ICCES, vol.16, no.1, pp.11-12

Computational Differentiation Enabled Fourth-Order Algebraic Monte Carlo Simulations

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Summary

Modeling uncertainty for nonlinear systems is often handled by developing a mathematical model, defining suitable parameters, establishing suitable initial conditions and numerically integrating the system response in order to study the behavior of the system. The potential range of behaviors that can be realized is assessed by varying the model parameters, integrating the response, and recording the changes in the system behaviors. In theory this process is straightforward for implementing. The only potential barrier to carrying out the repeated integrations of the system dynamics is the availability of powerful computer resources that can provide the density of sample points required in a time-frame suitable for impacting the design process for systems requiring engineering-level-of-fidelity simulations. This approach is presented in Figure I. The goal is to accumulate the spread in the behavior of the state response. Two potential problems can impact one's ability to carry the study of the system behavior: (1) Very expensive numerical integration effort arising from problem dimension, nonlinearity, and/or numerical stiffness in the equations; and (2) The number of simulations required to produce useful results. This paper presents an alternative method for evaluating the system behavior. The basic idea is to expand the system response in terms of problems initial conditions and parameters as a tensor-based power series. The required partial derivatives are described by state and parameter transition tensors.

The system response is assumed to be described by (1.1) where and .

The proposed tensor-based power series is assumed to be given by (1.2)

The (.) operation denotes tensor contraction. The transition tensors are evaluated along the reference values for state initial conditions and parameter values. Unlike the classical approach of Figure I, Eq (1.2) requires a single integration, where the state and transition tensors are numerically integrated. Estimates for the state variation subject to changes in the initial conditions and parameters are handled by evaluating Eq. (1.2), which only requires algebraic tensor contraction operations. It is anticipated that many problems in science and engineering will greatly benefit from this approach when thousands to millions of numerical integrations are required for propagating the system response. Computational differentiation techniques are presented for evaluating the n-th order jacobians required for modeling the higher-order state and parameter transition tensors.

Several numerical examples are presented for demonstating the effectiveness of the approach for handling problems of engineering interest. Turner's Object-Oriented

Coordinate Embedding Algorithm (OCEA) is used for generating the required system partial derivatives.