Implementation of 3D constitutive model on RC frame using EAS based lower order element in the elastic range

Amiya Kr. Samanta¹, Somnath Ghosh²

Summary

This paper deals with the implementation of hypo-elasticity based 3D constitutive model on Reinforced Concrete (RC) frame employing finite element technique, which uses lower order isometric solid elements HCiS18 with enhanced assumed strain (EAS) formulation to evaluate load-deformation, internal stresses produced in the elastic regime. Due attention has been paid to model concrete and the reinforcing steel with different physical and mechanical properties, which are combined together to represent its composite behaviour accurately in perfect bond situation. An in-house FORTRAN computer code has been developed for the purpose. The results of the finite element analysis are presented, compared and discussed.

keywords: Lower order elements, Three-dimensional, Enhanced assumed strain, Perfect bond, Finite element approach, RC frame.

Introduction

3D modeling may be a potential approach in order to achieve more realistic solution in general. In many applications, the standard quadratic 20-noded solid /hexahedral element has been used, which necessitates large computational time and cost. Since comparatively lower order elements have the advantages for 3D analysis due to easy mesh generation, data interpretation and lower computational time, improvement of such type of element performance has drawn attention of the researchers. Among the lower order elements, the linear isoparametric elements are the simplest constant strain elements. Wilson, Taylor presented a method of incompatible modes in this regard to improve the performance of the standard linear quadrilateral and hexahedral elements. Simo and Rifai [9] introduced a new concept of enhanced strain method, where the strain field is enhanced with the inclusion of additional variables. These additional variables, which are introduced in the calculations of the deformed state, have got no physical significance in numerical solution as it is eliminated at the element level. In case of 2D analysis Cesar et al. [5] also contributed to eliminate the volumetric locking. Not much attention was so far attributed to improve 3D analysis using enhanced strain lower order solid elements. A remarkable progress and accuracy has been obtained in this case by the element HCiS18 introduced by Sousa et al. [10, 11], even with the coarser meshes as it improves the original strain field in an additive way. In the present case, this element has been used to model the parent material i.e. concrete of the reinforced concrete structures.

¹Deptt. of CE., NIT, Durgapur -713,209, W.B. India. ksnitd@gmail.com

²Deptt. of Civil Engg., Jadavpur University, Kolkata - 70,032, W.B. India

Although plain concrete may be assumed as homogeneous medium in a macro scale for the sake of analytical modeling, RC structures are highly non-homogeneous medium due to discrete presence of the reinforcements. In general there are three methods available for modeling of reinforcement, e.g. the discrete, the smeared and the embedded approach. The first two methods are not generally applied to 3D applications, as they do not represent actual stiffness distribution of reinforcements over the parent element. This idea is well implemented by employing embedded formulation, proposed by Elwi et al. [8] and Barzegar et al. [3]. As a result, the mesh design becomes independent of reinforcement layout. Here the author has used the same method as proposed by Cheng et al. [6] and Barzegar [3] due to its simplicity to handle problems of 3D analysis of reinforced concrete structures in perfect bond situation. This paper simulates the elastic response of RC frame considering (1)concrete as a solid isotropic homogeneous medium, which uses hypoelasticity based constitutive model, (2) lower order solid elements to represent concrete medium, which reduces time and associated cost in terms of easy and simple mesh generation together with data interpretation, (3) reinforcements as 1D truss elements considering only the axial deformation in its exact spatial position without affecting the parent element mesh in perfect bond situation following embedded approach. Also a good effort has been attributed to develop a number of subroutines with the aid of Bathe et al. [4] for the purpose, specific to the problem and it doesn't uses any block available commercially. The present paper is an attempt on a continuing investigation [1, 2] of the finite element analysis of reinforced concrete members utilizing lower order solid hexahedral elements including assessment of the effect of reinforcement in perfect bond situation.

