 ICCES, vol.12, no.2, pp.53-64

Damage detection on beam structures based on fractal theory and wavelet packet transform

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Summary

A new damage identification method based on Fractal theory and wavelet packet transform is presented in the paper. The damage identification method utilize frequency-based domain message, time-based domain message and space based domain message effectively. NeXT method were first employed to reduce loading effect of random vibration measured from structures and then the vibration sinal were decomposed into the certain level wavelet packet components. For every time array of wavelet packet component then need to calculate the mean FD values as the ultimated FD values to reduce noise effect. The location of damage in the beam can be determined by the dramatic fluctuation appearing on contour of estimated FD and the extent of the damage can be estimated by FD-based damage index of δ. As a validation, the proposed method is applied to detect damage in a simply supported beam by numerical study. The successful detection of the damage in the beam demonstrates that the method is capable of assessing both the location and size of the damage. Noise stress tests are also carried out to demonstrate the robustness of the method under the influence of noise by considering all the information of every wavelet packet component synthetically.

keywords: Fractal theory; wavelet packet transform; FD-based damage index

Introduction

Damage present a serious threat to the performance of structures and for this reason damage identification have received growing attention over the last two decades and will be one of the important directions for future research. Damage identification has the advantage of identifying damage as soon as it is initiated, and it can maintain the safety and integrity of the structures so as to avoid loss of human life and money.

Vibration-based techniques for damage identification provide powerful methods allowing one to locate and size the damage.The mathematical tools to extract useful features from the vibration signals for damage identification are significant. Among many signal analysis methods, the wavelet transform based method for vibration signal analysis is used more and more widely in damage identification due to its good time-frequency localization[1-3].A possible drawback of the wavelet

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transform is that it have difficulties to discriminate signals containing high frequency components as the frequency resolution centralize in the low frequency region.

The wavelet packet transform (WPT) is a mathematical tool, which provides a complete level-by-level decomposition of signal [4]. It enables the extraction of features from the signals that combine the stationary and non-stationary characteristics with an arbitrary time-frequency resolution. Sun and Chang [5] proposed a wavelet packet component energy index which was combined with neural network models for damage assessment. S.S. Law et. al [6] have brought forward the sensitivity of wavelet packet transform component energy with respect to local change in the system parameters to identify damage. The wavelet energy rate index computed by decomposeing Dynamic signals measured from structures into the wavelet packet components was proposed by Jiangang Han et.al[7] which is used to locate damage.

In the last several years, damage identification methods based on nonlinear theory such as attractor and fractal theory have received growing attentions. Hadjileontiadis L J et al.[8] also brought forward a new technique for crack identification in beam structures based on fractal dimension analysis. The proposed approach adopts Katz's estimation of the fractal dimension employing the sliding window. The location and the size of the cracks are related to the fractal dimension measure which represent the local irregularity or nonlinear of mode shape. In the paper, a new arithmetic based on fractal theory and wavelet packet transform is proposed.

The damage identification method utilize frequency-based domain message, time-based domain message and space based domain message effectively. The predominance of the method is that it has the ability to location and sizing damage under heavy noise and errors. The free vibration of every sampling point were first obtained by NeXT method analysing Dynamic signals measured from structures and then The free vibration singles were decomposed into the wavelet packet components. The location of damage in the beam can be determined by the peak value appearing on the estimated FD profile and the extent of the damage can be estimated by$\delta$of every wavelet packet component.

Fractal theory and wavelet packet based damage identification

Theoretical background

Wavelet packet transform

Generally, an exact quadrature mirror filter $h(k)$ is defined as

$$\sum_k h(k-2i)h(k-2j) = \delta_{i,j}, \quad \sum_k h(k) = \sqrt{2} \tag{1}$$

If $\{u^i(t)\}$ is the wavelet function, there will be the following recursive equations:

$$u^{2i}(t) = \sqrt{2} \sum_{k=-\infty}^{\infty} h(k) u^i(2t-k) \tag{2}$$

$$u^{2i+1}(t) = \sqrt{2} \sum_{k=-\infty}^{\infty} g(k) u^i(2t-k) \tag{3}$$

where $g(k) = h(k+1)(-1)^k$. The above function is a dyadic scaling equation.

