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Non-Newtonian Lid-driven Cavity Flow Simulation by Mesh Free Method

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Summary

Non-Newtonian lid-driven cavity flow is studied in a wide range of Reynolds numbers. The algorithm of mesh free characteristic based split has been extended for solving non-Newtonian flow problems in meshfree context. It is assumed that the non-Newtonian fluid properties obey Carreau-Yasuda rheological model. The results obtained from mesh free characteristic based split algorithm have been compared to the results of other meshfree methods. Results have been obtained for the velocity profiles at Reynolds numbers as high as 1000 for a Carreau-Yasuda fluid.

Introduction

The lid-driven cavity flow of a Newtonian fluid was initially studied by Burggraf [1] in 1966. After that, many scientists became interested to solve this problem because lid-driven cavity flow contains many fluid flow phenomena such as interactions of complex vortexes [2]. In earlier papers the finite difference method was used for numerical solution of this problem [3]. The presence of corner singularities in this flow is potentially critical for high-order methods. The flexibility of mesh free method in using various and higher order shape functions is a good way to deal with complex problems such as lid-driven cavity flow. Zhang *et al.* studied lid-driven cavity flow of Newtonian fluid by mesh free method [4]. They used a least-squares mesh free method based on the first-order velocity–pressure–vorticity formulation for two-dimensional incompressible Navier–Stokes problem. Liu *et al.* studied lid-driven cavity flow at low Reynolds numbers using FPM and SPH meshfree methods. They concluded that the accuracy of SPH in simulating this fluid flow problem is obviously lower than that of FPM [5]. Shamekhi and Sadeghy studied lid-driven cavity flow of a Newtonian fluid by meshfree method. They solved this flow problem at different Reynolds numbers up to 10000 [6].

In this work mesh free method is used for solving two dimensional non-Newtonian lid-driven cavity flow in a wide range of Reynolds numbers. The algorithm of mesh free characteristic based split has been employed for this purpose [7]. This algorithm is the extension of general characteristic based split method which was initially introduced by Zienkiewicz and Codina in finite element framework [8]. It is assumed that the non-Newtonian fluid properties obey Carreau-Yasuda rheological model. Results have been obtained for the velocity profiles at Reynolds numbers as high as 1000.

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Governing Equations

Conservation laws for any fluid, whether Newtonian or non-Newtonian, compressible or incompressible, that is, in their most general form can be written as [8]:

$$\frac{\partial \rho}{\partial t} = \frac{1}{c^2}\frac{\partial p}{\partial t} = -\frac{\partial U_i}{\partial x_i}, \tag{1}$$

$$\frac{\partial U_i}{\partial t} = -\frac{\partial}{\partial x_j}(u_j U_i) + \frac{\partial \tau_{ij}}{\partial x_j} - \frac{\partial p}{\partial x_i} - \rho g_i \tag{2}$$

where U_i is the mass flow rate, u_i is the velocity components, ρ, p, g_i, c and τ_{ij} are the density, the pressure, the body forces, the speed of sound and the stress tensor, respectively. For purely-viscous non-Newtonian fluids, the stress tensor can be written as [8]:

$$\tau_{ij} = \eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right). \tag{3}$$

where δ_{ij} is the Kroenecker delta. In Eq. 3, η is the absolute viscosity of the fluid which is assumed to be shear-dependent. For a fluid obeying the Carreau-Yasuda model, the absolute viscosity can be related to the second invariant of the deformation-rate tensor as [9]:

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{\left[1 + \lambda^a \left|II_{2D}\right|^a\right]^{\frac{1-m}{a}}} \tag{4}$$

where λ , a, m are model parameters with η_0 and η_∞ being the viscosity at zero and infinite shear rates, respectively.

In this work, the algorithm of mesh free characteristic based split has been employed for solving the problem [7]. Moving least squares (MLS) shape functions has been used for construction of shape functions. The MLS approximation has two major features that make it popular: (1) the approximated field function is continuous and smooth in the entire problem domain; and (2) it is capable of producing an approximation with the desired order of consistency [10].

