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Magneto-electric laminates free vibration characterization by dual reciprocity BEM

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Summary

A dual reciprocity based boundary element approach for the analysis of magneto-electric laminates free vibration behavior is presented. The problem is formulated employing generalized displacements, that is displacements and electric and magnetic scalar potentials, and the corresponding generalized tractions. The generalized boundary integral representation is deduced by extending the reciprocity theorem to magneto-electro-elasticity problem and the multidomain boundary element technique is used to model multilayer structures. The magneto-electro-elastic static fundamental solutions are used jointly with the dual reciprocity method to transform the inertia domain integral into a boundary integral. Numerical results are presented focusing on the effects of the electro-magnetic poling directions.

Keywords: magneto-electric composites, boundary integral, free vibration, dual reciprocity BEM.

Introduction

Magneto-electric laminates have lately emerged as suitable for realizing advanced smart devices having valuable performances with respect to standard actuation materials, Priya Islam Dong and Viehland (2007). They show a wide range of potential applications including strain sensing/actuating devices, Ueno and Higuchi (2006), micro-power wireless generators, Bayrashev Robbins and Ziaie (2004), magnetic field sensor, Duc and Giang (2008), miniaturized antenna, Petrov et al. (2008), energy harvesters, Li et al. (2010), and much more, Bichurin Viehland and Srinivasan (2007). The feature of magneto-electric composites is the ability to convert energy among three distinct forms: the elastic, the electric and the magnetic one. It stems from the coexistence of the piezoelectric and piezomagnetic phases binding together in particulate or laminate configurations which leads to an elastic-mediated coupling between the electric and magnetic fields. Due to this inherent multi-physics nature, these composites demand efficient analysis and design tools to accurately predict the coupling effects and then take full advantage from their smart behavior.

In the present work, a boundary element model and its numerical implementation for the analysis of magneto-electro-elastic composites are presented with the

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aim to exploit their features in linear magneto-electro-elastic free vibration analysis. The multifield problem is formulated employing generalized variables whose corresponding boundary integral representation is obtained by the reciprocity theorem for magneto-electro-elasticity problem. The static fundamental solutions for magnetoelectroelasticity are used and the inertia terms are treated as body forces. The dual reciprocity method is used to transform the domain integral into equivalent boundary integral by employing radial basis functions as particular solution. The multidomain boundary element technique is implemented to deal with the analysis of multilayer configurations. Numerical results are presented focusing on the effects of magnetic and electric poling arrangements.

Boundary Element Model

The boundary integral formulation is developed for magneto-electro-elastic plane problems. To maintain a compact notation the Barnett and Lothe's formalism for piezoelectrics, Barnett and Lothe (1975), is extended to the magneto-electro-elastic problem by defining a generalized displacement vector **U,** collecting the displacement components**,** the electric potential and the magnetic potential. Consistently, the generalized compatibility operator **D** and the generalized constitutive matrix **R** are introduced, see for more details Milazzo Benedetti and Orlando (2006). By so doing a generalized Navier-like governing equation is obtained for magnetoelectroelasticity

$$\mathbf{D}^T\mathbf{R}\mathbf{D}\,\mathbf{U}+\mathbf{F}=\mathbf{0} \tag{1}$$

where **F** are the generalized body force.

By applying the reciprocity theorem to the generalized magneto-electro-elastic problem with the static magneto-electro-elastic fundamental solutions, the following boundary integral representation for the generalized displacements at the point P_0 is obtained, Milazzo Benedetti and Orlando (2006),

$$\mathbf{c}^*\mathbf{U}(P_0)+\int_{\partial\Omega}(\mathbf{T}^*\mathbf{U}-\mathbf{U}^*\mathbf{T})\,d\partial\Omega=\int_{\Omega}\mathbf{U}^*\mathbf{F}d\Omega \tag{2}$$

where **T** is the generalized traction vector, $\mathbf{U}^*$ and $\mathbf{T}^*$ are the fundamental solution kernels, $\mathbf{c}^*$ is the free term coefficient and $\partial\Omega$ is the boundary of the magneto-electro-elastic domain Ω. When collocated at the boundary, Eq. 2 provides the boundary integral equation which, coupled with the essential and natural boundary conditions, allows the problem solution.

In magnetoelectroelastic dynamic problems, assuming that the electric and magnetic potentials are quasi-static, the nonvanishing generalized body forces components are given by the inertial forces. Therefore, denoting by ρ the product of the material density by the 4×4 identity matrix in which the last two diagonal term are replaced by zeros, the generalized body forces are written as

$$\mathbf{F} = -\rho \ddot{\mathbf{U}} \tag{3}$$

where the overdot indicates time derivatives.

