Darcy-Stokes Equations with Finite Difference and Natural Boundary Element Coupling Method

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Summary

Numerical method is applied to investigate the Darcy-Stokes equations, which is governing the steady incompressible Stokes flow past a circular cavity in a porous medium. The free fluid flow is modeled by the incompressible Stokes equations, and the flow in the porous medium is imposed by Darcy equations. Based on domain decomposition method with D-N alternating iteration algorithm, the coupling method of finite difference method and natural boundary element method is studied for the coupling Darcy-Stokes equations under a certain pressure difference.

Divide the whole domain into two non-intersecting bounded subdomains and . The boundary of cavity is the artificial boundary . The fan shaped grid is used to mesh in polar coordinates. Then, the finite difference method is applied to resolve the Darcy' law in domain . Thus, the pressure at arbitrary node can be attained. According to the relationship of velocity and pressure, the radial velocity and tangential velocity on the artificial boundary would be acquired. Afterwards, based on Newton-Cotes numerical integration formula, utilize the velocity value on artificial boundary and calculate in domain with natural boundary element method. Remarkably, the initial pressure value should be given before iterating, and the new value on the artificial boundary can be obtained with finite difference method in domain . Finally, the natural boundary element method is applied in domain and the new pressure value will be educed again. Iterate in turn, the results will be achieved with rational precision.

As to the steady-state parallel flow with a void space, the velocity is increasing with the permeability coefficient under the same pressure difference. The horizontal velocity is rapidly decreasing with the raise of the distance from the center, and increasing slowly to the undisturbed velocity in order to fill the continuity condition on the artificial boundary.

The velocity vector distribution can also be obtained by the finite difference and natural boundary element coupling method.

The numerical results indicate that the finite difference and natural boundary element coupling method is efficient and convenient for the Darcy-Stokes problem of the steady-state parallel flow with a void space.