Bipolar coordinates, image method and method of fundamental solutions

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Summary

Green's functions of Laplace problems with circular boundaries are solved by using three approaches, analytical, semi-analytical and numerical solutions. For the analytical solution, we derive the Green's function by using the bipolar coordinates. Three kinds of semi-analytical solutions using (a). image method, (b). the null field BIE using the Green's third identity, and (c). the null field BIE in conjunction with superposition technique using the addition theorem are considered. A numerical method using the image concept is also employed to study the optimal location of MFS. It is interesting to find that the two frozen images appear on the two focuses in the bipolar coordinates. Besides, the optimal location of MFS sources are found to be dependent on the given source location and the geometry of circular boundaries. The image method can be seen as a special case of MFS with only three unknown strengths at known positions to be determined. Three examples including the eccentric annulus, the half-plane problem with a hole and infinite plane with two unequal holes were demonstrated to verify the finding. Agreement among the results is made.

keywords: bipolar coordinates, image method, method of fundamental solutions and Green's function

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