensile strength σ_T of the material, with tension considered as negative. The orientation of the fragmentation plane is given by

$$\theta = \tan^{-1}\left\{\frac{(\sigma_1 - \sigma_{xx})}{\sigma_{xy}}\right\} \tag{9}$$

where θ is the angle between the fragmentation plane and the global x-axis. In this case the θ-direction is aligned with the direction of maximum principal stress. The approach originates from concepts in rock mechanics, where tensile zone failure is limited to brittle fragmentation. In addition to the identification of criteria that determine viscoplastic flow and fragmentation, it is important to distinguish when one process or the other occurs in a region. This is largely governed by experimental evidence on material behaviour. In the present computational modelling it is assumed that brittle fragmentation will occur for all choices of σ_1 and σ_3. Other criteria include situations where fragmentation can occur if any one of the principal stress are tensile and viscoplastic flow will occur when both principal stresses

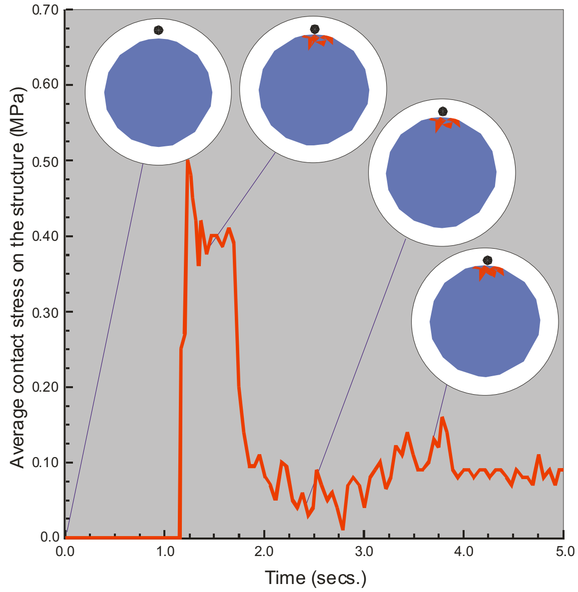

Figure 1: Development of average contact stresses at the contact zone between an impacting ice floe and a stationary object

are compressive. Once fragmentation occurs the manner in which individual fragments interact is based on Coulomb frictional responses of contact of the form where the stiffnesses are derived from Hertzian models. In addition to the constitutive properties, fragmentation criteria and interface characteristics, it is necessary to impose constraints on limits to fragmentation, which is required for computational and physical reasons. The limits to fragmentation are achieved by imposing a size-dependent tensile strength. Experimental evidence suggests that the tensile strength of most geomaterials will increase as the size decreases.

Computational modeling

We consider the dynamic interaction between a moving ice floe and a stationary object. The dynamic computational modelling of the problem accounts for the interaction, the occurrence of viscoplastic flow, the ability for fragmentation to take place in either compression or tension, the non-linear interactions between the individual fragments. The computational procedures for viscoplasticity and dynamic effects are incorporated in the discrete element procedure where these equations are solved for all fragments. The details of the problem will be described elsewhere [5]. Figure 1 illustrates the collinear interaction between a 10 m diameter ice floe of thickness 1m and the time history of development of contact stresses between the stationary object and the ice sheet. The approach velocity of the ice floe is 0.2 m.sec and the material parameters used in the simulations is given in [4].

Concluding remarks

The discrete element technique can be used to examine the process of continuum to fragmentation, which can occur in brittle geomaterials such as ice subjected to dynamic loads. The continued development of fragmentation is suppressed by the introduction of size-dependent strength criteria. This is a plausible constraint, which has a physical basis and can be used to limit the continued development of fragmentation. The time history of the interaction processes indicates that the peak value of the impact load is developed during elastic interaction between the ice floe and the stationary structure. The load is significantly reduced when fragmentation occurs.

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