# Computation of derivatives of stress intensity factors for two-dimensional anisotropic crack problems using fractal finite element method

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## Summary

Probabilistic fracture mechanics (PFM) blends the theory of fracture mechanics and the probability theory provides a more rational means to describe the actual behavior and reliability of structures. However in PFM, the fracture parameters and their derivatives are often required to predict the probability of fracture initiation and/or instability in cracked structures. The calculation of the derivatives of fracture parameters with respect to load and material parameters, which constitutes size-sensitivity analysis, is not unduly difficult. However, the evaluation of response derivatives with respect to crack size was a challenging task, since it requires shape sensitivity analysis [1]. Using a brute-force type finite-difference method to calculate the shape sensitivities was often computationally expensive. In that approach, large amount of deterministic finite element analyses were required for a complete reliability analysis. This particularly demands an efficient method to evaluate the sensitivity of fracture parameters accurately in the context of PFM. In this paper a new continuum shape sensitivity based method was presented for the evaluation of the fracture parameters and their derivatives for cracks in an anisotropic, linear-elastic material [2]. This method was based on the concept of fractal finite-element method (FFEM) [3]. Unlike virtual crack extension techniques, no mesh perturbation was needed in the proposed method to calculate the sensitivity of stress-intensity factors. Numerical results show that J-integral or stress intensity factors and first-order sensitivities of J-integral or stress intensity factors obtained by using the proposed methods are in satisfactory agreement with the reference solutions.

The FFEM is a recently developed method for calculating stress intensity factors (SIF) in linear elastic crack problems. Its original form, the fractal two-level finite element method (F2LFEM), was first proposed by Leung and Su in 1993. Since then, its efficiency in calculating SIF has been well demonstrated with a wide range of 2D crack problems under mode I and mode II loading. FFEM separates a 2D cracked elastic body into a regular and a singular region with the latter enclosing the crack tip. Both the regular and the singular regions are model by conventional, isoparametric finite elements. However, within the singular region an infinite num-

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ber of elements are generated by a self-similar, fractal process to capture the singular behaviour at the crack tip.

# References

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