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Numerical prediction and sequential process optimization in sheet forming based on genetic algorithm

Schmidt[1]

Summary

Genetic algorithm is an emerging technique used in engineering design activities to find an optimized solution which satisfy a number of design goals. Non-linear direct method of goal search use successive linearization techniques, which are sensitive to the chosen starting solution and quality of the objective function. The proposed technique can solve programming problems having non-convex regions, which are usually avoided in classical optimization problems. The efficacy of the proposed novel method is demonstrated by solving a number of test problems. The results suggest that the proposed method is effective and represents a practical tool for solving sheet forming problems.

Keywords: Sheet forming optimization, genetic algorithms, goal programming

Introduction

It is of interest to investigate the nature of the numerical prediction and optimization scheme and how the blank is affected by varying its forming process and die parameters. The optimization objective in the forming process is archived using genetic algorithms. These algorithms are known to be able to explore the entire functional space, thus they can detect the global optimal solution. Incremental data is used to further limit the space that is investigated. The gradient of the field calculated for simplex method eliminates candidates from the possible optimal solutions towards the global optimal solution and limits the scope of the data for the candidates obtained from generic algorithms.

Literature review

The finite element is adopted for metal forming since it provides detailed information about the domain being studied and is an essential component of computer-aided design. Kobayashi (Kobayashi 1987)applied a finite-element based backward tracing technique to design an optimized pre-form. However, this technique is largely inefficient in determining the optimal solution due to the presence of diverse

[1]Corresponding author. Department of Mechanical Engineering, Lipetsk State Technical University, Lipetsk, Russia. Tel.: +1 (773) 293-3115; fax: +1 (773) 213-9907. E-mail address: me@alexschmidt.net (A. Schmidt).

and multiple loading solution paths. John and Hwang (Joun and Hwang 1993)proposed derivative based approach to solve process optimization problems. In spite of potential of the proposed method, an initial guess required by this method can influence the search, and often can be stuck in a sub-optimal solution.

The optimization problem of finding the solution for the function $F(x)$ of the problem can be solved using a well-known quasi-Newton method proposed by physicist W.C. Davidon (Davidon 1991), which idea is based on the Newton's method where it is assumed that a function can be locally approximated around the optimal solution, but requires that the optimized function is computed for each iteration. The quasi-Newton method uses Hessian matrix and incremental analysis of the successive gradient vectors by imposing a simple constraint onto Hessian estimate.

The improved version, popularized by Fletcher and Powell known as method of Davidon-Fletcher-Powell (Davidon 1991)is no longer in wide use, which is based on the secant method, leads to a positive-definite matrix. Among the most common quasi-Newton algorithms today is Broyden-Fletcher- Goldfarb -Shenno (BFGS) (Nocedal and Wright 2006)method which uses McCormick criteria does not hold against DFP method, where it can lead to difficulties because the search space is unknown when multiple diverse loading paths are present. Geodesics by Rosenbroke's function and Himmelblau (Himmelblau 1972), which ideas are similar to optimization lines, are important in global minimum search problem because the error cost involved is forming the multi-minima potential in weight parametric space. Damped oscillator equation known as HBF is a widely-used technique to arrive at global minimum.

To address limitations of the existing methods, a number of authors are adopting genetic algorithms with embedded finite-element solver to automate the search of the optimized solution. Roy et al. (Roy 1996,) implemented an adaptive genetic algorithm for shape optimization of sheet forming process. This method can deliver good solutions, however, the GA-based approach using the finite element solver as embedded optimizer incurs severe computational costs since it require large number of solutions to converge.

Furthermore, we obtained a novel adaptive search using generic algorithm. To investigate the performance and potential of this novel descent, we applied direct method of descent based on simplex to the genetic functional space. The results show that our method determined the global minimums in our test problems. Our adaptive descent may be applied in different problems related to forming optimization, such as sheet metal forming.

Genetic algorithms in sequential process optimization

A review of the literature revealed that there is a growing interest in approaches for dealing with the sequential process optimization problems using Genetic Algorithms [(Deb 1998)]. Kobyashi[] at al. TODO: Add more. However, these attempts tend to consider the entire space of possible solutions as candidates for the optimal solution bounded by criterion space.