The WPT contains a complete decomposition at every level and hence can achieve a higher resolution no matter in the low frequency region or in the high frequency region. The recursive relations between the jth and the $(j+1)$th level component single are

$$f_j^i(t) = f_{j+1}^{2i-1}(t) + f_{j+1}^{2i}(t) \tag{4}$$

Where f_j^i is the ith component of j level wavelet packet. Therefore, the WPT has the ability to extract features from the signals including the stationary and non-stationary characteristics with an arbitrary time-frequency resolution.

Fractal dimension

From the definition of Fractal, the fractal dimension is directly connected to the complexity of the vibration signal across the beam length which is induced by the existence of damage.

Consider a homogeneous, uniform cross-sectional, one-dimensional simple supported beam with m elements (sampling interval). Assume that the input–output relationship of the beam is linear. A sliding window of n sample length is employed to estimate FD which is assigned to the midpoint of the sliding window x_p.The Katz's fractal dimension for point x_p can be then calculated using the following function [8,9]

$$FD(x_p) = \frac{\log_{10}(n)}{\log_{10}[\frac{d(x_p)}{L(x_p)}] + \log_{10}(n)} \tag{5}$$

Where n is the number of steps or sample length in the sliding window; $L(x_p)$ is the total length of the sliding window which can be defined as the sum of distances between successive sampling points in the pth sliding window, and $L(x_p)$ and n have the connection of $L(x_p) = n \times l$, l is the mean distance between successive sampling points, $d(x_p)$ is the farthest distance between the first and pth points of the sequence in a sliding window and can be written as $d(x_p) = \max[dist(1,q)]$.

Fractal theory and wavelet packet transform based damage localization method

The damage localization method is a four-step process designed to accurately locate damage.

The first step is to acquire displacement response data from the operating structure. Long time histories of continuous data are desired, provided the operating conditions are relatively stationary.

The second step is to calculate auto- and cross-correlation functions from these time histories using standard techniques [10]. Correlation functions are commonly used to analyze randomly excited systems, which can be treated as though they were free vibration responses—that is, sums of decaying sinusoids[11].

The third step of the damage identification method is decomposing correlation functions into the j level wavelet packet components by adopting DB5 mother wavelet functons. There will be 2^{j-1} wavelet packet components generated.

The fourth step is for every wavelet packet component to calculate the estimated FD value (see Eq.(5)) along the structure of every time point, then calculate the mean FD value along the structure as the final FD result of every wavelet packet component. The jth wavelet packet component have the final FD result as Eq. (6) when the sampling step number of displacement response data is S.

$$FD(x_p) = \frac{1}{S}\sum_{t=1}^{S} FD(t, x_p) \tag{6}$$

Fractal theory and wavelet packet transform based damage extent estimation method

For quantifying damage, a de-trend analysis is employed to remove the low trend and clearly reveal the characteristics of the FD peak of interest [12].In Figure 3, a example of the de-trend analysis is shown. the maximum of the FD peak along with the estimated trend is shown in (a), the de-trend analysis is applied in (b) and the FD peak is efficiently isolated from the underlying trend.

Comparing with the low trend, index δ for damage extent estimation can be propoed.

$$\delta = \frac{FD - FD_{\text{trend}}}{FD_{\text{trend,max}}} \tag{7}$$

where FD is the estimated fractal dimension value for the structure, FD_{trend} is the estimation of the trend using an interpolation Procedure; $FD_{\text{trend,max}}$ is the largest value of the estimated trend.

Numerical studies

Description of test beam

To validate the applicability of the Fractal theory and wavelet packet transform based damage identification method, numerical experiment of the simple supported beam was performed to examine the effectiveness of FD damage index by using the commercial software ANSYS and the pre-damage or post-damage modal parameters of the model including frequencies and mode shapes were obtained numerically. The model of the simple supported beam had the length of $L = 20$m and the rectangular cross-section of $b \times h = 2 \times 2$m. For modal analysis purposes the beam was divided into 80 3-D Elastic Beam elements and each element size was 0.25 m. The material properties including the elastic modulus, Poisson's ratio's and mass density were assigned to be $E = 30Gpa$; $\nu = 0.167$ and $\rho = 3000kg/m^3$, respectively. In the paper, the damage severities are modeled by reducing the stiffness of specific elements (see Fig.1).