The boundary integral formulation is numerically implemented by using the Boundary Element Method, Aliabadi (2002), and the Dual Reciprocity technique, Dziatkiewicz and Fidelinski (2007), which lead to the following equations of motion

$$\mathbf{M}\ddot{\Delta} + \mathbf{H}\Delta = \mathbf{GP} \tag{4}$$

where Δ and $\mathbf{P}$ are the vectors of the generalized displacements and boundary tractions nodal values, respectively. In Eq. 4, $\mathbf{H}$ and $\mathbf{G}$ are square influence matrix computed by integrating the fundamental solution kernels weighted by the shape functions Ψ employed to express the generalized displacements and tractions on the boundary, while $\mathbf{M}$ is the mass matrix whose computation is described in the next section. Eventually, in order to model magneto-electric laminated structures, the multidomain approach is implemented, Davì and Milazzo (2001). It requires to write the equation of motion for each of the N homogeneous sub-region

$$\mathbf{M}^{(k)}\ddot{\Delta}^{(k)} + \mathbf{H}^{(k)}\Delta^{(k)} = \mathbf{G}^{(k)}\mathbf{P}^{(k)} \quad k = 1, 2..., N \tag{5}$$

and the compatibility and equilibrium conditions along all the sub-region interfaces

$$\Delta^{(i)}_{\partial\Omega_{ij}} = \Delta^{(j)}_{\partial\Omega_{ij}} \,; \mathbf{P}^{(i)}_{\partial\Omega_{ij}} = -\mathbf{P}^{(j)}_{\partial\Omega_{ij}} \quad i = 1, ..., N-1; \; j = i+1, ..., N \tag{6}$$

where the superscript (k) denotes quantities pertaining to the k-th sub-region and the subscript $\partial\Omega_{ij}$indicates quantities associated with the nodes belonging to the interface between the *i-th* and *j-th* sub-regions.

Mass Matrix Computation

To compute the mass matrix $\mathbf{M}$ the dual reciprocity technique is employed, Partdrige, Brebbia and Wrobel (1992). Let us assume that the generalized displacement components can be approximated as a sum of the product of spatial functions $\boldsymbol{F}$ multiplied by time-dependent unknown functions α. The acceleration vector is written as

$$\ddot{\mathbf{U}} = \boldsymbol{F}\,\ddot{\alpha} \tag{7}$$

The selection of the spatial functions $\boldsymbol{F}$ is carried out so that they satisfy the Navier-type equation

$$\mathbf{D}^T \mathbf{R} \mathbf{D} \boldsymbol{G} + \boldsymbol{F} = \mathbf{0} \tag{8}$$

where the auxiliary function $\boldsymbol{G}$ is supposed to be a-priori assigned; a typical selection for the auxiliary function is given by third order radial basis functions depending on the distance between collocation and integration points, see Dziatkiewicz and Fidelinski (2007). It follows that the domain integral related to inertial terms in Eq. 2, reads as

$$\int_\Omega \mathbf{U}^* \mathbf{F} d\Omega = \int_\Omega \mathbf{U}^* \rho \mathbf{D}^T \mathbf{R} \mathbf{D} \boldsymbol{G} d\Omega \, \ddot{\alpha} \tag{9}$$

where Eqs. 2, 7 and 8 have been taken into account. The right-hand-side domain integral of Eq. 9 is transformed into a boundary integral by applying the reciprocity theorem

$$\int_\Omega \mathbf{U}^* \mathbf{F} d\Omega = \mathbf{c}^* \boldsymbol{G}(P_0) + \int_{\partial\Omega} (\mathbf{T}^* \boldsymbol{G} - \mathbf{U}^* \boldsymbol{H}) \, d\partial\Omega \, \rho \ddot{\alpha} \tag{10}$$

where $\boldsymbol{H}$ are the boundary tractions associated to the auxiliary displacements $\boldsymbol{G}$; if the shape functions Ψ used to express the generalized displacements and tractions on the boundary are employed to approximate $\boldsymbol{G}$ and $\boldsymbol{H}$ by means of their nodal values γ and η, respectively, it follows that Eq. 10 can be rewritten as

$$\int_\Omega \mathbf{U}^* \mathbf{F} d\Omega = \rho \left(\mathbf{H}\gamma - \mathbf{G}\eta \right) \, \ddot{\alpha} \tag{11}$$