Methods of the sequential process optimization are assuming initial points of the optimal solution are based on the knowledge of the physical problem and can be derived for the set of optimizing constraints. This makes the optimization algorithms susceptible to the correctness of the selection and whether the system can converge to the optimal solution, including the number of stages required to achieve the required convergence. In addition, this approach, while being exhaustive, requires significant computation time to obtain the optimal solution. It seems feasible to apply gradient descending method to a NSGA algorithm to the sheet forming with multiple stages based on the ideas from (Fonseca and Fleming 1993)(Srinivas and Deb 1994).

Moreover, the dependency between stages is modeled based on the results obtained during previous stage such that the optimization algorithm evaluates alternatives before moving into the next stage.

Given a matrix describing a problem and Dirichlet boundary conditions on an optimizing domain Ω_D, the goal is to solve the problem

$$-\triangle u = F\left(x\right) \text{ in } \Omega, \quad u = u_0 \text{ on } \partial\Omega_D$$

where $F\left(x\right)$ is the function of the problem on a polygonal domain, which $\in \Omega$, u is a solution on the domain Ω, u_0 is an initial solution on the domain Ω.

Let vector f_j^k denote a constraint of the objective function$F\left(x\right)$, and f be a continuous function. Then

$$F_i^k\left(x\right) = \int_{j=1}^{n} f_{ij}^k\left(x\right) \ |i = 1, 2, \ldots p$$

where

$$f_j\left(x\right) = \left\{f_{ij}\left(x_{1j}, x_{2j}, \ldots x_{mj}, x_{1k}, x_{2k}, \ldots x_{rk}\right)|, k = j-1, r < m\ \right\}$$

i is the ith objective, p is the number of objectives, m is the number of design variables at stage j, k is the constraint from the previous stage $j-1$, r is the number of constraints from the stage $j-1$that is taken into consideration at stage j, and n is the number of stages.

Genetic String Representation Printing Area

x_{11}	x_{21}		x_{m1}	x_{12}	x_{22}		x_{m2}		x_{1n}	x_{2n}		x_{mn}
100…	101…	…	111…	110…	100…	…	111…	…	001…	010…	…	001…
stage 1				*stage 2*					*stage n*			

Figure 1: String Structure of a Chromosome

The weighed pairing algorithm [(Courrieu 2009)] is used for selection of a chromosome to produce new offspring. The rank weighting is evaluated according to the following formula:

$$P_n = \frac{N_{keep} - n + 1}{\int_{n=1}^{N_{keep}} n}$$

Process of mating and producing offspring is performed from two parent's chromosomes. A crossover point is selected stochastically between the first and last bits of parents' chromosomes. Consequentially the offspring contains binary codes of both parents.

Mutation Operator

A polynomial probability distribution is used to derive next solution, based on a parent solution. Let parent solution be $y_i^{(j)}$. Then, the following process applies to the procedure of obtaining each element for next solution $y_i^{(j)+1}$. First, choose a random number u between 0 and 1. Next, calculate a parameter of mutation δ_q as follows:

$$\delta_q = \begin{cases} \left[2u + (1-2u)(1-\delta)^{\eta_m+1}\right]^{\frac{1}{\eta_m+1}} - 1, & \text{if } u \leq 0.5;; \\ 1 - \left[2(1-u) + 2(u-0.5)(1-\delta)^{\eta_m+1}\right]^{\frac{1}{\eta_m+1}}, & \text{otherwise} \end{cases}$$

where $\delta = \min\left[\left(y_i^{(j)} - y_i^l\right), \left(u_i^u - y_i^{(j)}\right)\right] / \left(y_i^u - y_i^l\right)$. Here parameter η_m is the distribution index for mutation. Lastly, calculate the mutation as follows:

$$y_i^{(j)+1} - y_i^j + \delta_q \left(y_i^u - y_i^l\right).$$

The mutation probability p_m is taken based on (Gao 1998)who evaluated it in the context of Markhov chain models. He showed that the probability of mutation and the smaller the population is reverse proportional to GA convergence rate. In our case, it is varied from $1/P$ till 1.0, which results into least parameter mutation gets in the beginning and most extensive parameter mutation get mutated at the end of simulation.

Algorithms of direct search.

