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Brittle Fracture under Dynamical Loading with of Accounting of the Crack Edges Contact Interaction

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Summary

The contact interaction of opposite faces of cracks in 2-D and 3-D solid is studied. The case of a normal time-harmonic wave loading is studied in more details. The distribution of stress intensity factors as functions of the wave number is investigated. The results are compared with those obtained for cracks without allowance for the contact interaction.

Introduction

In designing structures by methods of fracture mechanics, inertial effects due to rapidly applied loads may have a significant effect [1, 2]. The action of the dynamic load is transferred to the cracks by stress waves propagating through the material. When the waves and the cracks interact, the crack edges may come into close contact at some areas. It should be taken into account that during deformation of a solid, the opposite faces of cracks mutually interact with the unilateral contact forces in the normal direction and the frictional contact forces in the tangential direction. The contact zones, and the adhesion and sliding sub-zones appear on the faces of cracks. The boundaries between contact and non-contact zones, and also between adhesion and slipping sub-zones, are time dependant and unknown beforehand. It implies the significant transformation of the stress-strain state in the vicinity of crack front and the corresponding modification of the stress intensity factors distribution. An analysis of problems of static fracture mechanics demonstrates that taking into account the contact interaction of crack edges may affect significantly the fracture mechanics criteria in dynamic problems and the effect of this interaction may be much greater than in the static case.

The present paper is devoted to the solution of the 2-D and 3-D fracture dynamics problem for cracks under incident harmonically waves. Different mathematical formulations algorithm for the problem solution are presented. The problem is solved with allowance for the contact interaction of the crack faces. The distributions of stress intensity factors for different wave numbers are investigated.

Formulation of the Problem

Arbitrary Loading

Let an elastic body in three-dimensional Euclidean spaceR^3 occupy a volume V. The body's boundary ∂V is piece-wise smooth and consists of sections ∂V_p and ∂V_u, to which the vectors of surface load $\mathbf{p}(x,t)$ and displacements $\mathbf{u}(x,t)$, respectively, are assigned. There are N arbitrary oriented cracks with surfaces $\Omega_n^+ \cup \Omega_n^-$,

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where $\mathbf{\Omega}_n^+$ and $\mathbf{\Omega}_n^-$ are the opposite edges. The body may be subjected to volume forces $\mathbf{b}(x,t)$. The stress-strain state of the body is described by the displacement equations of the linear dynamic theory of elasticity

$$A_{ij}u_j + b_i = \rho \partial_t^2 u_i, \quad A_{ij} = \mu \delta_{ij} \partial_k \partial_k + (\lambda + \mu) \partial_i \partial_j \tag{1}$$

where ∂_i and ∂_t are derivatives with respect to a coordinate and time, respectively, λ and μ are the Lame constants, and ρ is the density of the material.

The initial and boundary conditions are

$$\begin{aligned} &u_i(\mathbf{x},t_o) = u_i^0(\mathbf{x}), \quad \partial_t u_i(\mathbf{x},t_o) = v_i^0(\mathbf{x}) \\ &p_i(\mathbf{x},t) = \sigma_{ij}(\mathbf{x},t) n_j(\mathbf{x}) = \psi_i(\mathbf{x},t), \quad \forall \mathbf{x} \in \partial V_p, \\ &u_i(\mathbf{x},t) = \varphi_i(\mathbf{x},t), \quad \forall \mathbf{x} \in \partial V_u. \end{aligned} \tag{2}$$

Here n_i is a unit vector normal to the boundary ∂V.

On the crack edges, the vectors of contact forces and displacement discontinuity must satisfy unilateral contact constraints with friction in the form [2]

$$\begin{aligned} &\Delta u_n \geq -h_o, \quad q_n \geq 0, \quad (\Delta u_n + h_o) q_n = 0 \\ &|\mathbf{q}_\tau| \leq k_\tau q_n \Rightarrow \partial_t \Delta \mathbf{u}_\tau = 0, \quad |\mathbf{q}_\tau| = k_\tau q_n \Rightarrow \partial_t \Delta \mathbf{u}_\tau = -\lambda_t \mathbf{q}_\tau, \end{aligned} \tag{3}$$

where $q_n, \mathbf{q}_\tau$ and $\Delta u_n, \Delta \mathbf{u}_\tau$ are the normal and tangential components of the vectors of contact forces and displacement discontinuity, respectively, h_0 is the initial crack opening, and k_τ and λ_τ are coefficients depending on the properties of the contacting surfaces $\Omega^+ = \bigcup_{n=1}^{N} \Omega_n^+$ and $\Omega^- = \bigcup_{n=1}^{N} \Omega_n^-$.

