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The energy localization by the rupture propagation

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Summary

The simple analytical model for the energy flux by the earthquake is proposed. The energy flux can be evaluated through the Umov-Pointing vector by the rupture propagation in media. Discontinuity of vector components is found. This discontinuity cause the change of an energy flow direction and localization of the energy field.

Keywords: Rupture, geomassif fracture, energy flow, stability area

Introduction

Defined scaling characteristics of mountain rock fracture or earthquake processes is a popular general topic today. At the same time, a detailed theory of mechanical behaviour of geological massifs in linear approximation does not bring sufficiently satisfactory results, since the real continuum of geomechanics is essentially non-linear (14, 15). On the other hand, the behaviour of geological structures under external affections is critically important. This is related to colossal scales of possible implications — both human and material casualties — at burst out of "geological" emergencies. Among such emergencies we can name earthquakes, mudflows and earth shell dynamics. Of considerable interest are mathematical problems of a non-linear continuum, connected with mineral exploration activities.

It is well known that energy redistribution by the rupture propagation play the important role by the possible earthquake development and cause some research interest, example (2) where the complex method for evaluating the lattice Green functions used in calculating the stress redistribution due to local ruptures. In present work we are propose the simple analytical method for the energy flow calculation for the well known media model. The broadening the method for more general media and complex stressed"=deformed state is possible.

Energy flux

It is known from the vector analysis that the flux of random vector field $\mathbf{F}(\mathbf{r})$ through surface S is defined as:

$$\mathcal{F} = \int_S \mathbf{F}(\mathbf{r}) d\mathbf{S}, \tag{1}$$

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where $(\mathbf{r})$ is vector of the position, $\mathbf{S} = S\mathbf{n}$ is the oriented surface and $\mathbf{n}$ is the normal to the surface. This definition is easily spread on tensors of random order and valence and on spaces of any mathematical structure. For simplicity, we consider rupture propagation in the medium without heat sources and neglect the traction forces. In this case, for the energy flux density (8) we have

$$\dot{W} = \lim_{\varepsilon\to 0}\int_{S_p}\left\{\sigma_{ij}\dot{u}_i + \left[\int^t\left(\sigma_{ik}\dot{\varepsilon}_{ik}\right)dt + \frac{1}{2}\rho\dot{u}_i\dot{u}_i\right]v_j\right\}n_j dS_\varepsilon =$$
$$\lim_{\varepsilon\to\infty}\int_{S_p}\left\{\sigma_{ij}\dot{u}_i + \left[\int^t\left(\sigma_{ik}\dot{\varepsilon}_{ik}\right)dt + \frac{1}{2}\rho\dot{u}_i\dot{u}_i\right]v_j\right\}d\mathbf{S} = \int_{S_p}\vec{\mathcal{E}}d\mathbf{S} \quad (2)$$

where σ_{ij} is the stress tensor, u_i is the displacement vector, ε_{ik} is the strain tensor, t is the deformation time, ρ is the material density and v_j is the vector of the crack surface S_ε speed. In this equation the integration is realized in small area ε in the proximity tip and

$$\vec{\mathcal{E}} = \sigma_{ij}\dot{u}_i + \left[\int^t\left(\sigma_{ik}\dot{\varepsilon}_{ik}\right)dt + \frac{1}{2}\rho\dot{u}_i\dot{u}_i\right]v_j \quad (3)$$

is the Umov"=Pointing vector. It defines the direction of energy propagation (3, 13). The mathematical details connected with the integration procedure of equation (2) can be found in (5). The Umov-Pointing vector can be presented in a bit different form:

$$\vec{\mathcal{E}} = -\sigma_q|\mathbf{q}|, \quad (4)$$

where $\sigma_q = \boldsymbol{\sigma}\mathbf{q}/|\mathbf{q}|$ is the vector of stresses on the pad with a normal directed along the displacement vector.

In the mechanics of deformed bodies, the physical value of this vector is connected with non-coaxiality of the stress and deformation tensors, which leads to localization of deformation (7). In the general case, we have a possibility to find $\vec{\mathcal{E}}$ out of (4), by using known solutions for σ in case of dynamic propagation (5, 10).

It is well-known that an important difference between a discrete and continuum model is the existence of radiation from the tip of the moving crack, due to existence of periodic modulation in the velocity in the presence of an underlying lattice (4). From this viewpoint, the considered model of crackon (8, 9) is a discretization of the classic continual model of the medium.

