Development of Intrinsic Formulation of W.-Z. Chien of the Geometrically Nonlinear Theory of Thin Elastic Shells

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Summary

Wei-zhang Chien (1944) derived the equilibrium equations and the compatibility conditions of the nonlinear theory of thin, isotropic elastic shells entirely in terms of the surface stress and strain measures associated with the shell base surface. This allowed Him to divide the complex boundary value problem (BVP) of nonlinear shell analysis into two disjoint and supposedly simpler steps: I) finding the surface stress and strain measures, and II) establishing displacements from already known surface strain measures. In the present paper some achievements of this formulation obtained during the last 66 years are reviewed, with special account of the results obtained by the author.

In the first part, using the error of the constitutive equations, we remind some consistent intrinsic BVPs proposed in the literature. These are, in particular: 1) the intrinsic BVP in terms of the surface strain measures, 2) the refined intrinsic BVP in terms of the surface stress resultants and the surface bendings, 3) the almost inextensional bending BVP, 4) the almost membrane BVP, and 5) the intrinsic bending BVP reduced to two PDE for the stress and deformation functions. The alternative set of the refined intrinsic shell equations formulated in the rotated surface base is also presented. All discussed BVPs contain the corresponding intrinsic natural and kinematic boundary conditions.

In the second part, recent achievements on determination of position of the deformed shell base surface from the surface strains and bendings are reviewed. Three methods of finding such a position are presented: a) by direct solving some vector ODE, b) through establishing the surface deformation gradient, and c) applying the right polar decomposition to the deformation gradient and then solving some ODE for the rotation tensor field.

Finally, we briefly discuss some problems related to the intrinsic formulation of thin shell theory. These are: i) determination of position of a surface from components of its two fundamental forms, ii) bifurcation buckling of the axially compressed circular cylinder, iii) determination of position of the deformed base surface from its surface strains and the height function, and iv) basic assumptions of the special class of flexible shells.

In conclusion, some open problems of the intrinsic nonlinear theory of shells are pointed out.