

## **BE analysis of delaminated Composite Structures repaired with Piezoelectric Active patches**

Alaimo A.<sup>1</sup>, Milazzo A.<sup>1</sup>, Orlando C.<sup>1</sup>

### **Summary**

The main target of the present paper is the analysis of the fracture mechanics behavior of delaminated composite structures actively repaired through piezoelectric patches. The aim at issue has been achieved by using a boundary element code implemented to study piezoelectric solids, including, as limiting case, the applicability to linear elastic anisotropic materials. The assembled structures, made up of the damaged composite components and the piezoelectric patches, have been modeled through the multidomain technique. To take into account for the adhesive layer among the host structure and the active patch, an interface spring-model has been also implemented. The multidomain technique coupled with the spring-model has also allowed the modeling of delamination cracks. The fracture mechanics behavior of bimaterial cracks has been characterized in terms of the total Energy Release Rate (ERR) and the phase angle  $\Psi$ , which provide information about the mix between mode I and II of fracture. The numerical results obtained have provided useful information on the design of piezoelectric devices used in the field of the active-repair technology.

### **Introduction**

It is well known that delamination represents one of the most dangerous failure mode of composite laminates [1]. It appears as an interlaminar fracture arising between two anisotropic fiber-composite as a consequence of the high interlaminar stresses caused by geometric discontinuities or mismatch in the material properties. Since it represents basically a 3D problem, for which no closed form solution exists, numerical methods, such as the finite element method [2, 3] and the boundary element method [4, 5] are used to analyze interlaminar stresses and consequently to better understand the behavior of delamination cracks. When left unattended, delaminations can grow at an alarming rate due to the singularity in stresses and strains near the damaged region. Delamination detection is then a primary concern in order to avoid the deterioration of the structural performances [6] and even catastrophic failures. In most conditions, the service lifetime of a damaged component can be extended with repairs instead of immediate replacement. The increasing development of the Smart Materials opens new opportunities for improved repair techniques as an alternative to the conventional repair methods like bonded or riveted patches. Among this emerging methods, the "active repair" technology appears to be the more attractive. Active repairs, obtained by using piezoelectric

---

<sup>1</sup>Dipartimento di Ingegneria Strutturale, Aerospaziale Egeotepnipa, Università di Palermo, Palermo, Italy.

patches, are based on the converse piezoelectric effect, according to which, the strain induced by an applied electric field across the piezoelectric patch can help the structure to reduce the crack opening and consequently to increase its fatigue life [7]. In the field of composite materials, the active repair technology has been recently applied to delaminated beam by Duan et al. [8]. They studied actively repaired delaminated beam through the finite element method instead of the simple beam theory proposed by Wang et al.[9]. In this paper, a multidomain boundary element code implemented for 2D piezoelectricity has been employed to study the fracture mechanics behavior of delaminated composite structures actively repaired through piezoelectric active patches. Parametric analyses, based on the patch position, have been performed on a drop-ply delaminated structure previously studied by Beuth [10] and Narayan et al. [11] to represent a bi-dimensional simplification of the adhesive joints between composite aircraft fuselage skins and stiffeners. The numerical results obtained have served to provide information on the design of piezoelectric materials for the active repair of delaminated structures.

### Numerical Model and adhesive modeling strategy.

The boundary integral formulation is developed for a two-dimensional piezoelectric domain  $\Omega$  with boundary  $\partial\Omega$  lying in the  $x_1$   $x_2$  plane under the hypothesis of generalized plain strain elasticity and in-plane electrostatic [5, 7]. The numerical model of the problem is then obtained by using the boundary element method, which provides a linear algebraic resolving system expressed in terms of generalized displacements and tractions nodal values  $\boldsymbol{\delta}$  and  $\mathbf{P}$ , respectively

$$\mathbf{H}\boldsymbol{\delta} + \mathbf{G}\mathbf{P} = \mathbf{0} \quad (1)$$

To achieve the modelling of assembled heterogeneous structures, made up by the damaged composite components and the piezoelectric repairs, the multidomain technique is implemented. It is based on the division of the original domain into homogeneous subregions so that Eqs. (1) still hold for each single subdomain [7]. The solution of the whole problem is then obtained by restoring the displacement continuity and traction equilibrium conditions along the interfaces between contiguous subdomains. The modelling of the adhesive layer among the host structure and the active repair can be achieved by assuming displacement jumps in normal and tangential directions across the interface that can be expressed, according to the spring-model [7], as function of the respective interface tractions through the compliance interface constants in normal and tangential direction,  $k_N$  and  $k_T$ .