In real systems, the energy introduced into the volume by external loads, is localized by the crack through various mechanisms. These can be, for example, acoustic emission (11), stress waves (16), or surface waves (12). Outgoing from the structure of the fissured body (the presence of free surfaces), the surface waves in particular are principally changing the energy relaxation picture in this case. In the case of geological structures, an essential role belongs also to Love waves, and the share of the energy taken away by all types of waves makes up to 5% of the overall deformation energy (15). In this case, microcracks are emitting insignificant energy. According to experimental data, for sandstone, the average dimension of a microcrack lies within the limits of $(1.4\dots28.4)\cdot10^{-6}$m, and the total energy of acoustic signals at formation of a submicroscopic crack makes $0.03 \div 58.25$nJ (17). A considerable share of the energy "pumped" into the volume is absorbed by inter-block motions, and by elastic and plastic deformation of individual blocks.

The energy field by the quasi-statatic rupture propagation

Lets investigate the quasi-static rupture propagation in media. It is well known that transition to the dynamical case demand some dynamical corrections as the propagation velocity function to receiving static results (5). Lets investigate mode I crack in plane stressed state in case of ideal plasticity. The stress field is essentially distinguished in three areas limited by the characteristic direction $\varphi_1 = 79,7^\circ, \varphi_2 = 151,4^\circ$ for the crack model with diffusive plastic flow (1, 6). In this description $\varphi = 0^\circ$ correspond to actual crack propagation direction.

For the stress field in the first area $\varphi \le \pm\varphi_1$ we obtain

$$\sigma_{\varphi\varphi} = 2\sigma_{rr} = \frac{2}{\sqrt{3}}\cos\varphi; \quad \sigma_{r\varphi} = \frac{1}{\sqrt{3}}\sin\varphi.$$

For the stress field in the second one $\varphi_1 \le \varphi \le \varphi_2$ we receive :

$$\sigma_{rr} = \frac{1}{4}(-1+3\cos2\varphi_1)+\frac{1}{4}(1+\cos2\varphi_1)\cos2(\varphi-\varphi_1)+\frac{1}{2}\sin2\varphi_1\sin2(\varphi-\varphi_1),$$

$$\sigma_{\varphi\varphi} = -\sigma_{rr}+\frac{1}{2}(-1+3\cos2\varphi_1),$$

$$\sigma_{r\varphi} = -\frac{1}{4}(-1+\cos2\varphi_1)\sin2(\varphi-\varphi_1)+\frac{1}{2}\sin2\varphi_1\cos2(\varphi-\varphi_1).$$

For the third area, $\varphi_2 \le \varphi \le \pi$ have:

$$\sigma_{rr} = \frac{1}{3}(1+\cos2\varphi), \quad \sigma_{\varphi\varphi} = -\frac{1}{2}(1-\cos2\varphi), \quad \sigma_{r\varphi} = \frac{1}{2}\sin2\varphi.$$

For the simplicity we believe that the material displacements in zone of the crack opening are along the axis, $\mathbf{u} = (0,1)$. Take into account the expressions for the physical components of the stress tensor:

$$\sigma_{11} = \sigma_{rr}\cos^2\varphi + \sigma_{\varphi\varphi}\sin^2\varphi - \sigma_{r\varphi}\sin2\varphi,$$

$$\sigma_{22} = \sigma_{rr}\sin^2\varphi + \sigma_{\varphi\varphi}\cos^2\varphi + \sigma_{r\varphi}\sin 2\varphi,$$

$$\sigma_{12} = \frac{1}{2}(\sigma_{rr} - \sigma_{\varphi\varphi})\sin^2\varphi + \sigma_{r\varphi}\cos 2\varphi.$$

we have essentially different behavior of the Umov"=Pointing vector (energy flux vector) in different domains near the crack tip. For the firs domain we obtain $\vec{\mathcal{E}} = (\mathcal{E}_1, \mathcal{E}_2)$ with components:

$$\mathcal{E}_1 = \frac{1}{2\sqrt{3}}\left(\cos\varphi\sin 2\varphi + 2\sin\varphi\cos 2\varphi\right),$$

$$\mathcal{E}_2 = \frac{1}{\sqrt{3}}\left(-2\cos\varphi + \cos\varphi^3 - \sin\varphi\sin 2\varphi\right) \tag{5}$$

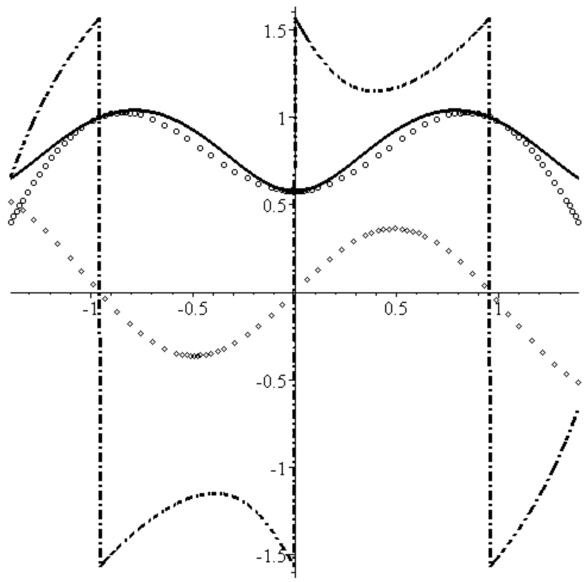

Figure 1: Energy flix versus angle in first domain. Solid line - magnitude of the flux vector, $\circ$ is $\mathcal{E}_1$ component, $\diamond$ is $\mathcal{E}_2$ component, dash-dotted line is the angle between energy flux vector and OX axis.