Concrete

Concrete is considered as the most important structural material in civil engineering. As highly heterogeneous medium, its behaviour is very complex. However in this paper, initial effort has been given only to model concrete at very low stress level. Hence concrete may be assumed to behave linearly elastic and isotropic even in multi-axial stress states for all engineering purpose. From this standpoint only two material parameters are required viz. Young's modulus (E) and Poisson's ratio (μ) for finite element modeling of concrete /parent material of reinforced concrete structures. A classical displacement based isometric formulation is followed with three translational D.O.F. at each node of 8-noded solid hexahedral elements to model the parent material (concrete) of the reinforced concrete. The element stiffness for the continuum in 3D stress state is derived in a very straightforward way as,

FE formulation

$$K_P^e = \sum_P B_P^T . T_{\varepsilon,gl}^T . D_P . T_{\varepsilon,gl} . B_P . dV_P.$$
(1)

Where, $[D_P]$ is the elasticity matrix of continuum /parent material, $\{B_P\}$ is the enhanced strain displacement matrix and $[T_{\varepsilon,gl}]$ is the transformation matrix for the volume of domain (V_P) . With the effect of volumetric locking, the standard 8-noded Serendipity (parent) element grossly underestimates structural response (deflection). Here an enhanced strain formulation proposed by Sousa et al. [11] is incorporated based on extra compatible modes of deformation. These extra modes of deformation are eliminated at the element level by static condensation method as described in Cook et al. [7]. In particular, element is designated as HCiS18, where 18 new additional variables are associated in addition to the usual strain field. With these 18 nos. enhanced strain components, the size of the element stiffness matrix becomes 42x42, which may be reduced to 24x24 by the method of static condensation before the assembly process for the entire domain. The material properties and related hypoelasticity based modeling of concrete behaviour may be found in the similar works /references of the author [1, 2].

Reinforcement

The reinforcement bars are modeled following classical embedded approach, as shown in Figure-1, using the same displacement field same as the parent element. The stiffness of the reinforcements is calculated as one-dimensional elements embedded in the space of parent element and is then super-imposed on the stiffness of the parent element and thus composite stiffness of an element is derived. The same strain-displacement matrix 'B_P' is utilized to evaluate the stiffness of the reinforcement(s), the strain displacement matrix has been computed at the respective gauss point(s) of the reinforcements expressed in terms of the intrinsic coordinates of the parent element. A Newton root finding algorithm in 3D is used for this purpose, where the known integration points of reinforcement in global coordinates are computed in local coordinates using an inverse mapping procedure based on iterative method. Thus with D_R as the elasticity matrix the stiffness contribution of reinforcement towards the element becomes

$$K_R^e = \sum_R B_P^T . T_{\varepsilon,gl}^T . D_R . T_{\varepsilon,gl} . B_P . dV_R,$$
⁽²⁾



By adding up equation (1) and (2), the total stiffness of a 3D reinforced concrete element is calculated, in case there is a reinforcement embedded in the parent /concrete element as

$$K^{e} = \sum_{P} B_{P}^{T} . T_{\varepsilon,gl}^{T} . D_{P} . T_{\varepsilon,gl} . B_{P} . dV_{P} + \sum_{R} B_{P}^{T} . T_{\varepsilon,gl}^{T} . D_{R} . T_{\varepsilon,gl} . B_{P} . dV_{R}$$
(3)

Case study and discussion

A single bay substitute frame as shown in Figure-2 with column section $125(b) \times 250(d)$ and beam section $125(b) \times 200(d)$ has been investigated. The beam is 2.0m(=L)long provided with 2 nos. ordinary ribbed reinforcing steel placed at the bottom. It subjected to two point loads (4.0 MT each) at quarter span of the beam only apart from its self-weight. The concrete has the characteristic strength of 25Mpa, elastic modulus (E_c) = 25000 MPa, Poisson's ratio $\mu = 0.17$ and the reinforcement bar has the elastic modulus E_s = 200000 MPa with effective cover (d') on both sides equal to 50mm.