Harmonic loading

Let us detail the case where an elastic body is under harmonic loading. For simplicity we consider an infinite region. In [1] it was shown that the contact-interaction vector is not harmonic and we cannot represent the unilateral conditions in a harmonic form, because of their nonlinearity. This fact complicates the problem significantly. The stress-strain components have to be expanded into Fourier series, which depend on the loading parameter ω,

$$p_i(\mathbf{x},t) = Re\{\sum_{-\infty}^{\infty} p_i^n(\mathbf{x}) e^{i\omega nt}\}, \quad u_i(\mathbf{x},t) = Re\{\sum_{-\infty}^{\infty} u_i^n(\mathbf{x}) e^{i\omega nt} \tag{4}$$

where

$$p_i^n(\mathbf{x}) = \frac{\omega}{2\pi} \int_0^T p_i(\mathbf{x},t) e^{-i\omega nt} dt, \quad u_i^n(\mathbf{x}) = \frac{\omega}{2\pi} \int_0^T u_i(\mathbf{x},t) e^{-i\omega nt} dt. \tag{5}$$

Substituting (4) into (1) and considering what has been said above, we obtain a set of differential steady-state elastodynamic equations:

$$A_{ij}u_j^n + \omega^2 n^2 u_i^* = 0, \quad \forall \mathbf{x} \in V,\ n = 0, \pm 1, \ldots, \pm\infty. \tag{6}$$

The boundary conditions on the crack surfaces have the form

$$p_i^n(\mathbf{x},t) = \begin{cases} q_i^n(\mathbf{x},t) & \forall \mathbf{x} \in \Omega_e, \quad \forall n \neq 1 \\ p_i^*(\mathbf{x},t) + q_i^n(\mathbf{x},t), & \forall \mathbf{x} \in \Omega_e, \quad \forall n = 1 \end{cases}, \tag{7}$$

where $\Omega_e = \Omega^+ \cap \Omega^-$ is the close-contact region.

With this an approach, the initial-boundary-value problem (1), (2) with the unilateral constraints (3) reduces to a countable set of boundary-value problems (6) with the parameter $\omega^2 n^2$ and the unilateral constraints (3).

Boundary Integral Equation

In [2] it was shown that the BIE that appears in many engineering applications may be written, on a smooth boundary, in the following form

$$\begin{aligned} \mp\frac{1}{2}u_i(\mathbf{y},\bullet) &= \int_V (p_j(\mathbf{x},\bullet) * U_{ij}(\mathbf{x}-\mathbf{y},\bullet) + u_j(\mathbf{x},\bullet) * W_{ij}(\mathbf{x},\mathbf{y},\bullet))dS \\ &\quad + \int_\Omega \Delta u_j(\mathbf{x},\bullet) * W_{ij}(\mathbf{x},\mathbf{y},\bullet)dS + \int_V b_j(\mathbf{x},\bullet) * U_{ij}(\mathbf{x}-\mathbf{y},\bullet)dV \\ \mp\frac{1}{2}p_i(\mathbf{y},\bullet) &= \int_V (p_j(\mathbf{x},\bullet) * K_{ij}(\mathbf{x},\mathbf{y},\bullet) + u_j(\mathbf{x},\bullet) * F_{ij}(\mathbf{x},\mathbf{y},\bullet))dS \\ &\quad + \int_\Omega \Delta u_j(\mathbf{x},\bullet) * F_{ij}(\mathbf{x},\mathbf{y},\bullet)dS + \int_V b_j(\mathbf{x},\bullet) * K_{ij}(\mathbf{x},\mathbf{y},\bullet)dV \end{aligned} \tag{8}$$

Here $\bullet$ indicates t for the time domain, ω for the frequency domain formulations and zero for the statically problems, respectively. The plus and minus signs in (8) are used for the interior and exterior problems, respectively. Sign $*$ indicates the convolution,

$$f(t) * g(t) = \int_{\Im} f(\tau)g(t-\tau)d\tau,$$

in the time domain BIE formulation and a multiplication of functions otherwise. In the case of scalar problem indices are omitted.

