

## A Numerical Solution of 2D Buckley-Leverett Equation via Gradient Reproducing Kernel Particle Method

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### Summary

Gradient reproducing kernel particle method (GRKPM) is a meshless technique which incorporates the first gradients of the function into the reproducing equation of RKPM. Therefore, in two-dimensional space GRKPM introduces three types of shape functions rather than one. The robustness of GRKPM's shape functions is established by reconstruction of a third-order polynomial. To enforce the essential boundary conditions (EBCs), GRKPM's shape functions are modified by transformation technique. By utilizing the modified shape functions, the weak form of the nonlinear evolutionary Buckley-Leverett (BL) equation is discretized in space, rendering a system of nonlinear ordinary differential equations (ODEs). Subsequently, Gear's method is applied for temporal discretization of the ODEs. Through numerical experiments, employment of a moderate viscosity seeks the efficacy of the solution when the diffusion term is important; moreover, application of a small viscosity confirms the potential of the approach for treatment of the problems involving steep gradient regions. The outcomes are verified by performing convergence tests using uniformly spaced particles. Consideration of non-uniform distribution of particles further demonstrates the virtue of the presented methodology in producing smooth profiles in the critical regions near the fronts.

**keywords:** Buckley-Leverett equation; Gradient reproducing kernel particle method; Steep gradient; Two-phase flow; Meshless; Nonlinear evolutionary partial differential equation

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