Figure 2: Single Bay Substitute Frame

The mesh of 40 elements of size $100 \times 100 \times 125$ for the beam and another 40 elements of size $100 \times 125 \times 125$ is generated with a preprocessing subroutine for the



Figure 3: Deformed Configuration of Frame

parent material. The central /mid-span deflection of the beam using the developed code has been obtained as 1.1715mm, whereas conventional frame (using line element along the center-line of the frame) analysis provides a value of 1.7457mm for the same against the same loading condition. It is may be inferred that conventional frame analysis overestimates the stress and deformation much on conservative side (approx. by 49%) leading to the section uneconomic. Also it fails to capture appropriate in-plane rigidity at the beam-column junction. The above substitute frame has also been analyzed using the FEM software ABAQUS to validate the prediction of load-deformation response and the output of the computer code, the author has developed for the purpose. It uses the lower order hexahedral and linear 3D stress elements (type C3D8I) using the incompatible mode. The frame has only been analyzed for linear elastic condition using concrete with similar material properties and loading /boundary condition. The maximum mid-span deflection from the ABAQUS frame analysis has been noted as 1.164 mm, which is within ± 0.94 % when compared to analysis output of the computer code (with EAS based hexahedral lower order element) the author has developed. Also it clearly depicts that the beam is stressed most at and near the supports of the beam (12.67 MPa), although the maximum deflection is taking place at the center of the beam.

Conclusion and outlook

The finite element formulation for the elastic analysis of simply supported RC

frame using linear hexahedral element has been presented, which utilizes additional assumed enhanced strain modes. The performance of this new enhanced strain element recently found in the literature is tested through the example and found very similar to the higher order element. Further to this effort of linear elastic 3D analysis reinforced concrete frame, this model may be upgraded to solve prestressed concrete structures and may be extended to the non-linear regime including the effect of bond slip also. Various long-term effects along with shrinkage and creep may also be included in the analytical model for further refinement. At the same time the large gap between the deflection /response evaluated by the proposed model and conventional frame analysis may also be studied and a measure could be proposed so as to enhance awareness of the designers.

References

- Amiya K. Samanta and Somnath Ghosh.(2008): "A 3D Computational Model of RC Beam Using Lower Order Elements with Enhanced Strain Approach in the Elastic Range." *Computers, Materials & Continua*, Vol. 8, No. 1, pp. 43-52.
- 2. Amiya Kr. Samanta and Somnath Ghosh.. (2008) : "A 3D Hypoelastic Computational Model of Reinforced Concrete Structure." *Icfai University Journal of structural Engineering*, Accepted for publication (in Press).
- Barzegar F. and Maddipudi S. (1994) : "Generating reinforcements in FE modeling of concrete structures." *Journal of structural Engineering, ASCE*, Vol. 120, No. 5, pp. 1656-1661.
- 4. Bathe K. J. and Wilson W.L. (1978) : *Numerical methods in finite element analysis*, Indian Reprint New Delhi, Prentice-Hall of India (P) Ltd.
- Cesar de sa J. M. A. and Natal Jorge R. M. (1999) : "New enhanced strain elements for incompressible problems." *International Journal for Numerical Methods in Engineering*, Vol. 44, pp. 229-248.
- 6. Cheng Y.M. and Fan Y. (1993) : "Modeling of reinforcement in concrete and reinforcement coefficient.", *Finite Element analysis and design*, Vol. 13, pp. 271-284.
- Cook, R. D., Mulkus D. S., Plesha M. E. and Witt R. J. (2003): Concepts and applications of finite element analysis, 4th edition, Singapore, John Wiley & Sons (Asia) Pte Ltd.
- Elwi A.E. and Hrudey T. M. (1989) : "Finite element model for curved embedded reinforcement.", *Journal of Engineering Mechanics, ASCE*, Vol. 115, No. 4, pp. 740-754.

- 9. Simo J. C. and Rifai M. S. (1990) : "A class of mixed assumed strain methods and the method of incompatible modes." *International Journal for Numerical Methods in Engineering*, Vol. 529, pp. 1595-1638.
- Sousa de R. J. A., Natal Jorge R. M., Valente A. F. Robert, Cesar de sa J. M. A., Arieas M. A. Pedro and Fernandes A. A. (2002) : "Lower order elements for 3D analysis." 5th world congress on Computational Mechanics July 7-12, 2002 ; Viena, Austria.
- Sousa de R. J. A., Natal Jorge R. M., Valente A. F. Robert and Cesar de sa J. M. A. (2003): "A new volumetric and shear locking-free 3D enhanced strain element." *Engineering computations*, Vol. 20, No. 7, pp. 896-925.