Methods of the direct search of an optimal solution are based on the evaluation of the objective function. These methods are usually based on the empirical deductions and don't hold strong mathematical background. Also, characteristics of the convergence of the direct methods and speed of convergence are not very well studied. However, these methods are carrying ideas similar to those in the methods of first and second order [(Abramson and Charles 2005) (Kolda, Lewis and Torczon 2003)]. In certain cases, this allows to assess the effectiveness of the algorithms of direct search in the context of certain classes of function. We argue that it is possible to perform the assessment by comparing experimental data with the data obtained theoretically and to perform comparative analysis of those results.

In order to achieve the objective of optimization, the mathematical apparatus of unconstrained method of BFGS used for the solution based on the Newton equation:

$$B_i D_j = -gradF_i^k\left(x_j\right)$$

where B_i is an approximation to the Hessian matrix, j is a stage of the optimization, and D_j is a search direction, $F_i^k\left(x_j\right)$ – function at stage j, -grad is antigradient of the function$F_i^k\left(x_j\right)$.

The complete Hessian matrix is calculated at the first stage and is updated iteratively at each stage. Finally, the next point x_{j+1} is obtained based on the line search in the direction D_j.

At first stage, the search direction P_k is calculated using the eigen matrix of the order n. Note that in this case, $p_1 = w_1 = -gradf\left(x_0\right)$, that is first stage is calculated similar to the one calculated for the method of fastest descent. On another hand, let $A_k = H - 1(x_k - 1)$ and $k_{sik} = 1$, then we are coming to the classic Newton method.

Lemma 1

Let the objective function$f\left(x\right)$ be a convex function and there exist a Hessian matrix $H(x)$ for every $x, y \in \mathbb{R}^n$ such that

$$H\left(x\right) - H\left(y\right) \leq L\left|x - y\right|$$

then, Newton method converges as a quadratic function and the following formula holds

$$\left|x^k - x^*\right| \leq \frac{L}{\lambda_1}\left|x^{k-1} - x^*\right|^2, \quad k \in \mathbb{N}$$

Proof

Given λ_1 is equal to minimal proper value of Hessian matrix $H(x)$of the objective function in the vicinity ofx^*, which containsx^{k-1}. Then, $H\left(x\right)$ is positive definite in this vicinity and we have

$$\lambda_1 |x|^2 \leq (H(x)x,x)$$

for each $x \in$ vicinity of x^*.

The above concludes the optimization process at stage$k$. The Hessian matrix at stage j is computed as B_{j+1} by the addition of two matrices according to the following:

$$B_{k+1} = B_k + U_k + V_k$$

where B_k– Hessian matrix at stage k, U_k and V_k are step-size matrices calculated at each stage.

Moreover, the convergence ratio of the highly convex objective function is described by quadratic function. Therefore, the algorithm of the quasi-Newton method exhibits convergence rate, if for each stage matrix A_k is selected such that it approximates to the matrix $H-1(x_k-1)$ at $x^{k-1}?\mathbb{R}^n$.The construction of the matrix $H-1(x_k-1)$ for matrix A_k considering gradient at x_{k-1} can be greatly simplified. This can be achieved by following the next steps. The sequence of the approximating matrices A_k is constructed according to

$$A_{k+1} = A_k + \triangle A_k, \quad k \in \mathbb{N}$$

where $\triangle A_k$ – correction matrix of order n. Let $f(x) = \frac{1}{2}(Q_x, x) + (c,x)$ be a quadratic function with positive define matrix Q. In this case, function $f(x)$ is highly convex function, and the following holds

$$grad f(x) = Qx + c$$

$$\Delta\omega^k = grad f\left(x^{k-1}\right) - grad f\left(x^k\right) = Q\left(x^{k-1} - x^k\right)$$

Finally,

$$Q^{-1}\Delta\omega^k = -\Delta x^k, k \in \mathbb{N}$$

where matrix Q coincides with the Hessian matrix $H(x)$ of the function $f(x)$.

Test Problem 1

We use the following goal programming problem:

goal $(f_1(\Delta,P) = x_1 \geq 230)$,

goal $(f_2(t) = x_2 < 1.06)$,

Subject to $95 \leq x_1 \leq 370$, $1.0 \leq x_2 \leq 1.06$.

