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Fracture behavior of vulcanize rubber on the variations of thickness to width ratios

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Summary

The purpose of this research is to determine fracture effects of vulcanize elastomer at various thickness to width ratios under opening mode fracture mechanics. The J-integral values from finite element method are compared with those from experiment. In this research, fracture tests perform under Single Edge Notch Tensile (S.E.N.T) tests using the multiple-specimen method. Compare to the experimental data, the numerical results are acceptable at the deep crack length, *i.e.* high crack length to width ratio. The k values from the 3-D numerical calculation show that thin sheet is dominated by plane stress state, but thick sheet is dominated by plane strain condition. The results reveal that, in 2-D approximation, the plane strain assumption can be used at the thickness to width ratio (t/w) at 0.88 up.

keywords: Vulcanize rubber; thickness variations; Fracture behavior; J-Integral.

Introduction

Rubber is a highly deformable material that is widely used in many industries. One of the problems in rubber industry is the crack growth which can be solved by fracture mechanics [Hertberg (1989)]. The fracture mechanics in rubbers is generally tackled by using a global approach introduced by Griffith, the first scientist who works on the fracture behavior characterization [Mirzaei (2006)]. Griffith assumes that a crack initiation begins from defects in the material structure. Considering some materials with linear elastic behavior, Griffith defines linear fracture toughness, G, as a change of the potential energy, $\prod$, to change of the crack area A:

$$G = -\frac{\partial \prod}{\partial A} \tag{1}$$

Equivalent to the G parameter in linear fracture mechanics, Rice's J-integral parameter can be used in non-linear elastic materials [Richard (1989)]. This integral defines as:

$$J = \int_\Gamma \left(W dy - T_i \frac{\partial u}{\partial x} ds \right) \tag{2}$$

where x and y are arbitrary positions in rectangular coordinates, W; the strain energy density function, T_i; the traction at the i component, Γ; a contour around the

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crack tip, ds; an infinitesimal line element on Γ, and u; a displacement from the traction in x direction on ds. In thin sheets, the fracture parameters, *i.e.* the G and J parameters, are strongly depended on the specimen thickness, but independent to that for thick samples [Perez (2004)]. J-integral can also be written in term of a dimensionless parameter k, strain energy density of an uncrack specimen W_0, and the crack length a in S.E.N.T test. which is written as [Trimbell, Wiehahn, Cook, and Muhr (2003)];

$$J = 2kW_0a \tag{3}$$

A problem of finding fracture toughness is greatly simplified if it can be assumed to be a 2-D problem. For example, the classical fracture mechanic problem of thin sheet is normally assumed to be a plane stress problem. However, in fact, the plane strain effects are also occurred, and there is no upper limit to define the minimum thickness that the plane stress condition can be assumed. The aim of this research is to study the fracture toughness related to the thickness effects on the plane stress and plane strain conditions in a rubber specimen.

Constitutive model

In this research, the strain energy density function W, used to describe the rubber mechanical behavior, is approximated by the order 2 polynomial hyperelastic model (N = 2) which is [ABACUS (2004)],

$$W = \sum_{i+j=1}^{N} C_{ij}(I_1-3)^i(I_2-3)^j \tag{4}$$

where i and j are integers, C_{ij} are constant coefficients of the material,I_1 and I_2 are the 1^{st} and 2^{nd} invariants of the deformation tensor. The C_{ij} coefficients are determined by pure uniaxial tests. The stress as a function of the stretch ratio λ can be derived from the strain energy density function W in Eq 4 [Gent (1992)] where,

$$\sigma = \lambda\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial \lambda} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial \lambda}\right) \tag{5}$$

Method

Experiment

The fracture tests are performed under S.E.N.T test [Hocine, Abdelaziz, Ghfiri, and Mesmacque (1996)] using crosshead speed at 200 mm/min. on the multiple-specimen method [Gdoutos (1993)]. The specimen dimensions of 150-mm width (w), 50-mm length (L) and 14 mm thick (t) are prepared for the different crack lengths (a) of 5, 10, 15, 20 and 25 mm by razor blade (see Fig. 1a).