$$\Delta\delta_N = k_N P_N, \quad \Delta\delta_T = k_T P_T \quad (2)$$

being  $\Delta\delta_N$  and  $\Delta\delta_T$  the displacement jump components in normal and tangential directions. The elastic interface conditions, expressed by Eqs. (2), represent an uncoupled behavior between interlayer tractions and displacement jump components

characterized by the compliance constants  $k_N$  and  $k_T$ . For this reason the modeling of the bonding layer through an equivalent zero-thickness elastic interface may be achieved by linking the mechanical properties of the adhesive to the compliance interface constants. Due to the small thickness of the bonding layer and by virtue of the generalized plane strain hypothesis, it may be assumed that the non zero components of the strain field, characterizing the adhesive layer, are  $\epsilon_{22}$  and  $\gamma_{12}$ . It follows that, by assuming the  $x_2$  axis along the thickness direction, the isotropic adhesive constitutive equations are written as

$$\begin{aligned} \epsilon_{22} &= S_{22}^* \sigma_{22} \text{ with } S_{22}^* = (1 + \nu)(1 - 2\nu)/E(1 - \nu) \\ \gamma_{12} &= S_{66}^* \tau_{12} \text{ with } S_{66}^* = 1/G \end{aligned} \quad (3)$$

In Eq.(3) E, G and  $\nu$  are the Young's Modulus, the Shear Modulus and the Poisson's ratio of the adhesive. The link between the aforementioned quantities and the spring interface compliance constants  $k_N$  and  $k_T$  can be obtained by comparing Eq.(2) and Eq.(3). Both equations represent in fact the adhesive constitutive laws which relates the peel and the shear stresses and tractions at the interface to the respective strains and displacements. It follows that the interface compliance elastic constants  $k_N$  and  $k_T$  can be obtained from  $S_{22}^*$  and  $S_{66}^*$  of Eq. (3), see [7], by considering that  $S_{22}^*$  and  $S_{66}^*$  represent compliance coefficients per unit thickness, thus one can write

$$k_N = t_a S_{22}^*, \quad k_T = t_a S_{66}^* \quad (4)$$

being  $t_a$  the adhesive thickness.

### **Numerical applications and discussion.**

The active repair performances of piezoelectric patches and the corresponding fracture mechanics behavior of the drop-ply delaminated composite structure, shown in Figure 1, are discussed in this section. As mentioned before, this kind of geometry has been chosen since it can be used for the modeling of skin-stiffener delamination problem [10, 11].

The geometry of the host structure is characterized by the following dimension:  $L_3=50.8\text{mm}$ ,  $L_2=35.6\text{mm}$ ,  $H_1=5.08\text{mm}$ ,  $H_2=3.18\text{mm}$ ,  $H_3=1.9\text{mm}$ . An edge delamination, having length  $a=10.2\text{mm}$ , is located at the interface between the stiffener flange and the skin. Moreover, the analyzed structure is clamped at the left side and a distributed shear force per unit length  $P=21.0 \text{ KN/m}$  is applied on the right hand side of the skin. For both the multi-layered and bimorph patch configurations analyzed, shown in Figure 1, a thickness  $h=1.5\text{mm}$  and a length  $L_p= 2a$  are considered. In the case of the multi-layered patch, obtained by stacking 10 piezoelectric layers in series configuration, the active repair is achieved through the elongation of the patch that induce a bending moment into the host structure. If the bimorph configuration is used, the applied voltage directly induce bending deformations allowing

