The Modified Collocation Trefftz Method and Exponentially Convergent Scalar Homotopy Algorithm for the Inverse Boundary Optimization Problem

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Summary

The inverse boundary optimization problem, which is governed by Helmholtz equation, is analyzed by the modified collocation Trefftz method (MCTM) and the exponentially convergent scalar homotopy algorithm (ECSHA). In the inverse boundary optimization problem, the position for part of boundary with given boundary condition is unknown, and the position for the rest of boundary with additionally specified boundary conditions is given. Therefore, it is very difficult to handle the boundary optimization problem by any numerical scheme. In order to stably solve the boundary optimization problem, the MCTM, one kind of boundary-type meshless methods, will be adopted in this study, since it can avoid the generation of mesh grid and numerical integration. In the boundary optimization problem governed by the Helmholtz equation, the numerical solution of MCTM is expressed as linear combination of the T-complete functions. When this problem is considered by MCTM, a system of nonlinear algebraic equations will be formed and solved by ECSHA which will converge exponentially. The evolutionary process of EC-SHA can find the unknown coefficients in MCTM and the position of the unknown boundary. Some numerical examples will be provided to demonstrate the ability and accuracy of the proposed scheme. Besides, the stability of the proposed meshless method will be proven by adding some noise into the boundary conditions.