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Sampling-reconstruction procedure of discrete Markov processes with continuous time

Yu.A. Goritskiy[1], V.A. Kazakov[2]

Summary

At the first time the statistical description of the Sampling-Reconstruction Procedure (SRP) of Discrete Markov Processes (Markov chains) with continuous time and with an arbitrary number of states is given. The mathematical models of Markov chains with continuous time are intensively used in the description of some real stochastic processes with jumps (in control systems and radio engineering), for instance, impulse noise [1, 2]. This is the reason that it is necessary to know: how to sample, how to reconstruct and how to calculate the reconstruction errors of such processes. (Jumps can be occurred in continuous time moments.) So, the usual method of the SRP investigation of continuous stochastic processes (i.e. the method of the conditional mathematical expectation rule) can not be applied directly.

Markov chain $\xi(t)$ with continuous time and with the states *1, 2,…,N,* is completely described by the intensities $\lambda_1, \ldots, \lambda_N$ and by the matrix of the transfer probabilities $P_{ij}(P_{ii} = 0)$ at the jumps moments. Time η_i of stay in state i has an exponential distribution with pdf $p_{\eta i} = \lambda_i \cdot \exp(-\lambda_i t)$, t >0. Let us designate $t_0, t_1, ..., t_n, t_{n+1}$ as sampling moments. Let us $\xi(t_n) = i$. It is necessary to find the time interval T_i determining the next sampling moment $t_{n+1} = t_n + T_i$ under the next conditions: 1) **condition of accuracy**: the variance $\mathrm{V}\hat{\tau}_{ij}$ of the estimation $\hat{\tau}_{ij}$ of the jump moment τ_{ij} from the state i into the state j ($j \neq i$) is not more than a given value σ^2 (the same for all i and j); 2) **condition of miss:** probability of state miss on interval $(t_n, t_n + T_i)$ is not more than a given value γ.

It is obtained conditional probability density for the jump moment under condition $\{\xi(t_n) = i, \xi(t_n + T_i) = j\}$:

$$p(t|i,j) = Ce^{-(\lambda_i - \lambda_j)t}, \quad 0 < t < T, \tag{1}$$

this is the cut exponential distribution (uniform distribution if $\lambda_i = \lambda_j$), $C$is the normalizing constant.

Estimation $\hat{\tau}_{ij}$ is the corresponding expectation

$$\hat{\tau}_{ij} = E\{\tau_{ij}|i,j\} == \begin{cases} \left[1 - \mu_{ij}T/\left(e^{\mu_{ij}T} - 1\right)\right]\mu_{ij}^{-1}, & \mu_{ij} \neq 0, \\ T/2, & \mu_{ij} = 0, \end{cases}, \quad \mu_{ij} \equiv \lambda_i - \lambda_j \tag{2}$$

[1]Moscow Power Engineering Institute, Moscow, Russia
[2]National Polytechnic Institute of Mexico, Mexico

Estimation variance is determined and interval $T' \equiv T'_{i\sigma}$ is obtained from the accuracy condition

$$\max_{j,\, j \neq i, P_{ij} \neq 0} \mathrm{V}\hat{\tau}_{ij} = \sigma^2,$$

it gives equation

$$\left[1 - (\mu_i^* T)^2 \left(e^{\mu_i^* T} + e^{-\mu_i^* T} - 2\right)^{-1}\right] (\mu_i^*)^{-2} = \sigma^2 \tag{3}$$

where $\mu_i^* = \min_{j,\, i \neq j, P_{ij} \neq 0} \left|\lambda_i - \lambda_j\right|$. The graph way for determination of $T \equiv T'_{i\sigma}$ is given. It is clear that the graph of transitions influences at the sampling procedure but not values P_{ij} of probability transitions.

Condition on probability of the state miss is reduced to the view

$$\max_{j, i \neq j, P_{ij} \neq 0} P\{\eta_i + \eta_j < T\} \equiv P\{\alpha(\eta_i + \eta_{j*}) < \alpha T\} \equiv F_{\alpha\Sigma}(\alpha T) = \gamma, \tag{4}$$

where maximum is achieved on the state $j*$. Distribution function of the sum $\alpha\Sigma \equiv \alpha(\eta_i + \eta_{j*})$ is depended from two parameters λ_I and λ_{j*}; it is reduced to the one-parameter family by substitution $\alpha = \max(\lambda_i, \lambda_{j*})$, $\beta = \min(\lambda_i, \lambda_{j*})$, $k = \beta/\alpha$, $0 < k \leq 1$:

$$F_{\alpha\Sigma}(t) = \begin{cases} 1 - \left(e^{-kt}/k - e^{-kt}\right) k/(1-k), & k \neq 1, \\ 1 - (1+t)e^{-t}, & k = 1. \end{cases}$$

Approximate value is $T_\gamma \approx \sqrt{2\gamma/(\lambda_i \lambda_{j*})}$; Approximation accuracy is better than 10%. From conditions (3), (4) sampling interval in i state is $T'_i = \min(T'_{i\sigma}, T'_{i\sigma})$. Illustrate example is given.

References

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