The kernels $U_{ij}(\mathbf{x}-\mathbf{y},\bullet)$, $W_{ij}(\mathbf{x},\mathbf{y},\bullet)$, $K_{ij}(\mathbf{x},\mathbf{y},\bullet)$ and $F_{ij}(\mathbf{x},\mathbf{y},\bullet)$ in the BIE (8) are fundamental solutions for the differential equations that correspond to the problem under consideration. The integrals with such kernels are divergent and need a special consideration in order to have some mathematical sense (see [2] for details).

Variational Formulation and Algorithm for the Problem Solution

Here will be considered variational formulations with emphasis on unilateral contact and friction phenomena.

Let's consider functional of Hamilton-Ostrogradskii

$$\Phi_H[u_i] = \int_{\Im} [E(\mathbf{u}) - K(\mathbf{v}) + < b_i, u_i >_V - < \psi_i, u_i >_{\partial V_p}] dt, \tag{9}$$

on the set of admissible displacements

$$\begin{aligned}\mathbf{K}_H[u_i] = \{ & u_i \in \mathbf{H}^{1,1}(V \times \Im), \quad \varepsilon_{ij} = 1/2(\partial_i u_j + \partial_j u_i), \quad \sigma_{ij} = c_{ijkl}\varepsilon_{kl}; \\ & v_i = \partial_t u_i, \quad g_i = \rho v_i, \quad u_i(\mathbf{x},t) = \varphi_i(\mathbf{x},t), \forall \mathbf{x} \in \partial V_u, \quad u_i(\mathbf{x},t_0) = u_i^0(\mathbf{x}) \}\end{aligned} \tag{10}$$

The following notations have been used here: $< \cdot, \cdot >$ denotes the duality pairing for $\mathbf{H}^{\alpha}(V)$ and its dual functional space $\mathbf{H}^{-\alpha}(V)$, $\mathbf{H}^{\alpha}(V)$ is a Sobolev's functional space of index $\alpha(\alpha > 0, \alpha = 0, \alpha < 0)$, $E(\mathbf{u}) = \int\limits_V 1/2 c_{ijkl}\varepsilon_{ij}(\mathbf{u})\varepsilon_{kl}(\mathbf{u}) dV$ is the total potential energy of the deformed body $K(\mathbf{v}) = \int\limits_V 1/2 \rho v_i v_i dV$ is the total kinetic energy, the expressions $< b_i, u_i >_V = \int\limits_V b_i u_i dV$ and $< \psi_i, u_i >_{\partial V_p} = \int\limits_{\partial V_p} \psi_i u_i dS$ represent body forces and surface traction work respectively.

Variational formulation of elastodynamic problem with unilateral restrictions and friction (2) consists in the following.

Find $u_i \in \mathbf{K}_H(u_i)$ such that

$$\Phi_{H,n,\tau}[\mathbf{u}] = \underset{\mathbf{u}^* \in \mathbf{K}_H(\mathbf{u})}{extr} \left\{ \Phi_L[\mathbf{u}^*] - \sup_{q_i^* \in \mathbf{K}_n^c(q_n) \cap \mathbf{K}_\tau^c(\mathbf{q}_\tau)} \Phi_i^c[q_i^*] \right\} \tag{11}$$

where

$$\Phi_i^c[q_i] = \Phi_n^c(q_n) + \Phi_\tau^c(\mathbf{q}_\tau), \quad \Phi_n^c(q_n) = \begin{cases} 0, & \text{if } q_n \geq 0 \\ \infty, & \text{otherwise} \end{cases},$$

$$\Phi_\tau^c(\mathbf{q}_\tau) = \begin{cases} 0, & \text{if } |\mathbf{q}_\tau| = k q_n \\ \infty, & \text{otherwise} \end{cases}$$

$$\begin{aligned}\mathbf{K}_n^*(q_n) &= \{ q_n \in \mathbf{H}^{-1/2,0}(\Omega \times \Im), q_n \geq 0, \forall \mathbf{x} \in \Omega, \forall t \in \Im \} \\ \mathbf{K}_\tau^*(\mathbf{q}_\tau) &= \{ \mathbf{q}_\tau \in (\mathbf{H}^{-1/2,0}(\Omega \times \Im))^2, |\mathbf{q}_\tau| \leq k_\tau q_n, \forall \mathbf{x} \in \Omega, \forall t \in \Im \}\end{aligned} \tag{12}$$

The algorithm for solution of the contact problem with friction consists on two parts. The first part of algorithm is a solution of the problem without unilateral constrains and friction. The second part consists in iterative process, which continued until the solution fulfill to the unilateral constrains and friction.