Numerical comparison

An ABAQUS finite element program [ABAQUS (2004)] is used for the numerical study of the fracture toughness and validated with the experiment results. The full 3-D model is shown in Fig. 1b using 1,728 singularity elements at the crack tip vicinity and 4,864 quadrilateral elements with eight nodes located far from the crack (see Fig. 1c). The material is assumed virtually hyperelastic, homogeneous, isothermal, and incompressible.

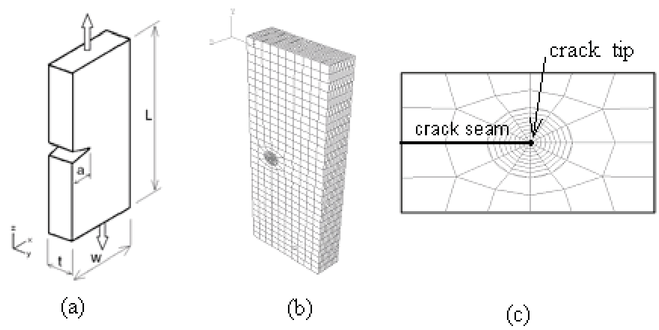

Figure 1: (a) specimen dimensions, (b) model meshing, (c) crack tip.

Result and discussion

Fig. 2 illustrates *J*-integral values from experiment relative to the strain at the crack length to width ratios (*a*/*w*) from 0.1 to 0.5 by using the multiple-specimen method. From the figure, the *J*-integral values increase with the strain and crack length per width ratios. Fig. 3 compares the *J*-integral data from the experiment with those from the numerical for the crack length per width ratios at 0.1 and 0.5. At low crack length per width ratio (a/w= 0.1), the numerical result underestimate the experimental one by half for all strain, while at high crack length per width ratio (a/w= 0.5), the numerical result seems acceptable. The major error at the small crack length per width ratio is caused by the mathematical correlation used to estimate the *J*-integral values from the experiment data. These results confirm previous work by Hocine, Abdelaziz, Ghfiri, and Mesmacque (1996).

Fig. 4 shows relation of the dimensionless k and the position along the specimen thickness (y/t) at various thickness to width ratios, *t*/*w*, from numerical simulation at 10 percent strain and a/w= 0.5. From the figure, the plane strain condition clearly occurs at the middle of the specimen (y/t = 0 up to 0.25) and the plane stress phenomena occurs at y/t = 0.25 and up to 0.5 (the free surface). At *t*/*w* = 0.88 or more this effect is more obvious since k is independent to *y*/*t* at the middle of the specimen. To represent k along the thickness, a bulk value of k is approximated by its average k_{ave}. The relations of k_{ave} with the strain in the specimen for crack size a/w = 0.5 at various *t*/*w* has been shown in Fig. 5. k_{ave} increases with the specimen

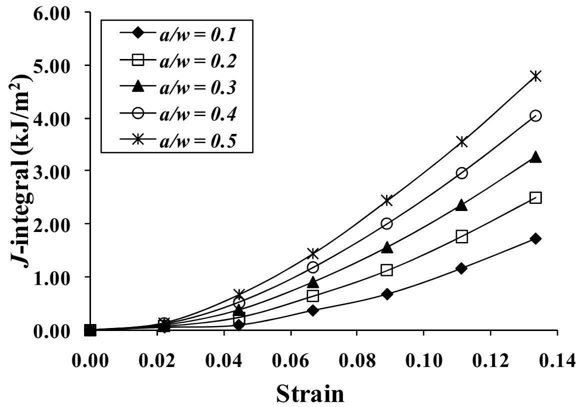

Figure 2: J-integral experimental data versus strain at various a/w.