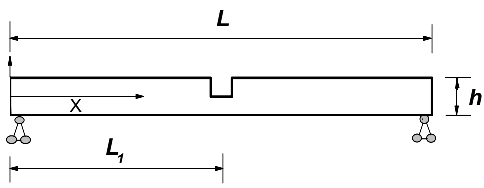

Figure 1: Simple supported beam with a crack at position of L_1

Damage identification by Fractal theory and wavelet packet transform based damage identification method

Damage localization

The load step was set to be 5000, and the simulative ambient excitation was acts on the location of one quarter and three quarter of the beam. The estimated FD profile of components for the third level wavelet packet transform versus location of the damaged beam with 40% damage at multiple positions of 6.00-6.25m and 10.00-10.25m with no noise are shown in Fig. 2. In these subfigures, it is demonstrated that the profile of estimated FD of all component have reveal damage information and can localize multiple damage but with different estimated FD value of different component at damage location, which is caused by different wavelet packet component energy.

The profile of Estimated FD of the free vibration responses which employing no wavelet packet transform are presented in Fig. 3, it is demonstrated that damage at position of 6.00-6.25m is not easy to localize as the peak value is much smaller than the peak value at position of 10.00-10.25m.

It is shown that for the case that employ no wavelet packet transform, the damage information may not display wholly sometimes. However, this problem can be settled by wavelet packet transform.

Damage quantification

It is easy concluded that index δ is comply with damage extent of structure one and only, and have little connection with the damage position. As shown in Figure. 4, the curve estimated δ versus stiffness loss ratio when damage uniformly distributed in a sampling interval of 0.25m is obtained which give the feasibility for

