Hybrid and mixed variational principles for the fully nonlinear analysis of shells

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Summary

This work addresses the development of some alternative hybrid and mixed variational formulations for the geometrically-exact three-dimensional first-order-shear shell boundary value problem [1,2]. In the framework of the complementary-energy-based formulations, a Legendre transformation is used to introduce the complementary energy density in the variational statements as a function of the cross-sectional resultants only. The corresponding variational principles are shown to feature stationarity within the framework of the boundary-value-problem. The main features of the principles are highlighted, giving special attention to their relationships from both theoretical and numerical point of view.

Variational principles constitute the core of the development of numerical methods in Solid Mechanics. The utility of such principles is two-fold: first, they provide a very convenient method for the derivation of the governing equations and natural boundary conditions of the boundary value problem and, second, they provide the mathematical foundation required to produce consistent numerical approximations. In this second role, the variational methods have been most useful in Computational Solid Mechanics [3].

Many different variational principles can be constructed depending on the equations enforced in the weak form. Perhaps the most remarkable variational principle is the Principle of Stationary Total Potential Energy, which states that, among all kinematically admissible displacement fields, those that satisfy the equilibrium conditions in the domain and at the boundary, lead to a stationary value of the total potential energy functional.

In this paper, after the presentation of many mixed variational principles for the nonlinear analysis of shells, we derive a hybrid form of the Two-field Principle of Total Complementary Potential Energy, which is used on a totally consistent derivation of equilibrated co-rotational finite elements [4].

References

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