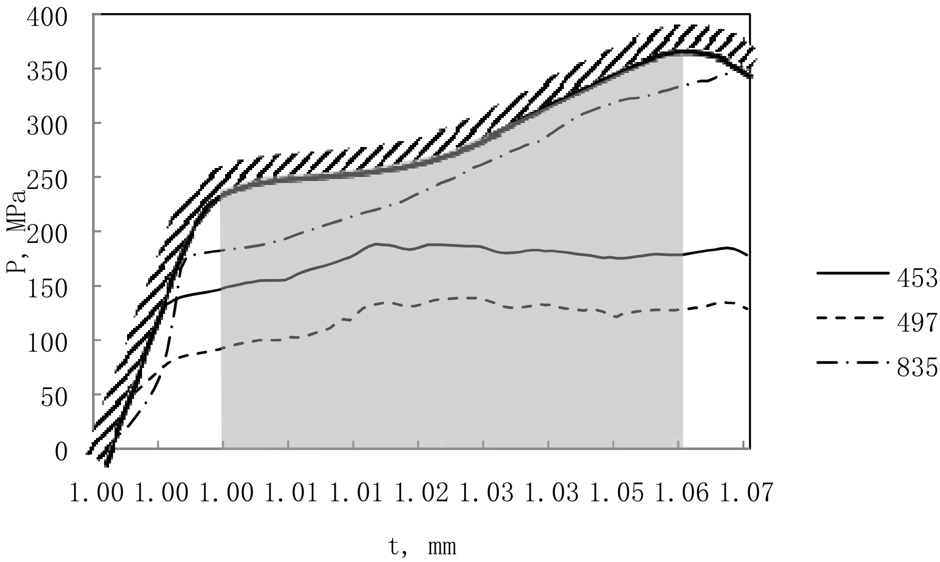

Figure 2: Solution is shown on the problem space of thickness vs. stress dependency.

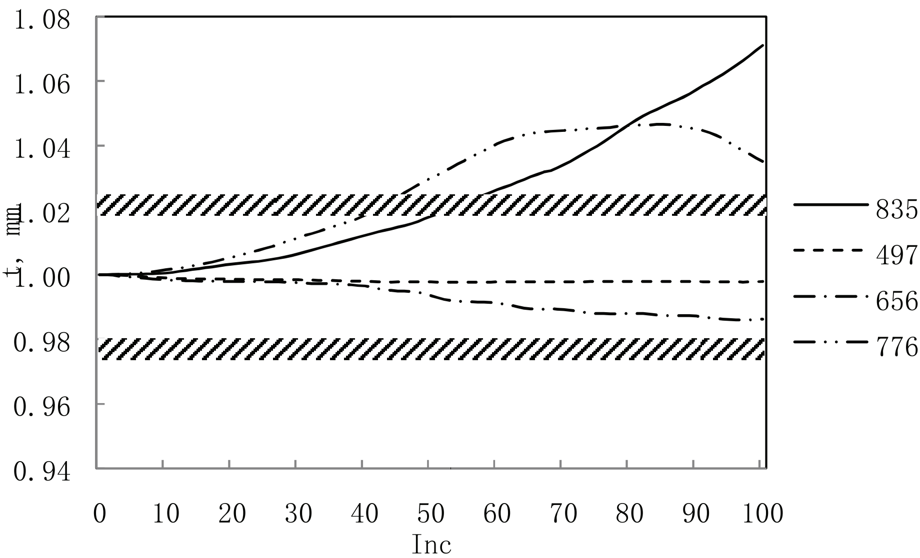

Figure 3: Solution is shown on the problem space of increment vs. thickness dependency.

The programming problem includes two goals, the first goal is of greater-than-equal-to type and the second goal is of less-than-equal-to type.

The feasible decision space is shown in Fig. 2 and the criterion space is the line between points A and B. Each discreet solution in this region corresponds to a goal programming problem. The solutions to this problem for the function 1 represent plastic deformation of the body. The varying problem for the function 2 represents possible thickness of the deformable body undergoing plastic deformation, depending on the weight factors used. To solve the described problem using NSGA, it is converted into an equivalent two-objective optimization problem as follows:

Minimize $\langle 230 - f_1(x_1) \rangle$,

Minimize $|f_2(x_2) - 1.06|$,

Subject to $95 \le x_1 \le 370, 1.0 \le x_2 \le 1.06$.