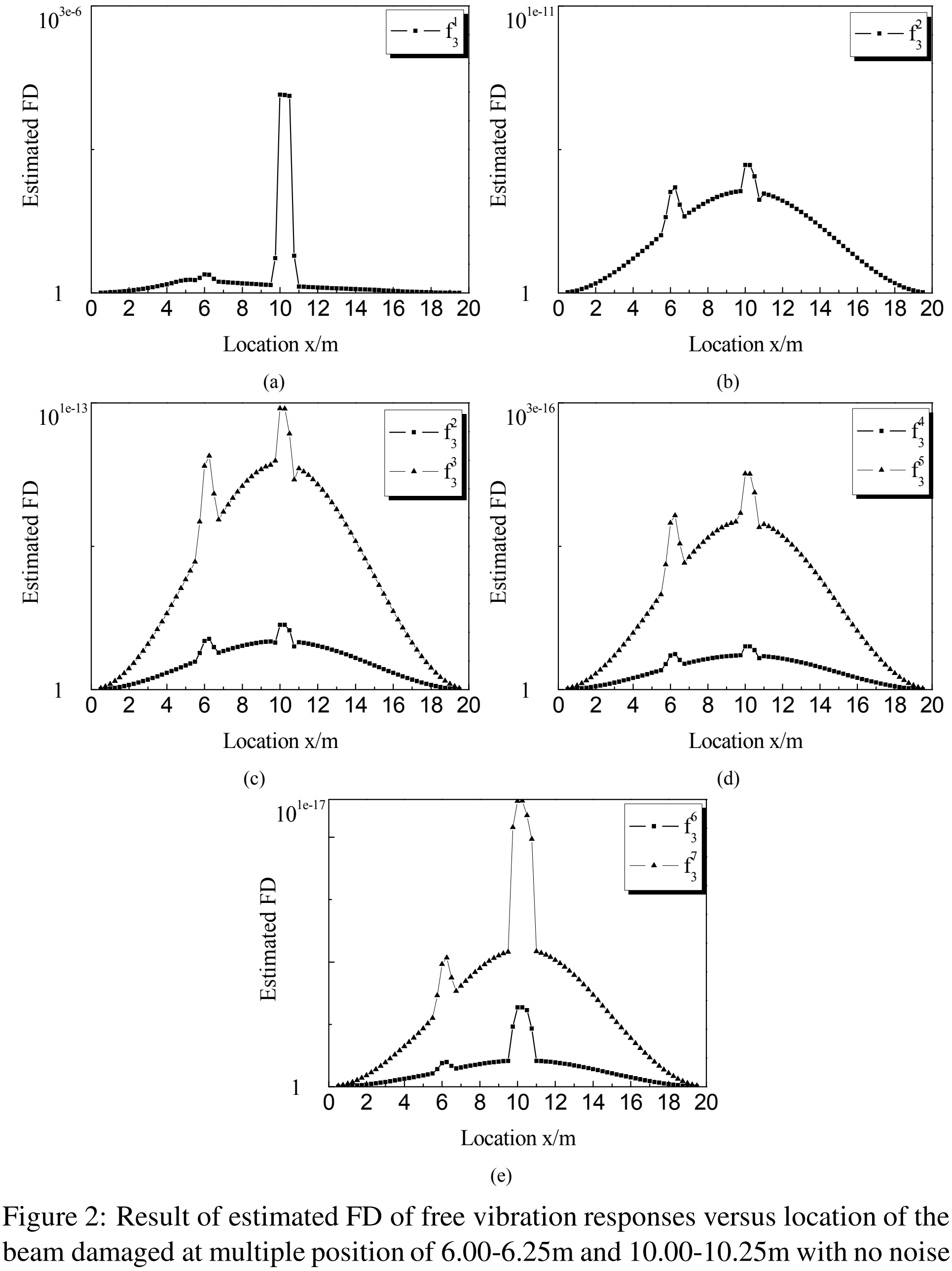

Figure 2: Result of estimated FD of free vibration responses versus location of the beam damaged at multiple position of 6.00-6.25m and 10.00-10.25m with no noise when employing the third level wavelet packet transform.

quatify damage accuracy in this case.

The value of index δ and the estimated stiffness loss resulted from the curve of Figure 4 for every wavelet packet component of the case of the previous section is obtaioned as Table.1 and Table.2. It is seen that the estimated stiffness loss for position of 10.00-10.25m is changeful, but a majority of results is close to the real stiffness loss, so we can average the most similar results to estimate damage extent. The final estimation result is 32.3%. Nevertheless, different frequency range have little influence on estimating damage extent when damage position is 6.00-6.25m which is not at mid-span, the estimated damage severity value for this position for every wavelet packet component is approximately similar, So the average of the results which is can be regard as estimated damage extent. The final estimation result is 33.6%. So it is seen that employing $\delta$is an effective method for estimation damage extent when combining with fractal theory and wavelet packet transform based damage identification method.

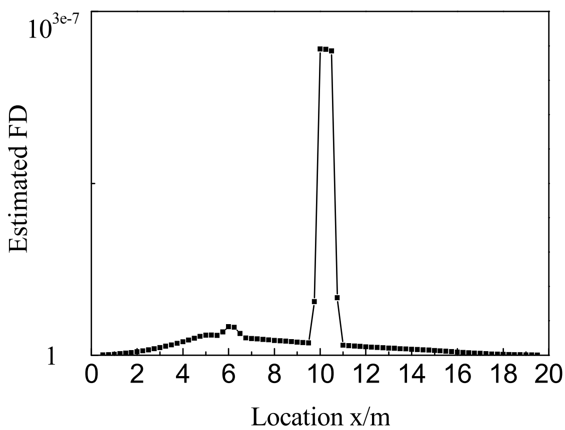

Figure 3: Result of estimated FD of free vibration responses versus location of the beam damaged at multiple position of 6.00-6.25m and 10.00-10.25m with no noise when employing no wavelet packet transform

Table 1: Damage quantification results of test beam at position of 10.00-10.25m

		f_3^1	f_3^2	f_3^3	f_3^4
δ	0.9770	13.662	0.2726	0.3703	0.2499
Estimated $\Delta EI/EI$	53%	88%	24%	30%	23%
		f_3^5	f_3^6	f_3^7	f_3^8
δ		0.3792	0.3765	0.7917	0.6672
Estimated $\Delta EI/EI$		30%	30%	47%	43%

Noise effect test

It is known that the Components energies of vibration response for the certain level wavelet packet transform is diverse, but for Gaussian noise the components energies for the third level wavelet packet transform is more uniform. There will