For the second domain we have energy flux vector in form :

$$\mathcal{E}_1 = \frac{1}{2}\left(\frac{1}{2}(1+\cos\varphi_1)\cos(2\varphi - \varphi_1) + \sin\varphi_1\sin(2\varphi - \varphi_1)\right)\sin 2\varphi +$$
$$+ \left(\frac{1}{4}(1+\cos\varphi_1)\sin(2\varphi - \varphi_1) + \frac{1}{2}\sin\varphi_1\cos(2\varphi - \varphi_1)\right)\cos 2\varphi$$

$$\mathcal{E}_2 = \left(-\frac{1}{4} + \frac{3}{4}\cos\varphi_1 + \frac{1}{4}(1+\cos\varphi_1)\cos(2\varphi-\varphi_1) + \frac{1}{2}\sin\varphi_1\sin(2\varphi-\varphi_1) \right)\sin^2\varphi$$

$$+ \left(-\frac{1}{4} + \frac{3}{4}\cos\varphi_1 - \frac{1}{4}(1+\cos\varphi_1)\cos(2\varphi-\varphi_1) - \frac{1}{2}\sin\varphi_1\sin(2\varphi-\varphi_1) \right)\cos^2\varphi +$$

$$+ \left(\frac{1}{4}(1+\cos\varphi_1)\sin(2\varphi-\varphi_1) + \frac{1}{2}\sin\varphi_1\cos(2\varphi-\varphi_1) \right)\sin 2\varphi \quad (6)$$

The sudden change the angle of energy flux vector slope α follow from the $\mathcal{E}_2$

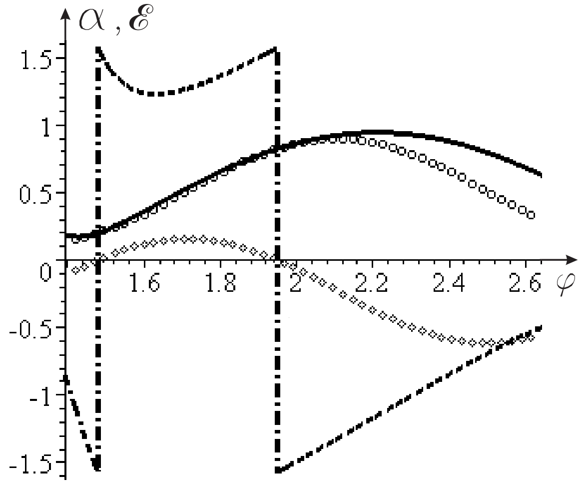

Figure 2: Energy flux in the second domain

component behavior. The energy flux vector try the discontinuity caused the change regime: energy flow into crack tip domain change to energy flow out of domain. Nevertheless the magnitude of the energy flux is the smooth function.

In the third domain we have constant physical components of the stress tensor $\sigma_{rr}, \sigma_{\varphi\varphi}, \sigma_{\varphi r} \neq const$. It means that energy flow is equal to zero.

Results and discussion

The obtained analytical results about Umov-Pointing vector behavior affirm that the energy is localized by the rupture propagation. The deformed body initially homogeneous internal energy distribution transferred to inhomogeneous by the rupture (crack) propagation. It means that the regions (channel) preferred dissemination exists. This fact is coincide with well known situation of energy localization and complex structure of energy flux near the rock excavation in elastic massif (13) or in layered media. Possibility of transition from accumulation to energy radiation can be found as a result of computer modeling for dynamical crack, too (4).

Existence of the energy channels in earth crust cause possibility to predict areas relative stability by the planned earthquake on the base of analysis possible local magnitudes by the initial homogeneous deformation of the seismic basin.

With account of possibility to regulate the rupture trajectory, a practical problem arises to take preventive measures to control the direction of energy emission of an earthquake through preliminary creation of the field of defects of a definite structure. Naturally, this idea requires detailed additional investigation and demand very huge amount of computation because the real structure of geomassifs are essentially inhomogeneous and real media displacement is more complicated as taken by our analytical investigations.

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