The Fig. 3 and Fig. 4 shows that NSGA finds the potential optimal solutions, which are converging to the global optimal solution.

Test Problem 2

The problem is to minimize the delta between the optimal blank and thickness variation throughout the blank in the sheet forming process. The multi-objective optimization problem is considered in two-dimensional space using Pareto front for generic algorithm optimization. The programming problem includes two goals, where the first is of less-than-equal-to type and the second is of greater-than-equal-to type.

Where primary y-axis shows shape error μ_{error} is defined as the mean value of the shape difference between the current deformed contour of the flange and target contour. The shape error is calculated by taking the arithmetic mean of the absolute values of the difference between the nodes on the current deformed contour and the nearest nodes on the target contour. The total value of the shape error therefore can be expressed as follows:

$$\mu_{error} = \frac{1}{n} \int_{i=1}^{n} |d_i|$$

where d_i- difference between the nodes of deformed contour on the current stage and the nearest nodes on the target contour; n- total number of nodes located on the contour of the blank at current stage. The thickness distribution is plotted against the profile of the cup in the secondary y-axis. The distribution shows the effectiveness of the modeling and goal is to minimize this parameter's deviation. The blank is drawn for 70 mm and initial thickness of the blank is taken as 1.0 mm.

Minimize $\langle 0.15 - f_1(x_1) \rangle$,

Minimize $|f_2(x_2) - 1.0|$,

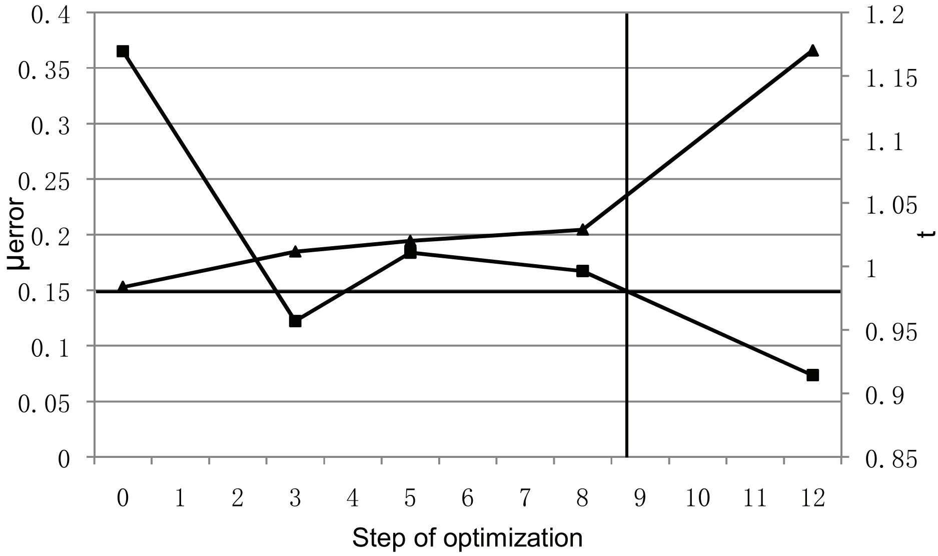

Figure 4: Solution is shown on the problem space.

Subject to $0 \leq x_1 \leq 0.15, 0.85 \leq x_2 \leq 1.15$.

Since there exists no solution with error μ_{error} greater than 0.15, and the thickness distribution is less than 0.02, the resulting solution is supposed to be the optimized solution. Fig. 5 shows that this solution converges to the following parameters:

μ_{error}=0.07, t = 1.0

The solution for these parameters is very close to the global optimal solution.

Conclusion

The method of the sequential optimization of a sheet forming process was reviewed and its limitations were brushed aside. It was shown that the generic algorithms are capable to deliver better solution compared to the traditional iterative approach based on the knowledge of the physical problem. Furthermore, the improved NSGA algorithm can be applied to the investigated problem in anticipation of optimal solution. The related portions of creating a chromosome, obtaining a crossover, and performing a mutation were mapped into problem space. The direct methods of global descent were used to improve the convergence speed. Finally, two test problems were presented to prove the validity of the ideas described in the paper. The results are these test problems indicated feasibility of the proposed approach as their results converged nicely into optimized solutions.

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