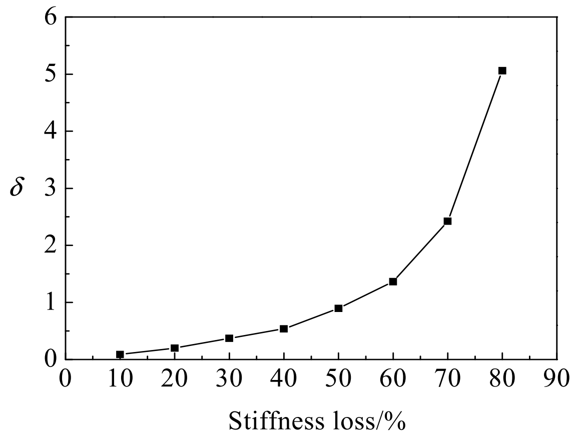

Figure 4: Estimated FD versus stiffness loss ratio when damage uniformly distributed in a sampling interval of 0.25m

Table 2: Damage quantification results of test beam at position of 6.00-6.25m

	f_0	f_3^1	f_3^2	f_3^3	f_3^4
δ	0.5086	0.2792	0.3803	0.390	0.3767
Estimated $\Delta EI/EI$	38%	24%	31%	32%	30%
		f_3^5	f_3^6	f_3^7	f_3^8
δ		0.5078	0.5073	0.5041	0.5061
Estimated $\Delta EI/EI$		38%	38%	38%	38%

be some Components that the noise effect is relative smaller. Furthermore, as the number of load steps becomes bigger, the noise effect will be decreased for this method. the So it is feasible to reduce the noise effect on damage identification through wavelet packet transform.

To introduce noise effect the additive zero-mean Gaussian noise was constructed in such a way that ensured the SNR values in all original vibration signal of the damaged location (LSNR). If no wavelet packet transform was employed as shown in Fig 5, the estimated FD at damage position of 6.00-6.25m will be submerged by the fluctuation induced by noise. So damage can not been localized from the time history responses completely as LSNR=50dB.

Contrarily, in Fig.6 there are the profiles of estimated FD of components for the third level wavelet packet transform with the same case as before. Only For the first two components, the peak value of estimated FD induced by the damage at the location of 6.00-6.25m is indistinguishable from the others caused by the noise. But for the third to the eighth component the damage can be clearly localized by the peak value of estimated FD at damage location when SNR=50dB. It is demonstrated that wavelet packet transform advance the efficiency of FD-based damage

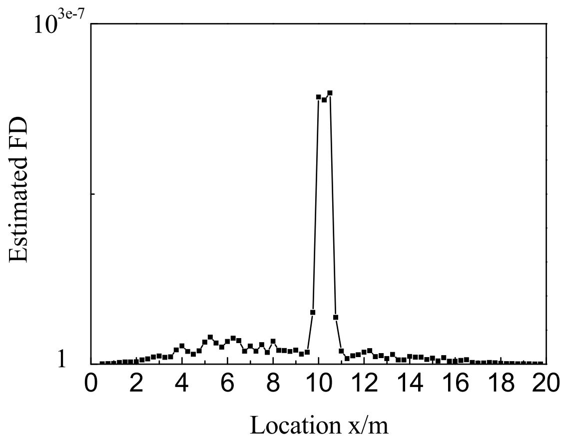

Figure 5: Result of estimated FD of free vibration responses versus location of the beam damaged at multiple position of 6.00-6.25 and 10.00-10.25 with with noise lever of SNR=50dB when employing no wavelet packet transform

identification method.

Conclusions

In this paper, the fundamental issues of Fractal theory and wavelet packet transform based damage identification method are addressed.

This present method is simple to be implemented and does not require the knowledge of the healthy structure. The method is implemented on the displacement response data, the free vibration responses of a damaged beam is first obtained analytically by Natural Excitation Technique, which then decomposed into the $j$level wavelet packet components by adopting DB5 mother wavelet functions, for every time array of wavelet packet component then need to calculate the mean fractal dimension value along the structure. The location of damage in the beam can be determined by the peak value appearing on the estimated FD profile and the extent of the damage can be estimated by δ of every wavelet packet component.

The effectiveness and robustness of this method under the influence of noise has been demonstrated by the noise stress tests. Compared with existing damage detection methods, the method is capable of assessing both the location and size of the damage under heavy noise by considering all the information of every wavelet packet component synthetically. It is easy predict that as the number of load step and decomposition level becomes bigger, the robustness of this method under the influence of noise is better.