This algorithm is based on variational principles (11) and consists in the following steps

1. the initial distribution of the contact forces on the contact surface $q_i^0(\mathbf{x},t), \quad \forall \mathbf{x} \in \Omega$, $\forall t \in \Im$ is assigned;
2. the problem without constrains is solved and the unknowns quantities on the region and /or on the boundary and also on the contact surfaces $u_i(\mathbf{x},t)$ are defined;
3. the normal and tangential components of the vector of contact forces are corrected to satisfy the unilateral restrictions and friction

$$q_n^1(\mathbf{x},t) = P_n[q_n^0(\mathbf{x},t) - \rho_n(\Delta u_n^1(\mathbf{x},t) - h_0(\mathbf{x},t))],$$

$$\mathbf{q}_\tau^1(\mathbf{x},t) = \mathbf{P}_\tau[\mathbf{q}_\tau^0(\mathbf{x},t) - \rho_\tau \mathbf{u}_\tau^1(\mathbf{x},t)] \qquad (13)$$

where

$$P_n[q_n] = \begin{cases} 0, & \text{if } q_n \leq 0 \\ q_n, & \text{if } q_n > 0 \end{cases}, \quad \mathbf{P}_\tau[\mathbf{q}_\tau] = \begin{cases} \mathbf{q}_\tau, & \text{if } |\mathbf{q}_\tau| \leq k_\tau q_n \\ k_\tau q_n \frac{\mathbf{q}_\tau}{|\mathbf{q}_\tau|}, & \text{if } |\mathbf{q}_\tau| \leq k_\tau q_n \end{cases}$$

are operators for the orthogonal projection onto the sets $q_n \geq 0$ and $|\mathbf{q}_\tau| \leq k_\tau q_n$, coefficients ρ_n and ρ_τ are chosen based on the conditions that give the best convergence for the algorithm;
4. then proceed to the second step of the iteration.

Numerical Results

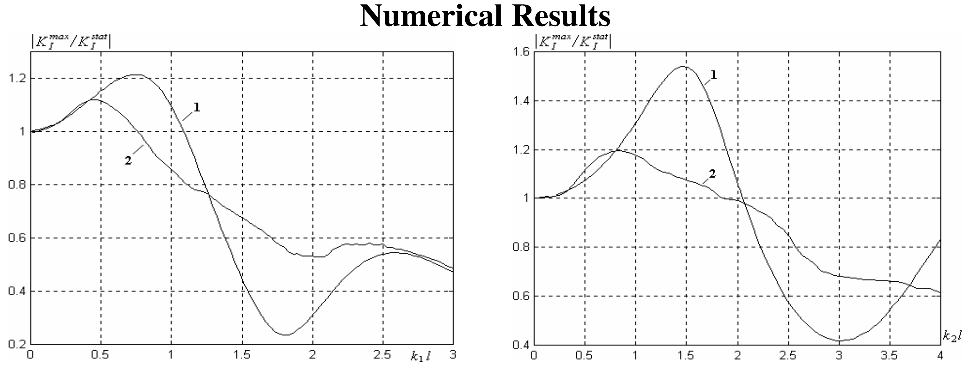

Figure 1:

Distribution of the normalized stress intensity factor versus wave number for flat crack in plane and penny-shaped crack in space are presented in fig.1. Curves 1 and 2 correspond to solution without counting and with counting contact interaction of the crack faces.

The results presented here confirm the significance of taking into account the contact interaction of crack faces. It should be taken into account in strength analyses of structures by methods of fracture mechanics.

References

1. Guz, A.N., Zozulya, V.V. (2001): "Fracture dynamics with allowance for a crack edges contact interaction", *International Journal of Nonlinear Sciences and Numerical Simulation,* 2001, Vol. 2, pp. 173-233.
2. Guz, A.N.; Zozulya, V.V. "Elastodynamic unilateral contact problems with friction for bodies with cracks", *International Applied Mechanics*, Vol. 38, pp. 895-932.