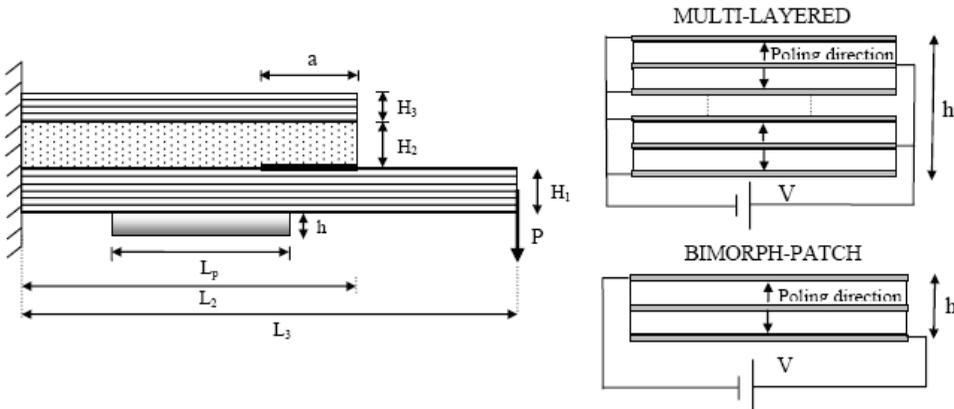


Figure 1: Drop-ply configuration and piezoelectric patch configurations.

the active repair with less shear stress transfer at the interface between the patch and the damaged structure. The material properties of the adhesive used for the bonding refer to the typical value characterizing epoxy resin,  $E=3\text{GPa}$  and  $\nu=0.4$ . By considering an adhesive thickness  $t_a=0.1\text{mm}$  and according to the adhesive modeling strategy adopted in the present paper [7] the compliance interface constants corresponding to the material properties of the adhesive layer are  $k_N=1.56 \cdot 10^{-5}\text{m/GPa}$  and  $k_T=9.33 \cdot 10^{-5}\text{m/GPa}$ . The material properties of both the piezoelectric patch and the unidirectional lamina are listed in Table 1.

Table 1: Material properties.

Elastic const. (GPa)	PZT-4		Graphite-Epoxy	
	Piezo const. (C/m <sup>2</sup> )	Dielectric const. (pC/Vm)	E, G GPa	$\nu$
$C_{11} = 139$	$e_{21} = -5.2$	$\epsilon_{11} = 6447$	$E_{11} = 134$	$\nu_{12} = 0.3$
$C_{22} = 115$	$e_{22} = 15.1$	$\epsilon_{22} = 5617$	$E_{22} = 10.2$	$\nu_{13} = 0.3$
$C_{12} = 74.3$	$e_{14} = 12.7$	$E_{33} = 10.2$	$\nu_{32} = 0.49$	
$C_{13} = 77.8$			$G_{12} = G_{13} = 5.52$	
$C_{44} = 25.6$			$G_{23} = 3.43$	

The boundary element model of the analysed structure is obtained by using the multidomain technique. The boundary element mesh for the host delaminated structure consists of 434 linear elements and is obtained by assembling 3 subregions. Mesh refinement is also used at the crack tip in order to caught the stress singularities and to obtain an accurate prediction of the total energy release rate. The multi-layered patch model consists of 250 linear elements and 10 subregions are used to stack the active repair. On the other hand, 50 linear elements are used to model the bimorph patch.

### Multi-layered patch configuration

With the aim of determining the best repair conditions for the multi-layered patch, the active repair performances and the fracture mechanics behavior of the repaired structure are investigated for different position of the piezoelectric device with respect to the tip.