Results and Discussion

The technique described above has been used to simulate steady lid-driven cavity flow of a purely-viscous non-Newtonian fluid obeying the Carreau-Yasuda model. The cavity of interest has been shown schematically in Fig. 1 and is seen to have a dimension of $[1\times 1]$. Figure 2 presents a comparison between $u-$velocity profiles along the line x = 0.5 obtained using different meshfree methods including CBSM (present method), SPH, FPM and the finite volume method at Re $\sim$ 0 for Newtonian fluids. The Reynolds number is defined as:

$$Re = \frac{\rho U_{lid} H}{\eta_0} \tag{5}$$

where U_{lid} is the lid velocity, H is the cavity height, and η_0 is the viscosity at zero shear rate.

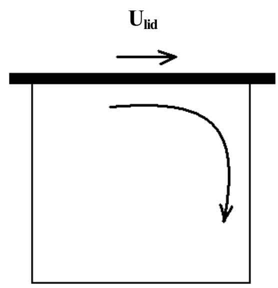

Figure 1: Two-dimensional lid-driven cavity flow in a square 2D cavity [1×1].

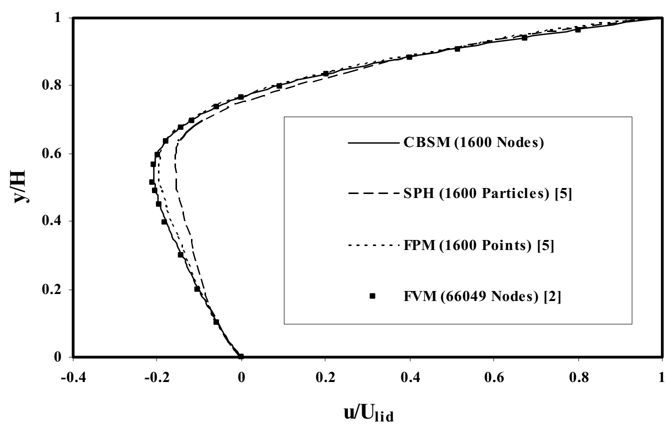

Figure 2: A comparison between $u-$velocity profiles along the line x = 0.5 obtained using CBSM, SPH, FPM and the finite volume method at Re $\sim$ 0 for Newtonian fluids.

Figs. 3 and 4 show the effect of the parameter λ on the u- and v-velocity components along the mid-planes x = 0.5 and y = 0.5, respectively. To obtain these results, the following parameters have been used in the Carreau-Yasuda model: $m = 0.5$; $a = 2.0$; $\kappa = \eta_0/\eta_\infty = 5.0$.

Figure 5 and 6 show the effect of the Reynolds number on the velocity profiles along the line x = 0.5 and y = 0.5, respectively. These figures suggest that by an increase in the Reynolds number, the flow exhibits more and more the structure of a boundary layer near the walls.

Conclusion

The CBSM (characteristic based split meshfree) method has been used for

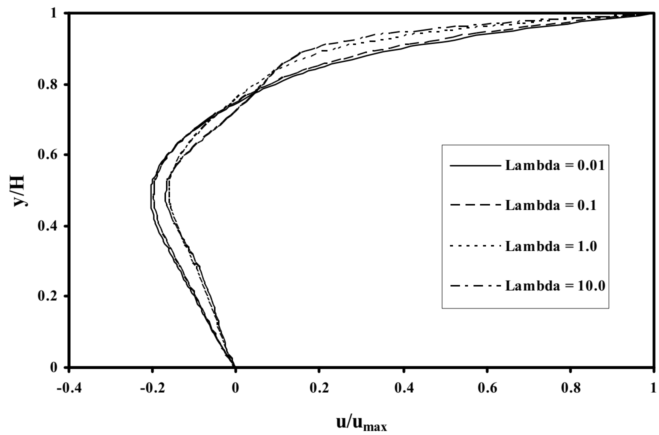

Figure 3: The effect of the parameter λ in the Carreau-Yasuda model on the u−velocity component along the line x = 0.5 obtained using CBSM method at Re = 400.