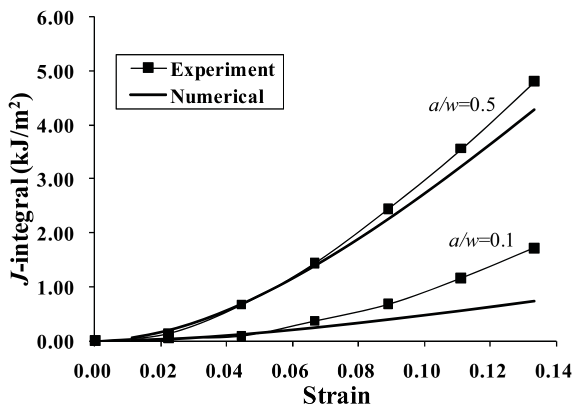

Figure 3: J-integral values from the experiment compare with those from the numerical for different crack length per width ratio (a/w).

thickness to width ratio t/w. Fig. 6 illustrates k_{ave}. as a function of the thickness to width ratio (t/w) at a/w = 0.5 at 10 percent strain. At the beginning, k_{ave} greatly depends ont/w ratio and less dependent on the t/w ratio at the t/w = 0.88 or more. This means that, for very thin plate, the specimen is plane stress dominated, and for thick plate, the specimen is plan strain dominated. This phenomenon confirms the 2-D classical fracture mechanic assumptions.

Conclusion

Strain energy release rate represented by J-integral for a crack in rubber can be approximated by using ABAQUS, a finite element commercial program. Rubber can be characterized by the polynomial order-2 material function. J-integral values from simulation agree with the experiment data when deep crack ($a/w \geq 0.5$). The

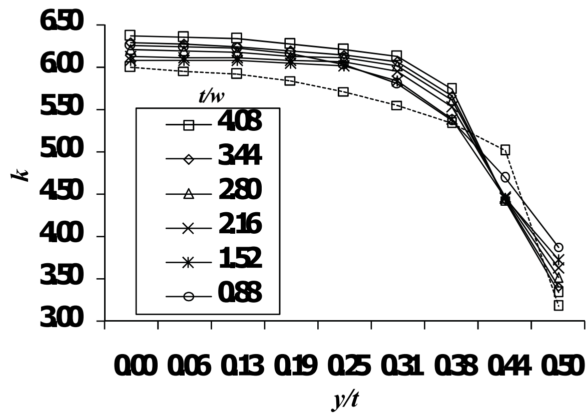

Figure 4: k along the thickness for different t/w at $a/w = 0.5$ and 10 percent strain.

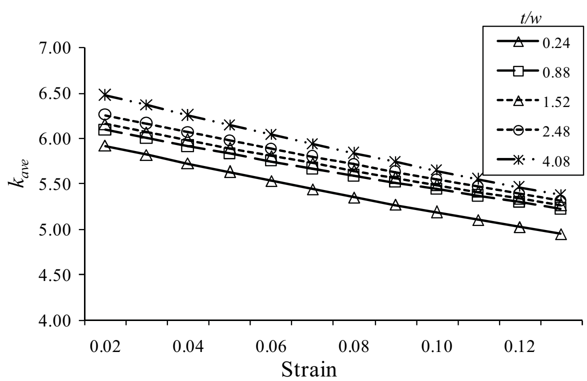

Figure 5: effect of k_{ave} with strain at $a/w = 0.5$.

k value versus *y/t* plot shows that the material exhibits plane stress characters at the edge of the specimen, while the plane strain presents at the center. Thus, this confirms that the thin sheet is dominated by the plane stress, and the thick sheet is dominated by the plane strain assumption. The critical value of the t/w ratio is about 0.88.

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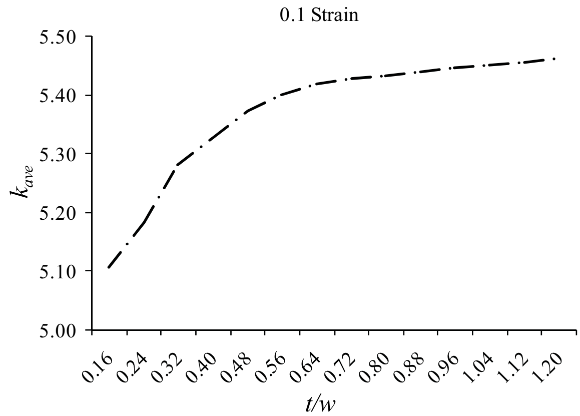

Figure 6: k_{ave} versus thickness and width ratio at a/w = 0.5 and 10 percent strain.

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