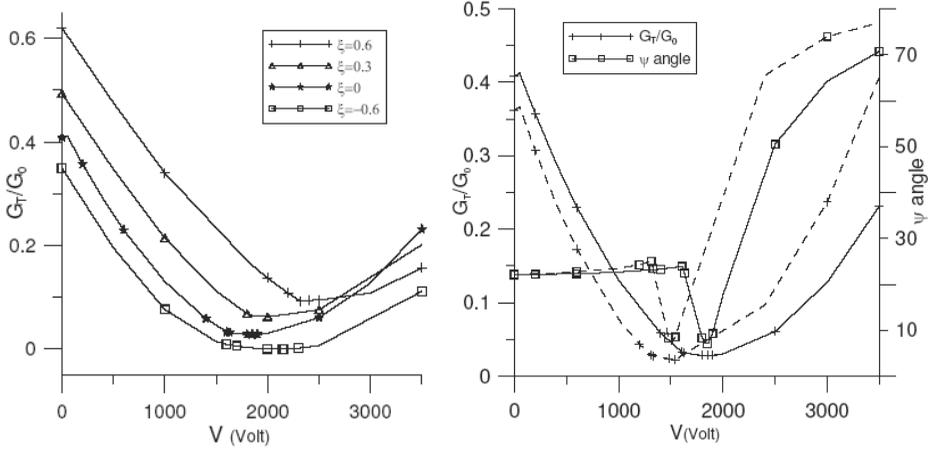


Figure 2: ERR distr. (left);  $G_T/G_0$  and  $\Psi$  for  $\xi=0$  (right: solid adhesive, dashed perfect).

Parametric studies are then performed in terms of the parameter  $\xi = (X_G - X_T)/a$ , representative of the relative position between the crack and the patch.  $X_G$  and  $X_T$  are, in fact, the coordinates along the  $x_1$  axis of the crack tip and of the patch geometric centre, respectively. The fracture mechanics response of the repaired structure is studied in terms of the total energy release rate ( $G_T$ ), see Figure 2 (left), and, according to the Energy based repair criterion, the structure is considered to be repaired when  $G_T$  reaches its minimum value. Moreover, due to the mode mix usually associated to delamination at bi-material interface, the phase angle  $\Psi$  is also investigated, see Figure 2 (right). In Figure 2 (left) the total energy release rate distribution versus the applied voltage is plotted for different values of the distance parameter  $\xi$ . The total ERR is expressed in dimensionless unit by dividing  $G_T$  to  $G_0$ , being  $G_0$  the total ERR characterizing the fracture mechanics behavior of the unrepaired structure. It is worth nothing that the computed value of  $G_0 = 1.43 \cdot 10^{-4} \text{MPa}\cdot\text{m}$  agrees very well to that found by Narayan et al.[11] through a finite element analysis. From Figure 2 it can be pointed out that the best position for the patch corresponds to  $\xi=-0.6$  since for a given applied voltage the minimum value of  $G_T$  is obtained. The effect of the adhesive layer between the host structure and the patch, shown in Figure 2 (right) for  $\xi=0$ , is to reduce the repairing performance of the piezoelectric device by increasing the repair voltage,  $V_r$ , of about the

20%.

### Bimorph patch configuration

As previously discussed, the bimorph patch undergoes bending deformation under an applied voltage and for this reason, considering the same number of piezoelectric laminae to arrange the repair, these configuration appears more effective than the multilayered one. To quantify the differences between the two configurations let's consider a bimorph and a bi-layer patch at  $\xi = 0.3$ . The two patches have the same number of plies but the repair voltage of the bimorph results 64% less than the bi-layer one. For a better understanding of the bimorph patch repair mechanism, the total energy release rate distributions versus the applied voltage are plotted in Figure 3. It can be pointed out that the best location for the bimorph device is at  $\xi = 0.3$  because the maximum reduction of the energy release rate is obtained with the lowest repairing voltage. From the same figure it appears that, by considering the bimorph patch located at  $\xi = -0.6$ , once the total ERR reaches its minimum value it remains constant even when the applied voltage exceeds the repairing one.