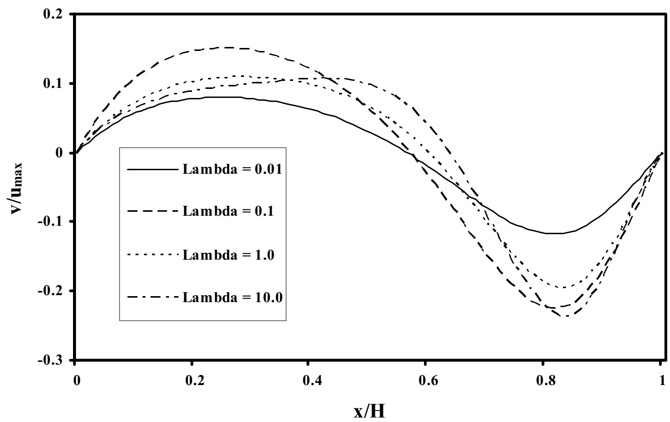

Figure 4: The effect of the parameter λ in the Carreau-Yasuda model on the v−velocity component along the line y = 0.5 obtained using CBSM method at Re = 400.

numerical simulation of the lid-driven cavity flow for both Newtonian and non-Newtonian fluids. For Newtonian fluids, results obtained from our method show good agreement with benchmark results published in the literature. They are also consistent with recent results obtained using other meshfree methods such as FPM and SPH with the advantage that the new method has better accuracy with the same number of nodes or particles. Results obtained for the Carreau-Yasuda model

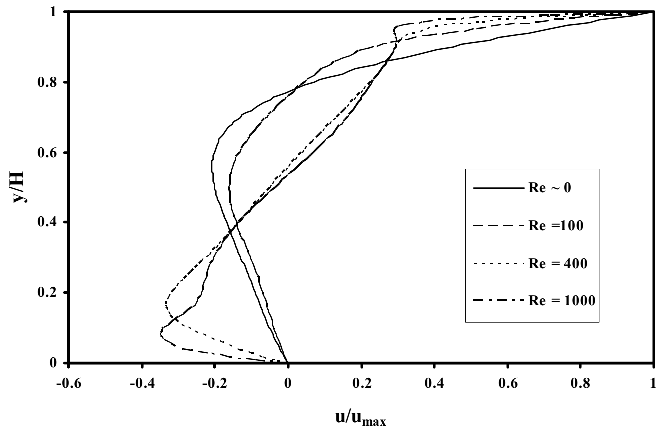

Figure 5: The effect of the Reynolds number on the u-velocity component along x = 0.5 computed using CBSM method.

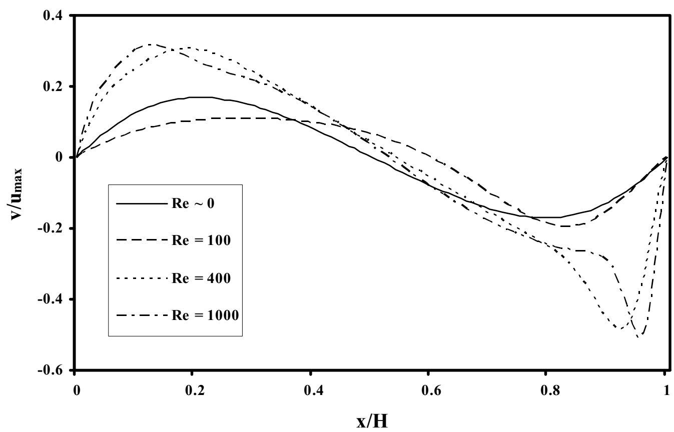

Figure 6: The effect of the Reynolds number on the v-velocity component along y = 0.5 computed with CBSM method.

reveals the strong effect of the shear-thinning behavior of the fluid on its flow kinematics within the cavity.

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