By collocating Eq.7 at the discretization nodes one obtains

$$\ddot{\alpha} = \bar{\boldsymbol{F}}^{-1} \ddot{\Delta} \tag{12}$$

where $\bar{\boldsymbol{F}}$ is the collocation matrix. Finally, substituting Eq. 12 into Eq. 11 leads to the expression of the mass matrix involved in the BEM equation of motion for magneto-electro-elastic domains

$$\mathbf{M} = \rho \left(\mathbf{H}\gamma - \mathbf{G}\eta \right) \, \bar{\boldsymbol{F}}^{-1} \tag{13}$$

Results

The analysis of a bimorph configuration is presented to show the soundness of the proposed approach in terms of accuracy and effectiveness. The laminate is realized by stacking a piezoelectric $BaTiO_3$ layer and a piezomagnetic $CoFe_2O_4$ layer, whose properties are given in Milazzo Orlando and Alaimo (2009). The bimorph length is $L = 0.3\,m$ while its overall thickness is $h = 0.02\,m$. Natural frequencies computed for the cantilever configuration are reported in Tab. 1 in comparison with finite element results, taken from Annigeri Ganesan and Swarnamani (2007), and with analytical results obtained by Milazzo Orlando and Alaimo (2009).

Table 1: Cantilever bimorph natural frequencies [Hz]

Mode	**DRBEM**	**FEM**	$100 \times \left(1 - \frac{DRBEM}{FEM}\right)$	**Analytic**	$100 \times \left(1 - \frac{DRBEM}{Analytic}\right)$
1	188.62	188.70	0.0424	189.63	0.5326
2	1138.13	1154.65	1.4307	1159.42	1.8363
3	2904.15	3120.80	6.9421	3129.14	7.1902
4	4093.75	4335.11	5.5676	4420.84	7.3988
5	5857.47	5838.36	-0.3273	5841.67	-0.2705

It appears from Tab.1 that the proposed approach is accurate in computing the natural frequencies of the magnetoelectric laminates. The maximum percentage discrepancy between the DRBEM simulation and other results is always about 7%. The same stands for higher order modes, not reported for the sake of conciseness.

The second application deals with a tri-layered piezomagnetic/piezoelectric/piezomagnetic composite. The attention is focused on the influence of the smart layer poling directions on the laminate free vibrations. A sketch of the four magnetization-polarization arrangements, characterizing the laminate (longitudinal *L–L*, transverse *T–T*, or longitudinal-transverse *L–T* or *T–L*) working modes, is reported in Fig.1.

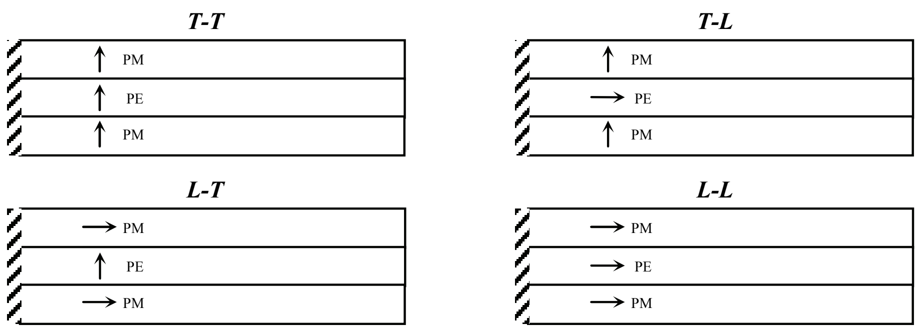

Figure 1: Tri-layered magnetoelectric composite working modes arrangements

In Fig.1 the arrows represent the magnetization (*PM*) and the electric polarization (*PE*) directions. The natural frequencies of the first five modes of vibration are given in Tab.2 for all the plies polarization arrangements. It gathers that a longitudinal magnetization of the piezomagnetic plies affects the magnetoelectric composite behavior by slightly reducing the fundamental frequencies.

Table 2: Natural frequencies [Hz] for the tri-layered device

Mode	*T-T*	*T-L*	*L-T*	*L-L*
1	278.59	279.54	271.12	272.02
2	1671.97	1675.43	1625.95	1628.83
3	3736.29	3757.14	3644.04	3667.14
4	4748.26	4781.86	4629.22	4671.98
5	7939.65	7988.07	7754.79	7805.22

Conclusion

In this paper a boundary element method for the free vibration analysis of magneto-electroelastic laminates has been presented. The static fundamental solutions of the plane magneto-electro-elasticity problem have been employed and the inertia loads have been treated as body loads. The arisen domain integral has been transformed to boundary integral by using the dual reciprocity method and the multi-domain technique has been implemented to model multilayer configurations. The results obtained show the accuracy and effectiveness of the proposed method to characterize the dynamic behaviour of magneto-electric laminated composite.

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