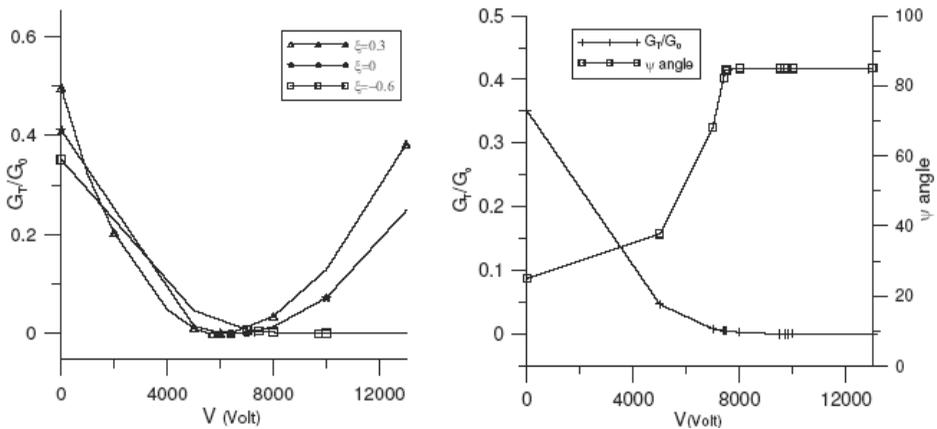


Figure 3: ERR distr. (left);  $G_T/G_0$  and  $\Psi$  for  $\xi = -0.6$  (right).

### Conclusion

The fracture mechanics behaviour of delaminated composite structures actively repaired through piezoelectric patches has been investigated by using a boundary element code. The assembling of the host structures with the active repairs has been achieved through the multidomain technique and a spring interface model has been also implemented in order to take into account the adhesive layer. The analyses performed on both the configurations and the positioning of the patches have provided useful guidelines for the design of active repairs for damaged composite structures.

## References

1. Tay, T.E. (2003): "Characterization and analysis of delamination fracture in composites: An overview of developments from 1990 to 2001", *Appl Mech Rev*, Vol.56, (1).
2. Wang, S.S., Stango R.J. (1983): "Optimally Discretized Finite Elements for Boundary Layer Stresses in Composite Laminates", *AIAA Journal*, Vol.21, (6), pp. 614-620.
3. Whitcomb, J.D., Raju, I.S., Goree, J.G. (1982): "Reliability of the Finite Element Method for Calculating Free Edge Stresses in Composite Laminates", *Computers and Structures*, Vol.15, (1), pp. 23-37.
4. Davì, G., Milazzo, A. (1997): "Boundary Element Solution for the Free Edge Stresses in Composite Laminates", *Journal of Applied Mechanics*, Vol.64, (4), pp. 877-884.
5. Alaimo, A., Milazzo, A., Benedetti, I. (2007): "Analysis of composite laminates with imperfect bonding conditions", *Proceedings of the VIII International Conference on Boundary Element Techniques, July 2007, Naples*.
6. Todoroki, A., Ueda, M. (2006): "Low cost delamination monitoring of CFRO beams using electrical resistance changes with neural network", *Smart Materials and Structures*, Vol. 15, N75-84.
7. Alaimo, A., Milazzo, A., Orlando, C. (2008): "Boundary element analysis of Adhesively Bonded Piezoelectric Active Repair", *Engng Fract Mech* doi:10.1016/j.engfracmech. 2008.10.008.
8. Duan, W.H., Quek, S.T., Wang, Q. (2008): "Finite element analysis of the piezoelectric-based repair of delaminated beam". *Smart Materials and Structures*, Vol. 17, pp. 1-7.
9. Wang, S.S., Yuan, F.G. (1983): "A hybrid finite element approach to composite laminate elasticity problem with singularity". *J. Appl. Mech*, Vol. 50, pp. 835-844.
10. Beuth, J.L. (1996): "Separation of crack extension modes in orthotropic delamination models". *International Journal of Fracture*, Vol. 77, pp. 305-321.
11. Narayan, S.H., Beuth, J.L. (1998): "Designation of mode mix in orthotropic composite delamination problems", *International Journal of Fracture*, Vol. 90, pp. 383-400.