The damage identification method utilize frequency-based domain message, time-based domain message and space based domain message effectively. The present method can be used efficiently and effectively in damage identification and

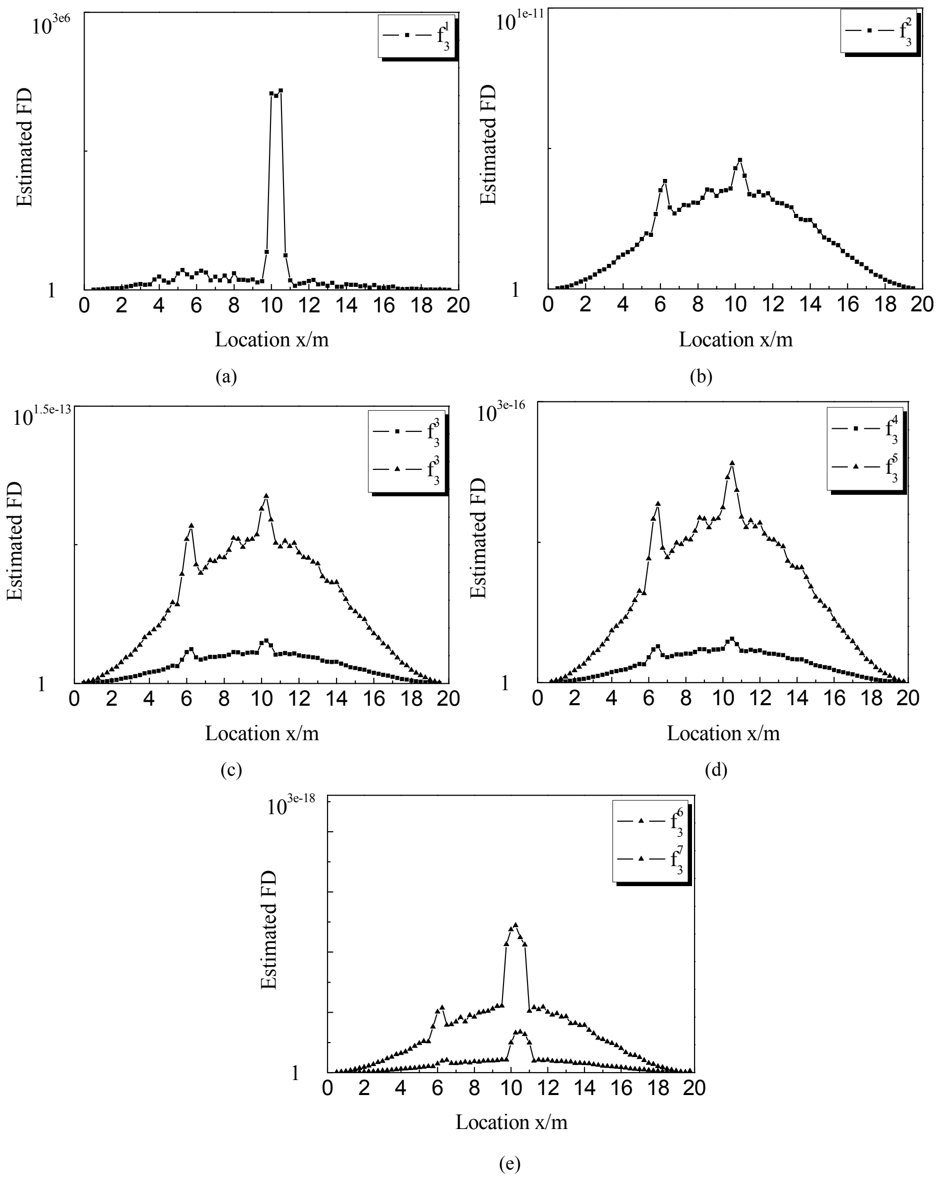

Figure 6: Result of estimated FD of free vibration responses versus location of the beam damaged at multiple position of 6.00-6.25m and 10.00-10.25m with noise lever of SNR=50dB when employing the third level wavelet packet transform

health monitoring of beam-type structures but also can extended to more complex structure.

Acknowledgement

This research is financially supported by NSFC (Grant Nos. 50525823, 50538020 and 50278029) and by the Ministry of Science and Technology, China (Grant Nos. 2006BAJ03B05, 2007AA04Z435 and 2006BAJ13B03).

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